Conference on Boundary Value Problems

Mathematical Models in Engineering, Biology and Medicine

Santiago de Compostela, Spain
September 16-19, 2008

BVP 2008

PROGRAMME AND BOOK OF ABSTRACTS

SPONSORS

Mathematical Models in Engineering, Biology and Medicine.
Conference on Boundary Value Problems.
September 16-19, 2008, Santiago de Compostela, Spain.

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- Juan J. Nieto (USC, Spain), **Chairman**.
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- Gerardo Rodríguez-López (USC, Spain).
- Rosana Rodríguez-López (USC, Spain).

GENERAL INFORMATION

The topics of the meeting cover theory of differential equations in broad sense, with special attention to nonlinear and singular phenomena arising in the Mathematical models that appear in Engineering, Biology and Medicine.

The conference will be held at the Faculty of Mathematics of the University of Santiago de Compostela under the auspices of the International Federation of Nonlinear Analysis. The official language is English.

ACCESS TO INTERNET: The Faculty is equipped with a WiFi system which provides access to Internet inside the building. The participants of BVP2008 can benefit from these facilities and access to Internet using their own laptops by selection BVP2008 among the available networks. After you have selected the appropriate network, you must open the browser and write in the identification page the following information:

LOGIN: userbvp
PASSWORD: n1KkniFv

The participants can also use the equipment of the Computer Rooms, using the following information:

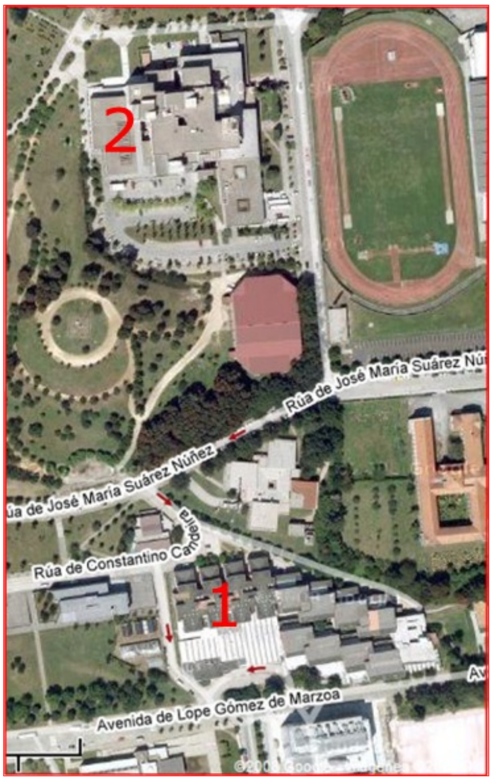
SISTEMA OPERATIVO: WINDOWS
LOGIN: nonpersoal.7
PASSWORD: licenza
DOMINIO: RAI

LUNCH: Lunch is served at the Restaurant located at the "Monte da Condesa" building, which is about 5 minutes walking from the Faculty. (See the map, location 2). Please do not forget the tickets provided with the documentation. Additional tickets for accompanying persons can be purchased at the Registration Desk.

EXCURSION: On the afternoon of Wednesday 17 we are planning a guided tour by the University's Heritage. For details, see the additional information inside your bags.

BANQUET DINNER: The conference dinner will be on Thursday 18 at the Hotel Monumento San Francisco (see the map, location 4) which is near the Cathedral (location 5). The banquet is scheduled to be served at 21:00 hours. Special dietary needs, such as vegetarian, will be met upon request.

PROCEEDINGS: The proceedings of the Conference will be published by the American Institute of Physics. All contributions will be refereed, and should be submitted before October 30, 2008. They must be sent in a pdf file to the e-mail address bvp2008@gmail.com with the subject Proceedings contribution. The length of the paper cannot exceed ten pages and it must follow the AIP instructions given on the web page http://proceedings.aip.org/proceedings/6x9.jsp



1. Faculty of Mathematics
2. Lunch (Monte da Condesa)
3. Meeting Point (Pazo de Fonseca)
4. Banquet Dinner (Hotel Monumento San Francisco)
5. Cathedral of Santiago de Compostela

Short Programme

TUESDAY 16 SEPTEMBER

	Aula Magna
09:30-10:00	OPENING

	Aula Magna
	S1 *(Ch.: A. Cabada)*
10:00-11:00	J. MAWHIN

COFFEE BREAK

	Aula Magna	**Salón Graos**
	S1 *(Ch.: V. Mañosa)*	S2 *(Ch.: P. Torres)*
11:30-12:00	S. Ibáñez	I. Rachůnková
12:00-12:30	J. Campos	S. Staněk
12:30-13:00	A. Gasull	M. Tvrdý

LUNCH (MONTE DA CONDESA)

	Aula 10	**Aula 9**	**Aula 8**
	S1 *(Ch.: M. Tvrdý)*	S2 *(Ch.: F. Zanolin)*	S3 *(Ch.: T. Faria)*
15:00-15:20	T. Jankowski	L. Recke	L. Berezansky
15:20-15:40	R. Hakl	N. Vieira	E. Braverman
15:40-16:00	P. Nečesal	E. Schiavi	J.-P. Lessard
16:00-16:20	G. Holubovà	B. Coll	G. Röst
16:20-16:40	Z. Opluštil	M. E. Filippakis	C. Núñez

COFFEE BREAK

	Aula 10	**Aula 9**	**Aula 8**
	S1 *(Ch.: A. Gasull)*	S2 *(Ch.: S. Staněk)*	S3 *(Ch.: R. Rodríguez)*
17:00-17:20	J. L. Bravo	Y. Nakata	M. Afshar
17:20-17:40	A. Buică	M. M. Rodrigues	E. Ahmady
17:40-18:00	M. Caubergh	V. Taddei	N. Ahmady
18:00-18:20	A. Makhlouf	A. Matas	T. Allahviranloo

WEDNESDAY 17 SEPTEMBER

	Aula Magna	**Salón Graos**
	S1 *(Ch.: E. Liz)*	S2 *(Ch.: D. Franco)*
9:00-10:00	P. DRÁBEK	
10:00-10:30	I. Győri	J. R. L. Webb
10:30-11:00	T. Faria	F. Minhós

COFFEE BREAK

	Aula Magna	**Salón Graos**
	S1 *(Ch.: A. Cabada)*	S2 *(Ch.: D. Franco)*
11:30-12:00	V. Jiménez	C. T. Cremins
12:00-12:30	V. Mañosa	M. Zima
12:30-13:00	S. Stević	G. Infante

LUNCH (MONTE DA CONDESA)

	Aula 10	**Aula 9**	**Aula 8**
	S1 *(Ch.: S. Ibáñez)*	S2 *(Ch.: J. J. Nieto)*	S3 *(Ch.: G. Infante)*
15:00-15:20	C. Alonso	L. F. Seoane	C. Trenado
15:20-15:40	X. Zhang	M. El-Shahed	J. C. Graña
15:40-16:00	S. M. Aleixo	E. Estévez	W. Greenberg
16:00-16:20	J. J. Oliveira	J. Otta	P. Lima

VISIT TO THE CITY

THURSDAY 18 SEPTEMBER

	Aula Magna	**Salón Graos**
	S1 *(Ch.: J. J. Nieto)*	S2 *(Ch.: R. L. Pouso)*
9:00-10:00	R. P. AGARWAL	
10:00-10:30	J. Andres	M. Frigon
10:30-11:00	L. Sanchez	R. Precup

COFFEE BREAK

	Aula Magna	**Salón Graos**
	S1 *(Ch.: I. Rachůnková)*	S2 *(Ch.: I. Győri)*
11:30-12:00	P. J. Torres	F. Hartung
12:00-12:30	J. Perán	A. Domoshnitsky
12:30-13:00	J. Tomeček	M. Pituk

LUNCH (MONTE DA CONDESA)

	Aula 10	**Aula 9**	**Aula 8**
	S1 *(Ch.: R. L. Pouso)*	S2 *(Ch.: A. Domoshnitsky)*	S3 *(Ch.: M. Pituk)*
15:00-15:20	A. I. Santos	Y. Enatsu	V. Tkachenko
15:20-15:40	R. Figueroa	R. Cibulka	Y. V. Rogovchenko
15:40-16:00	A. Gómez	Z. Varga	A. Boichuk
16:00-16:20	P. Stehlik	I. López	M. Langerova
16:20-16:40	N. Dilna	J. Chrobak	P. Vodstrčil

COFFEE BREAK

	Aula 10	**Aula 9**	**Aula 8**
	S1 *(Ch.: J. Campos)*	S2 *(Ch.: F. Sadyrbaev)*	S3 *(Ch.: G. Röst)*
17:00-17:20	L. Ferracuti	D. V. Georgievskii	F. Beiginejad
17:20-17:40	F. Papalini	K. N. Soltanov	C. Eslahchi
17:40-18:00	A. Rontó	YA. Shahmurova	M. Habibi
18:00-18:20	A. Calamai	V. Shakhmurov	H. Poormohammadi

21:00	CONFERENCE DINNER

FRIDAY 19 SEPTEMBER

	Aula Magna	**Salón Graos**
	S1 *(Ch.: V. Otero)*	S2 *(Ch.: F. Minhós)*
9:00-10:00	J. LLIBRE	
10:00-10:30	P. Omari	M. Benchohra
10:30-11:00	F. Zanolin	M. R. Grossinho

COFFEE BREAK

	Aula Magna	**Salón Graos**
	S1 *(Ch.: L. Sanchez)*	S2 *(Ch.: J. A. Cid)*
11:30-12:00	F. Sadyrbaev	J. M. Almira
12:00-12:30	S. Tersian	J. Diblík
12:30-13:00	R. Enguiça	P. Jebelean

LUNCH (MONTE DA CONDESA)

	Aula 10	**Aula 9**
	S1 *(Ch.: J. B. Ferreiro)*	S2 *(Ch.: I. Bajo)*
15:00-15:20	J. Bouchala	R. Rodríguez
15:20-15:40	Z. El Allali	L. Jahanshahloo
15:40-16:00	T. Gerasimov	N. A. Kiani
16:00-16:20	E. Moulay Ely	

Extended Programme

TUESDAY 16 SEPTEMBER:

09:30-10:00		OPENING
10:00-11:00	**S1**	Plenary Talk: JEAN MAWHIN *Bounded solutions: differential vs difference equations*
11:00-11:30		COFFEE BREAK
11:30-12:00	**S1**	Invited Talk: SANTIAGO IBÁÑEZ *Local organizing centers in diffusively coupled dynamical systems*
	S2	Invited Talk: IRENA RACHŮNKOVÁ *Bubble-type solutions of nonlinear singular problems*
12:00-12:30	**S1**	Invited Talk: JUAN CAMPOS *Fastness and continuous dependence in front propagation in Fisher-KPP equations*
	S2	Invited Talk: SVATOSLAV STANĚK *Boundary value problems depending on parameters*
12:30-13:00	**S1**	Invited Talk: ARMENGOL GASULL *Limit cycles for some one dimensional non-autonomous differential equations*
	S2	Invited Talk: MILAN TVRDÝ *Multiplicity results for singular periodic problems*
13:00-15:00	**MC**	LUNCH

15:00-15:20	**S1**	Tadeusz Jankowski *Boundary value problems for second order differential equations*
	S2	Lutz Recke *Periodic-Dirichlet Problems for dissipative hyperbolic systems and laser modeling*
	S3	Leonid Berezansky *On exponential stability for a linear delay differential equation*
15:20-15:40	**S1**	Robert Hakl *Periodic boundary value problem for third order functional differential equations*
	S2	Nelson Vieira *Parametrices of regular hypoelliptic boundary-value problems associated to the linear Schrödinger equation*
	S3	Elena Braverman *On oscillation and stability of equations with a distributed delay*
15:40-16:00	**S1**	Petr Nečesal *On the variety of the Fučík spectrum structure*
	S2	E. Schiavi *Multiphase systems for medical image region classification*
	S3	Jean-Philippe Lessard *A general fixed point method for the study of dynamical systems*
16:00-16:20	**S1**	Gabriela Holubová *Solvability of four-point boundary value problem with jumping nonlinearities*
	S2	Tomeu Coll *A PDE model applied to conditional image filtering*
	S3	Gergely Röst *On the global dynamics of the Nicholson blowflies and the Mackey-Glass equations*
16:20-16:40	**S1**	Zdeněk Opluštil *Nonpositive solutions of a certain functional differential inequality*
	S2	Michael E. Filippakis *Nodal and multiple constant sign solutions for equations concerning p-Laplacian*
	S3	Carmen Núñez *Exponential stability for a nonautonomous stage-structured population growth model*

16:40-17:00		COFFEE BREAK
17:00-17:20	**S1**	<u>José Luis Bravo</u> *Existence of limit cycles for rigid planar vector fields*
	S2	<u>Yukihiko Nakata</u> *Contractivity and global asymptotic stability for nonautonomous Lotka-Volterra systems with piecewise constant arguments*
	S3	<u>M. Afshar Kermani</u> *New method for solving fuzzy partial differential equations*
17:20-17:40	**S1**	<u>Adriana Buică</u> *Limit cycles of a quadratic polynomial planar system: the third order Melnikov function*
	S2	<u>M. M. Rodrigues</u> *Exact and approximate solutions of reaction-diffusion-convection equations*
	S3	<u>Elham Ahmady</u> *A new method for solving fuzzy linear differential equations*
17:40-18:00	**S1**	<u>Magdalena Caubergh</u> *Hilbert's Sixteenth Problem for Liénard equations*
	S2	<u>Valentina Taddei</u> *Multivalued boundary valued problems in Banach spaces*
	S3	<u>Nazanin Ahmady</u> *Numerical methods for hybrid fuzzy differential equations*
18:00-18:20	**S1**	<u>Amar Makhlouf</u> *Bifurcation of limit cycles from a 2-dimensional center in control systems*
	S2	<u>Aleš Matas</u> *Weak solutions of doubly degenerate diffusion equations*
	S3	<u>T. Allahviranloo</u> *Improved predictor corrector method for solving fuzzy initial value problem*

WEDNESDAY 17 SEPTEMBER:

09:00-10:00	**S1**	Plenary Talk: PAVEL DRÁBEK *Quasilinear model for phase transitions in one space dimension*
10:00-10:30	**S1**	Invited Talk: ISTVÁN GYŐRI *On delay differential equations related to biological compartmental systems*
	S2	Invited Talk: JEFF WEBB *Positive solutions of a nonlocal boundary value problem of conjugate type*
10:30-11:00	**S1**	Invited Talk: TERESA FARIA *Boundedness and stability for delayed n-dimensional models in biology*
	S2	Invited Talk: FELIZ MINHÓS *Lower and upper solutions: an appropriate method for BVP in Medicine and Engineering*
11:00-11:30		COFFEE BREAK
11:30-12:00	**S1**	Invited Talk: VÍCTOR JIMÉNEZ LÓPEZ *The Y2K problem of difference equations revisited*
	S2	Invited Talk: CASEY T. CREMINS *An eigenvalue theorem for semilinear equations*
12:00-12:30	**S1**	Invited Talk: VÍCTOR MAÑOSA *Global dynamics of discrete systems through Lie Symmetries*
	S2	Invited Talk: MIROSŁAWA ZIMA *Applications of coincidence equations to boundary value problems*
12:30-13:00	**S1**	Invited Talk: STEVO STEVIĆ *On behavior of a class of difference equations with maximum*
	S2	Invited Talk: GENNARO INFANTE *Nonlocal impulsive boundary value problems with solutions that change sign*
13:00-15:00	**MC**	LUNCH

15:00-15:20	**S1**	Clementa Alonso-González *Blow-up topologically equivalent singularities*
	S2	Luís F. Seoane *Modelling the evolution in time of two languages in competition*
	S3	Carlos Trenado *Multiscale model of auditory selective attention and habituation neural correlates*
15:20-15:40	**S1**	Xiang Zhang *Analytic normalization of analytic integrable systems*
	S2	Moustafa El-Shahed *Positive solutions for boundary value problems of nonlinear fractional differential equations*
	S3	José C. Graña *Numerical integration of nonlinear boundary value problems arising in nucleation theory*
15:40-16:00	**S1**	Sandra M. Aleixo *Population growth model proportional to beta densities*
	S2	Emilio Estévez *Dirichlet problems in graphs*
	S3	William Greenberg *Boundary value problems for abstract equilibrium transport equations*
16:00-16:20	**S1**	José J. Oliveira *Global stability for delayed neural network models*
	S2	Josef Otta *Boundary value problem for stationary case of generalized bi-stable equation*
	S3	Pedro Lima *Numerical methods for singular boundary value problems involving the one-dimensional p-laplacian*
17:00		VISIT TO THE CITY

THURSDAY 18 SEPTEMBER:

09:00-10:00	**S1**	Plenary Talk: RAVI P. AGARWAL *Singular boundary value problems with real world applications*
10:00-10:30	**S1**	Invited Talk: JAN ANDRES *Bound sets approach to boundary value problems*
	S2	Invited Talk: MARLÈNE FRIGON *The method of solution-tube applied to first, second and third order systems of differential equations*
10:30-11:00	**S1**	Invited Talk: LUIS SANCHEZ *On the existence of heteroclinic trajectories for some non-autonomous equations*
	S2	Invited Talk: RADU PRECUP *Componentwise compression-expansion conditions for systems of nonlinear operator equations and applications*
11:00-11:30		COFFEE BREAK
11:30-12:00	**S1**	Invited Talk: PEDRO J. TORRES *Existence and stability of periodic solutions of the relativistic oscillator*
	S2	Invited Talk: FERENC HARTUNG *On linearized stability of neutral differential equations with state-dependent delays*
12:00-12:30	**S1**	Invited Talk: JUAN PERÁN φ*-Laplacian Functional Equations*
	S2	Invited Talk: ALEXANDER DOMOSHNITSKY *Maximum principles and nonoscillation for first order functional differential equations*
12:30-13:00	**S1**	Invited Talk: JAN TOMEČEK *On nonlinear second order boundary value problems arising in hydrodynamics*
	S2	Invited Talk: MIHÁLY PITUK *Difference equations of Poincaré type*
13:00-15:00	**MC**	LUNCH

15:00-15:20	**S1**	Ana Isabel Santos *Solvability of an elastic beam fully nonlinear equation in presence of a sign-type Nagumo control*
	S2	Yoichi Enatsu *Permanence for n-dimensional nonautonomous Lotka-Volterra cooperative systems with loop structure*
	S3	Viktor Tkachenko *Global stability of difference equations satisfying Yorke condition*
15:20-15:40	**S1**	Rubén Figueroa Sestelo *Discontinuous first-order functional boundary value problems*
	S2	Radek Cibulka *Local controllability of nonlinear dynamical systems*
	S3	Yuri V. Rogovchenko *Seasonal fluctuations and harvesting in the Pearl-Verhulst population model*
15:40-16:00	**S1**	Ana Gómez González *Multiple positive solutions in the sense of distributions of singular BVPs on time scales*
	S2	Zoltán Varga *Control and observation in population models*
	S3	Alexander Boichuk *Dichotomous and normally resolvable operator in Banach space*
16:00-16:20	**S3**	Petr Stehlik *Basic properties of partial dynamic operators*
	S2	Inmaculada López *Observation and control in a model of a cell population affected by radiation*
	S3	Martina Langerova *Bifurcation conditions for perturbed Fredholm impulsive boundary value problems*
16:20-16:40	**S1**	Nataliya Dilna *The stability of a unique symmetric and periodic solution of the ordinary differential equation*
	S2	Joanna Chrobak *A mathematical model of cancer as a competition*
	S3	Petr Vodstrčil *On solvability of a three-point boundary value problem for second order nonlinear functional differential equations*

16:40-17:00		COFFEE BREAK
17:00-17:20	**S1**	Laura Ferracuti *Boundary value problems for strongly nonlinear multivalued equations involving different Φ-Laplacians*
	S2	Dimitri V. Georgievskii *Asymptotic analysis in the Prandtl′ problem*
	S3	F. Beiginejad *Protein complex and functional module prediction using PCA method*
17:20-17:40	**S1**	Francesca Papalini *Strongly nonlinear multivalued systems involving a singular Φ-Laplacian*
	S2	Kamal N. Soltanov *On a nonlinear parabolic equation with implicit degeneration not in divergence form*
	S3	Changiz Eslahchi *Secondary structure assignment using entropy*
17:40-18:00	**S1**	András Rontó *On linear singular functional differential equations*
	S2	Aida Shahmurova *Nonlinear abstract boundary value problems modelling atmospheric dispersion of pollutants*
	S3	Mahnaz Habibi *The analysis of protein structures based on graph theory*
18:00-18:20	**S1**	Alessandro Calamai *A general approach for front-propagation in functional reaction-diffusion equations*
	S2	Veli Shakhmurov *Maximal regular elliptic-convolution equations*
	S3	Hadi Poormohammadi *A heuristic algorithm for haplotype inference by pure parsimony*
21:00		CONFERENCE DINNER

FRIDAY 19 SEPTEMBER:

09:00-10:00	**S1**	Plenary Talk: JAUME LLIBRE *On the limit cycles and the integrability of the Liénard polynomial differential equations*
10:00-10:30	**S1**	Invited Talk: PIERPAOLO OMARI *Positive solutions of a prescribed mean curvature problem*
	S2	Invited Talk: MOUFFAK BENCHOHRA *Recent results for nonlinear differential inclusions with fractional order*
10:30-11:00	**S1**	Invited Talk: FABIO ZANOLIN *Complex dynamics in a Lotka-Volterra predator-prey model*
	S2	Invited Talk: MARIA DO ROSÁRIO GROSSINHO *Solvability of a stationary nonlinear Black-Scholes equations under conditions on the potential*
11:00-11:30		COFFEE BREAK
11:30-12:00	**S1**	Invited Talk: FELIX SADYRBAEV *Two-parameter nonlinear eigenvalue problems*
	S2	Invited Talk: JOSÉ MARÍA ALMIRA *Norbert Wiener and the Engineers: Some gems of Harmonic Analysis and its applications*
12:00-12:30	**S1**	Invited Talk: STEPAN TERSIAN *Existence of solutions for fourth-order ordinary differential equations in water wave models*
	S2	Invited Talk: JOSEF DIBLÍK *Positive solutions of delayed differential and discrete equations*
12:30-13:00	**S1**	Invited Talk: RICARDO ENGUIÇA *Second order non-autonomous homoclinics*
	S2	Invited Talk: PETRU JEBELEAN *Radial solutions for systems involving mean curvature operators in Euclidian and Minkovski spaces*
13:00-15:00	**MC**	LUNCH

15:00-15:20	**S1**	Jiří Bouchala *Boundary weak solution of the semicoercive 2D contact problem*
	S2	Rosana Rodríguez-López *Periodicity of solutions for fuzzy differential equations*
15:20-15:40	**S1**	Zakaria El Allali *Existence of solutions for some quasilinear elliptic systems*
	S2	L. Jahanshahloo *Fuzzy optimization model for land use change*
15:40-16:00	**S1**	Tymofiy Gerasimov *On the clamped grid problem in polygonal domains*
	S2	N. A. Kiani *Toward fuzzy differential inclusions*
16:00-16:20	**S1**	E. Moulay Ely *The behavior of a population under the effects of the propagation of an epidemie with fixed infection length*

Plenary Talks

Mathematical Models in Engineering, Biology and Medicine.
Conference on Boundary Value Problems.
September 16-19, 2008, Santiago de Compostela, Spain.

Singular boundary value problems with real world applications

RAVI P. AGARWAL

Keywords:
MSC2000 Classification:

Abstract

We shall provide easily verifiable sufficient conditions which guarantee the existence of solutions to some singular boundary value problems over finite and infinite intervals. The motivation of these problems comes from real world applications. The importance of our theory is that it provides lower and/or upper solutions to each problem we shall discuss. These lower and upper solutions have been used very effectively to construct approximate solutions.

Ravi P. Agarwal
Florida Institute of Technology, USA

Mathematical Models in Engineering, Biology and Medicine.
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Quasilinear model for phase transitions in one space dimension

PAVEL DRÁBEK, RAUL MANÁSEVICH AND PETER TAKÁČ

Keywords: *generalized Cahn-Hilliard and bi-stable equations, p-Laplacian, nonunique continuation for the spatial problem, phase plane analysis, first integral, uniqueness for the gradient flow, slow dynamics*
MSC2000 Classification: 35J20, 35B45, 35P30, 46E35.

Abstract

In this lecture we show striking differences in pattern formation produced by the Cahn-Hilliard model with the p-Laplacian and a $C^{1,\mu}$ $(0 < \mu \leq 1)$ potential W in place of the regular (linear) Laplace operator and a C^2 potential. The corresponding energy functional

$$\mathcal{J}_\varepsilon(u) = \int_0^1 \left(\frac{\varepsilon^p}{p} \, |u_x|^p + W(u) \right) \mathrm{d}x, \quad u \in W^{1,p}(0,1), \tag{1}$$

with $p \neq 2$ exhibits multi-dimensional continua ("polyhedra") of critical points as opposed to the classical case with the Laplace operator $(p = 2)$ and the C^2 potential W. Each of these continua is a finite--dimensional, compact $C^{1,1}$ manifold with boundary. Some of the critical points are local minimizers of the energy functional in the topology of the Sobolev space $W^{1,p}(0,1)$, whereas others are only saddle points. The former are interior points of the corresponding continuum (viewed as a compact manifold with boundary), while the latter are boundary points. These facts offer an explanation of the "slow dynamics" on the attractor for the dynamical system generated by the corresponding time-dependent parabolic problem for bi–stable equation

$$u_t = \varepsilon^p \left(|u_x|^{p-2} u_x \right)_x - W'(u) \quad \text{for } 0 < x < 1 \text{ and } t > 0, \tag{2}$$

subject to the boundary conditions

$$u_x = 0 \quad \text{at } x = 0,\, 1, \quad \text{for } t > 0, \tag{3}$$

see [2]. This explanation is different from the one based on the "classical" semilinear model presented e.g. in [1] and [3].

References

[1] J. Carr and R. L. Pego, Metastable patterns in solutions of $u_t = \varepsilon^2 u_{xx} - f(u)$, *Comm. Pure Appl. Math.* **42**, (1989), 523–576.

[2] P. Drábek, R. Manásevich and P. Takáč, Slow Dynamics in a Quasilinear Model for Phase Transitions in One Space Dimension, *to appear*.

[3] G. Fusco and J. K. Hale, Slow-Motion Manifolds, Dormant Instability, and Singular Perturbations, *J. Dynamics Diff. Eqs.* **1**, (1989), 1, 75–94.

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Conference on Boundary Value Problems.
September 16-19, 2008, Santiago de Compostela, Spain.

On the limit cycles and the integrability of the Liénard polynomial differential equations

JAUME LLIBRE

Keywords: *Limit cycles, algebraic limit cycles, integrability, polynomial Liénard systems.*
MSC2000 Classification: Primary 34C40, 51F14; Secondary: 14D05, 14D25.

Abstract

One of the more studied differential systems are the so–called *generalized Liénard equation*

$$\ddot{x} + f(x)\dot{x} + g(x) = 0, \tag{1}$$

which were studied by many researchers. Such dynamical systems appear very often in several branches of the sciences, such as biology, chemistry, mechanics, electronics, etc. The differential equation (1) of second order can be written as the equivalent 2–dimensional Liénard differential system of first order

$$\dot{x} = y, \quad \dot{x} = -g(x) - f(x)y. \tag{2}$$

When $g(x) = x$ the Liénard differential systems (1) are called the *classical Liénard systems*.

The main objective of this talk is to present some old and new results on the limit cycles of the Liénard systems (1) algebraic or not, and on their polynomial and analytical first integrals depending on the functions f and g. These results are based in the papers [1–8].

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Bounded solutions : differential vs difference equations

JEAN MAWHIN

Keywords: *Bounded solutions, lower and upper solutions, guiding functions.*
MSC2000 Classification: 34C11, 39A12.

Abstract

We compare some existence results for solutions bounded on the whole real axis for differential systems

$$x' = f(t,x)$$

or for second order differential equations

$$u'' + cu' = f(t,u)$$

to the corresponding ones for solution bounded over $\mathbb{Z}$ for systems of difference equations

$$\Delta x_n = f_n(x_n) \quad (n \in \mathbb{Z})$$

or for second order difference equations

$$\Delta^2 u_{n-1} + c\Delta u_n = f_n(u_n) \quad (n \in \mathbb{Z}).$$

In particular, we compare the results based on guiding functions in the case of first order systems, and Landesman-Lazer-type conditions in the case of second order equations.

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Invited Talks

Mathematical Models in Engineering, Biology and Medicine.
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Norbert Wiener and the Engineers: Some gems of Harmonic Analysis and its applications

JOSÉ MARÍA ALMIRA

Keywords: *N. Wiener's work in harmonic analysis, filters, prediction theory, cybernetics.*

Abstract

Norbert Wiener (1894-1964) is well known between mathematicians because of his fundamental contributions in so different areas such as stochastic processes (where he presented a nice formalization of Brownian motion), Fourier Analysis, Potential Theory, Number Theory, Information Theory, Mathematical Logic, etc. He also is well known for the electrical and electronic engineering community because he created the basic tools for communications theory, including a concept for Entropy and the basic (and not so basic) tools for the study of filters and prediction theory. Moreover, he also is considered the creator of Cybernetics. This last work put him in touch with a great variety of science people, working in so diverse areas such as medicine, biology, electrical engineering, etc.
In this talk we will present a survey of Wiener's work related with Harmonic Analysis and its applications. This will include some gems we will try to explain with detail. In conclusion, we will see how Harmonic Analysis is not only a beautiful mathematical discipline but also has deep connections with Nature and human's activities.

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Bound sets approach to boundary value problems

JAN ANDRES

Keywords: *Bound sets, bounding functions, differential equations and inclusions.*
MSC2000 Classification: 34A16, 34B15, 47H04.

Abstract

Continuation principles for the solvability of boundary value problems, depending on degree arguments, require the fixed point free boundaries of given sets of candidate solutions (see e.g. [6]). One of the techniques to satisfy the related transversality conditions is relied on the construction of (Liapunov-like) bounding functions, whence the title.

In our talk, which is based on the recent papers [1]–[5], we will rather demonstrate the essential ideas of this technique than concrete results in [1]–[5]. We will also point out main difficulties and indicate how to overcome them.

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Recent results for nonlinear differential inclusions with fractional order

MOUFFAK BENCHOHRA AND SAMIRA HAMANI

Keywords: *Differential inclusions, fractional integral, fractional derivative, existence, fixed point.*
MSC2000 Classification: 26A33.

Abstract

Differential equations of fractional order have recently proved to be valuable tools in the modeling of many phenomena in various fields of science and engineering. Indeed, we can find numerous applications in viscoelasticity, electrochemistry, control, porous media, electromagnetic, etc. (see [8,9]). Very recently Benchohra *et al* (see [1-7]) have considered various classes of initial and boundary value problems for differential equations and inclusions involving Riemann-Liouville and Caputo fractional derivatives. We present recent results for initial and boundary value problems for nonlinear functional and neutral functional differential inclusions with fractional order. The topological structure of the set of solutions of the above cited equations and inclusions is considered. Our results will be obtained by means of fixed point argument.

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Fastness and continuous dependence in front propagation in Fisher-KPP equations

JUAN CAMPOS

Keywords: *reaction-diffusion equations, travelling wave solutions, wave speed, fast heteroclinic, continuous dependence, sharp solutions, degenerate diffusivity.*
MSC2000 Classification: 35K57, 34B40

Abstract

We investigate the continuous dependence of the minimal speed of propagation and the profile of the corresponding travelling wave solution of Fisher-type reaction-diffusion equations

$$\vartheta_t = (D(\vartheta)\vartheta_x)_x + f(\vartheta)$$

with respect to both the reaction term f and the diffusivity D. Where f is a so-called Fisher-KPP reaction term, i.e. a Lipschitz function $f : [0,1] \to \mathbb{R}$ satisfying $f(0) = f(1) = 0$ and $f(s) > 0$ for $s \in (0,1)$, and the diffusion term $D(s)$ is a C^1-function on $[0,1]$ with $D(s) > 0$ for $s \in]0,1]$. We refer to the monographs [2], [3] and [4] and to the references there included.
We also introduce and discuss the concept of fast heteroclinic as in [1] to this context, which allows to interpret the appearance of sharp profile in the case of degenerate diffusivity ($D(0) = 0$).

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An eigenvalue theorem for semilinear equations

CASEY T. CREMINS

Keywords: *Semilinear equations, eigenvalues, cones.*
MSC2000 Classification: 34B10, 34B18.

Abstract

A norm-type expansion of a cone result is used to establish an existence theorem concerning eigenvalues for semilinear equations of the form $Lx = \lambda Nx$. Our results use the theory of A-proper Fredholm operators of index zero.

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Positive solutions of delayed differential and discrete equations

JOSEF DIBLÍK, MÁRIA KÚDELČÍKOVÁ AND ZDENĚK SVOBODA

Keywords: *Delayed equation, functional equation, positive solution.*
MSC2000 Classification: 34K15, 34K25, 39A10, 39A11.

Abstract

The phenomenon of existence of a positive solution of differential or difference equations often arises when we analyse mathematical models describing various processes. We discuss the existence of a positive solution of delayed differential equations of the form

$$\dot{y}(t) = -f(t, y_t)$$

and

$$\dot{y}(t) = -f(t, y_t, \dot{y}_t)$$

for $t \to +\infty$ and delayed discrete equations of the form

$$\Delta u(n) = -f(n, u(n), u(n-1), \ldots, u(n-k))$$

for $n \to +\infty$. Some linear consequences are considered as well. Importance of results consist also in the fact that appearance of positive solutions is caused by delay (which is with respect to the independent variable quite usual, e.g., in biological models) involved in the considered equations. If the delay is missing, formulated results lose any sense.

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Maximum principles and nonoscillation for first order functional differential equations

ALEXANDER DOMOSHNITSKY

Keywords: *Maximum principles, differential inequalities, nonoscillation, exponential stability, Green's functions, general boundary conditions.*
MSC2000 Classification: 34K15.

Abstract

We consider functional differential equation of the form

$$x'(t) - (Ax)(t) + (Bx)(t) = f(t),\ t \in [0, \infty),$$

where A, B are linear positive bounded operators acting from the space of continuous functions to the space of essentially bounded functions.
Many classical questions in the theory of functional differential equations such as nonoscillation, differential inequalities and stability were studied without connection between them. As a result, although assertions about the maximum principles for such equations can be interpreted in a corresponding case as analogs of corresponding classical concepts in the theory of ordinary differential equations, they do not imply important corollaries, reached on the basis of this connection between different notions. For example, results associated with the maximum principles in contrast with the cases of ordinary and partial differential equations do not add so much in problems of existence, uniqueness and comparison of solutions to boundary value problems. One of the goals of this talk is to propose a concept of the maximum principles for functional differential equations through description of the connection between nonoscillation, positivity of Green's functions for these equations. New results in every of these topics are also proposed. Analogs of Bohl-Perron theorem allow us to build on this basis an approach to studying exponential stability.

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Second Order Non-Autonomous Homoclinics

RICARDO ENGUIÇA AND LUÍS SANCHEZ

Keywords: *second order, homoclinic, non-autonomous.*
MSC2000 Classification: 34B40, 34C37.

Abstract

We study the existence of positive solutions for the differential equation

$$u''(x) = a(x)u(x) - u(x)^3,$$

with $u(\pm\infty) = 0$, where a is a positive monotone continuous function. The main objective is to generalize some of the well known results for the autonomous case where a is constant.

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Boundedness and stability for delayed n-dimensional models in biology

TERESA FARIA AND JOSÉ OLIVEIRA

Keywords: *delay, delayed population model, global asymptotic stability, global exponential stability.*
MSC2000 Classification: 34K20, 34K25, 34K60.

Abstract

We consider multiple species Lotka-Volterra type models of the form

$$x_i'(t) = r_i(t)x_i(t)\left[1 - b_i x_i(t) - \sum_{j=1}^{n} l_{ij}\int_{-\tau}^{0} x_j(t+\theta)\,d\eta_{ij}(\theta)\right], \quad i = 1,\dots,n, \tag{1}$$

where $b_i, l_{ij} \in \mathbb{R}, \tau > 0$, $r_i(t)$ are positive continuous functions and $\eta_{ij} : [-\tau, 0] \to \mathbb{R}$ are normalized bounded variation functions, for which we assume the existence of a positive equilibrium x^*. Conditions for the boundedness of all positive solutions of (1) are given, by imposing the existence of instantaneous negative feedbacks, which dominate the delayed competition effect. These conditions can also be used to study the global exponencial stability of more general systems of the form

$$\dot{x}_i(t) = -\rho_i(t, x_t)[b_i(x_i(t)) + f_i(t, x_t)], \quad t \geq 0, \quad i = 1,\dots,n, \tag{2}$$

where $b_i : \mathbb{R} \to \mathbb{R}, \rho_i, f_i : [0,\infty) \times C([-\tau, 0]; \mathbb{R}^n) \to \mathbb{R}$ are continuous, with $\rho_i(t, \varphi)$ positive, $i = 1,\dots,n$ for which the negative feedback condition $b_i(x)x > 0$ for $x \neq 0$, $i = 1,\dots,n$, is satisfied.
For (1), we also study the local and global asymptotic stability of x^*. Special attention is given to (1) with all functions η_{ij} being monotone, in which case sharper conditions for global stability are presented. This work generalizes known results for discrete delays [3] to systems with distributed delays.

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The method of solution-tube applied to first, second and third order systems of of differential equations

MARLÈNE FRIGON

Keywords: *System of differential equations, solution-tube, initial and boundary conditions.*
MSC2000 Classification: 34B15, 34A60, 34A34, 34M05.

Abstract

We show how the method of upper and lower solutions of differential equations can be extended to systems of differential equations by the method of solution-tube. This can be done for first, second and even for third order systems. It can also be applied to systems with maximal monotone terms. We show that using the method of solution-tube, existence and multiplicity results can be obtained. Finally, inspired by this notion, we present an existence result for analytical solutions to systems of differential equations in a complex domain.

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Limit cycles for some one dimensional non-autonomous differential equations

ARMENGOL GASULL

Keywords: *Abel equations, periodic orbits, limit cycles.*
MSC2000 Classification: 34C25, 37C10, 37C27.

Abstract

Consider non-autonomous differential equations of the form

$$\frac{dx}{dt} = S(x,t), \tag{1}$$

where $x, t \in \mathbb{R}$ and $S : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ is a T-periodic function in the variable t. We study the number of solutions satisfying $x(0) = x(T)$. Note that equation (1) can be considered on the cylinder $\mathbb{R} \times [0, T]$ and so the solutions satisfying $x(0) = x(T)$ are usually called *periodic solutions*. A periodic solution which is isolated from other periodic solutions of (1) is called a *limit cycle* of the differential equation. One of the interests of studying such solutions is because of its relation with the Hilbert sixteenth problem about the number of limit cycles of planar polynomial autonomous equations

We take special forms for the function $S(x,t)$ like $A(t)x^n + B(t)x^m + C(t)x$, $x^k + A(t)x^2 + B(t)x + C(t)$ or $A(t)|x| + B(t)$ and study whether the maximum number of limit cycles can be determined or not. This talk summarizes some of the results given in the references.

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Solvability of a stationary nonlinear Black-Scholes equations under conditions on the potential

Maria de Fátima Fabião-Ribeiro, Maria do Rosário Grossinho and Onofre Simões

Keywords: *Nonlinear Black-Scholes equation, condition on the potential, positive stationary solutions, upper and lower solutions.*
MSC2000 Classification: 35J25, 35J65, 35R60.

Abstract

In this work, we consider a nonlinear problem suggested by the Black–Scholes model for option pricing with stochastic volatility, namely,

$$\begin{cases} \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 f}{\partial S^2} + \frac{1}{2}\sigma^2 V^2 \frac{\partial^2 f}{\partial \sigma^2} + \rho\sigma^2 VS \frac{\partial^2 f}{\partial S \partial \sigma} - \frac{1}{2}\rho\sigma^2 V \frac{\partial f}{\partial \sigma} + rS\frac{\partial f}{\partial S} = r\,\gamma(f) & \text{in } \Omega, \\ f(S,\sigma) = h(S,\sigma) & \text{on } \delta\Omega. \end{cases}$$

where the variables S and σ are respectively the asset value and the market volatility ([1], [3]). In [2], a problem of this type with $\gamma(f) = g(f)f$ has been studied by an iterative procedure under the hypothesis that γ is nondecreasing. We prove the existence of a positive solution f assuming certain conditions on the potential Γ of γ. The method of the proof, which is based on the construction of upper and lower solutions, obtained as solutions of an auxiliary initial value problem, also yields information on the localization of f.

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Mathematical Models in Engineering, Biology and Medicine.
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On delay differential equations related to biological compartmental systems

István Győri

Keywords: *Compartmental systems.*
MSC2000 Classification: 34C, 34K, 92B05.

Abstract

In this lecture we study the qualitative properties of the solutions in a special class of delay differential equations. Our investigations are essentially motivated by the biological compartmental systems and their applications in the pharmacology

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On linearized stability of neutral differential equations with state-dependent delays

FERENC HARTUNG

Keywords: *Exponential stability, linearization, state-dependent delays.*
MSC2000 Classification: 34K20, 34K40.

Abstract

In this talk we present some recent result on exponential stability of constant or periodic steady-state solutions of several classes of delay and neutral differential equations with state-dependent delays. Sufficient and in some cases necessary stability conditions are formulated by means of linearization.

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Local organizing centers in diffusively coupled dynamical systems

FÁTIMA DRUBI, SANTIAGO IBÁÑEZ AND JOSÉ ÁNGEL RODRÍGUEZ

Keywords: *Coupled dynamical systems, singularities, unfoldings, chaotic dynamics, synchronization.*
MSC2000 Classification: 37D45, 37G10, 34C28, 34C60.

Abstract

Diffusively coupled dynamical systems arise in a wide range of fields. As an example, they are useful as models for biochemical reaction networks [1,5]. We will discuss two topics in this context: synchronization and chaotic dynamics.

In [2] we proved that a network consisting of two identical systems (which model a specific chemical reaction) contained strange attractors. The idea was to show the generic occurrence of a four-dimensional nilpotent singularity of codimension four. Such singularity unfolds generically three-dimensional nilpotent singularities of codimension three, which are nowadays well-known organizing centers of chaotic dynamics emanating from Shilnikov bifurcations (see [4]).

Most recently [3], yet working with the same model, we have reported the existence of other interesting local organizing centers, namely, the existence of several types of Hopf-pitchfork singularities. We will see that some of them can explain again the emergence of chaotic behaviour. On the other hand, we will show their role in the understanding of mechanisms related with synchronization. A remarkable fact is that Hopf-pitchfork singularities are expectable in many other coupling-based models. Particular attention will be devoted to that point, including some examples.

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Nonlocal impulsive boundary value problems with solutions that change sign

GENNARO INFANTE AND PAOLAMARIA PIETRAMALA

Keywords: *Impulsive differential equation, nontrivial solution, cone, fixed point index.*
MSC2000 Classification: Primary 34B37, secondary 34A37, 34B10.

Abstract

We discuss the existence of nonzero solutions for some second order impulsive boundary value problem subject to nonlocal boundary conditions. Our conditions are quite general and include the well-known multi-point boundary conditions, studied by other authors.
Our approach relies on the classical fixed point index for compact maps.

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Radial solutions for systems involving mean curvature operators in Euclidian and Minkovski spaces

PETRU JEBELEAN

Keywords: *Mean curvature operator, radial solution, Schauder fixed point theorem.*
MSC2000 Classification: 35J45, 34B15.

Abstract

Based on joint work with C. Bereanu and J. Mawhin.
We are concerned with the existence of radial solutions for systems of type

$$(1)\qquad \begin{cases} \operatorname{div}\left(\frac{\nabla v_1}{\sqrt{1 \pm |\nabla v_1|^2}}\right) = f_1(|x|, v_1, ..., v_m, \frac{dv_1}{dr}, ..., \frac{dv_m}{dr}) \\ . \\ . \\ . \\ \operatorname{div}\left(\frac{\nabla v_m}{\sqrt{1 \pm |\nabla v_m|^2}}\right) = f_m(|x|, v_1, ..., v_m, \frac{dv_1}{dr}, ..., \frac{dv_m}{dr}) \\ v_1 = ... = v_m = 0 \quad \text{on } \partial\Omega \end{cases}$$

in $\Omega \subset \mathbb{R}^N$, where the domain Ω is the unit ball or an annular domain. It is our aim to extend the results obtained in [1] for $m = 1$, to problem (1). The system (1) is transformed into a fixed point problem which is a special case of a class of systems of differential equations which includes, among others, radial p-Laplacian systems. The main tool in proving the existence results is the Schauder fixed point theorem.

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The Y2K problem of difference equations revisited

VÍCTOR JIMÉNEZ LÓPEZ

Keywords: *Difference equations of rational type, dominance condition, global attractor.*
MSC2000 Classification: 39A11, 37C70.

Abstract

The second order difference equation

$$x_{n+1} = \frac{p + qx_n}{1 + x_{n-1}}$$

(with $p, q > 0$) has the unique positive equilibrium

$$u = \frac{(q-1) + \sqrt{(q-1)^2 + 4p}}{2}.$$

For a long time it has been conjectured that u attracts all positive solutions of the equation. Indeed this has become one of the most challenging open problems in the field, having been dubbed "the year 2000 problem" of difference equations by G. Ladas, editor-in-chief of *Journal of Difference Equations and Applications*.

The conjecture is known to be true in the cases $q < 1$ [4] and $p \leq q$ [2]. Under the assumptions $q \geq 1$ and $q < p$ it has been proved in the progressively more general settings $p \leq 2(q+1)$ [3], $p \leq 2(q+1) + 4/(q-1) + 2(q^4-1)^{1/2}/(q-1)^2$ (which in particular implies the case $q = 1$ for all p) [7], and $p \leq 2q(q^2+1)/(q-1)^2$ [6]. The paper [5] purportedly provides a full proof of the conjecture but in fact has a rather basic mistake.

In this work we use a modified version of a tool introduced in [1] (so-called the *dominance condition*) to give a unified proof on the conjecture in the cases listed above and improve the bounds from [6].

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Global dynamics of discrete systems through Lie Symmetries

ANNA CIMA, ARMENGOL GASULL, AND VÍCTOR MAÑOSA

Keywords: *Integrable vector fields and maps, Lie Symmetries, topology of invariant sets, difference equations, rotation numbers.*
MSC2000 Classification: 37C05, 37C27, 37E10, 39A20.

Abstract

In this talk we show how to use Lie Symmetries to characterize the dynamics of a discrete dynamic system in the integrable case, and how to construct these Lie Symmetries.

A Lie symmetry of a discrete dynamic system given by a map F (defined in an open set of $\mathbb{R}^n$), is a vector field X such that F preserves the orbital structure of X (this is characterized by the relation $X(F) = DF\,X$). In the integrable case (when there exist $n-1$ independent first integrals of F), the existence of a Lie symmetry characterizes the dynamics of F over the energy levels, see [3]. But even in the non–integrable case the existence of a Lie Symmetry of F can give useful information to study the topology of the possible invariant sets, as well as other dynamical information.

Through the talk we will refer to the celebrated Lyness' family of maps

$$F(x_1,\ldots,x_k) = (x_2,\ldots,x_k,(a+\sum_{i=2}^{k} x_i)/x_1),$$

as an illustrative example of how this techniques can be applied, and which are their limitations, see [1,2] and [4].

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Lower and upper solutions: an appropriate method for BVP in Medicine an Engineering

FELIZ MINHÓS

Keywords: *Lower and upper solutions, Ambrosetti-Prodi problems, functional problems.*
MSC2000 Classification: 34B10, 34B15, 34K10, 34K12.

Abstract

The method of lower and upper solutions provides, as well as results of existence, other important properties such as location of solution, extremal solutions,..., which have been underused and, moreover, their potential has not been optimized, either in theory or in applications. This talk will present two cases to emphasize both items:

The third order periodic equation

$$u'''(x) + f(x, u(x), u'(x), u''(x)) = s\, p(x)$$

for $x \in [0,1]$, $f : [0,1] \times \mathbb{R}^3 \to \mathbb{R}$, $p : [0,1] \to \mathbb{R}^+$ continuous functions and $s \in \mathbb{R}$, where it is obtained some sufficient conditions for an Ambosetti-Prodi type discussion on s, to have existence, non-existence and multiplicity of solutions. An application to the homeostatic thyroid-pituitary mechanism used to prevent catatonic diseases will be shown.

A functional problem composed by the fully fourth order nonlinear functional equation

$$-\frac{d}{dt}(\phi \circ u''')(t) = f(t, u''(t), u'''(t), u, u', u''), \text{ for a.a. } t \in I = [a,b],$$

where $\phi : \mathbb{R} \to \mathbb{R}$ is an increasing homeomorphism, $f : I \times \mathbb{R}^2 \times \mathcal{C}(I)^3 \to \mathbb{R}$, and the boundary conditions

$$\begin{aligned} B_1(u(b), u, u', u'') &= 0 = B_2(u'(b), u, u', u''), \\ B_3(u''(a), u''(b), u'''(a), u'''(b), u, u', u'') &= 0 = L_2(u''(a), u''(b)), \end{aligned}$$

with $B_i,\ i = 1,2,3,$ and L_2 suitable functions. The functional dependence allows a generalization to a large number of problems including equations with deviated arguments, delays, advances, equations with maxima, integrodifferential equations,..., and different types of boundary conditions such as separated, nonlinear, multipoint, nonlocal, with maximum or minimum conditions,... An application to a continuous nonlinear model to study the deformation of the human spine under some forces and a technique to obtain extremal solutions will be referred.

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Positive solutions of a prescribed mean curvature problem

PIERPAOLO OMARI

Keywords: *Positive solution, Dirichlet problem, prescribed mean curvature equation.*
MSC2000 Classification: 34B18, 35J25.

Abstract

We discuss existence and multiplicity of positive solutions of the one-dimensional prescribed curvature problem

$$-\left(u'/\sqrt{1+u'^2}\right)' = \lambda f(t,u), \quad u(0)=0,\ u(1)=0,$$

depending on the behaviour at the origin and at infinity of the potential $\int_0^u f(t,s)\,ds$. Besides solutions in $W^{2,1}(0,1)$, we also consider solutions in $W^{2,1}_{loc}(0,1)$ which are possibly discontinuos at the endpoints of $[0,1]$. Our approach is essentially variational and is based on an elliptic regularization scheme. Recent extensions to the N-dimensional problem

$$-\mathrm{div}\left(\nabla u/\sqrt{1+\|\nabla u\|^2}\right) = \lambda f(x,u) \text{ in } \Omega, \quad u=0 \text{ on } \partial\Omega.$$

are described too. These results can be found in [1], [2], [3], [4] and [5].

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φ-Laplacian Functional Equations

JUAN PERÁN

Keywords: *φ-Laplacian, Lower and upper solutions.*
MSC2000 Classification: 34B15, 34K10.

Abstract

This talk is about the one-dimensional φ-Laplacian equation

$$(\varphi \circ u')' = F(u),$$

where $\varphi : \mathbb{R} \to \mathbb{R}$ is an increasing homeomorphism and F is an operator defined over the space of piecewise continuously differentiable functions on an interval.
Our attention is focused on abstract Nagumo-type conditions, as that obtained when the (possibly empty) set

$$\bigcup_{0<\lambda<1} \left\{ u : (\varphi \circ u')' = \lambda F(u) \right\}$$

is required to be bounded. We also consider lower and upper solutions and functional boundary conditions.

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Difference equations of Poincaré type

MIHÁLY PITUK

Keywords: *Difference equation of Poincaré type, limiting equation, asymptotic behavior.*
MSC2000 Classification: 39A11.

Abstract

In this talk we review some of our recent results on difference equations of Poincaré type. The results include asymptotic expansions of the solutions in terms of the eigenvalues of the limiting equation obtained jointly with Professor Ravi P. Agarwal (Florida Institute of Technology, USA) and a Poincaré type theorem for the nonoscillatory solutions of second order equations obtained jointly with Professor Rigoberto Medina (Universidad de Los Lagos, Chile).

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Componentwise compression-expansion conditions for systems of nonlinear operator equations and applications

RADU PRECUP

Keywords: *Nonlinear operator system, semilinear elliptic system, singular boundary value problem, positive solution, radial solution, fixed point, cone.*
MSC2000 Classification: 34A34, 34B15, 35J25, 47J05.

Abstract

This talk is based on a new method to treat systems of nonlinear operator equations which was established in [1], namely the vector version of Krasnoselskii's cone fixed point theorem. The method allows us to localize a fixed point $u = (u_1, u_2)$ of a vector-valued operator $N = (N_1, N_2)$ in a vector conical shell: $r_1 \leq \|u_1\| \leq R_1$, $r_2 \leq \|u_2\| \leq R_2$, under the assumption that N_1, N_2 satisfy compression-expansion boundary conditions, independently. This makes possible (see [1–3]) that the two components $N_1(u_1, u_2)$, $N_2(u_1, u_2)$ may have different (sub or super-linear) behaviors in u_1 and u_2.
As an application, we discuss the existence, localization and multiplicity of the positive solutions to a system of singular second-order differential equations , and, in particular, of the positive radial solutions to semilinear elliptic systems.

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Bubble-type solutions of nonlinear singular problems

IRENA RACHŮNKOVÁ

Keywords: *nonlinear singular BVP, mixed conditions on infinite interval*
MSC2000 Classification: 34B16, 34B40.

Abstract

We investigate a singular boundary value problem which originates from the Cahn-Hillard theory in hydrodynamics. Here ρ denotes the density and $\mu(\rho)$ the chemical potential of a non-homogeneous fluid. If the motion of the fluid is zero, the state of the fluid in $\mathbf{R}^N$ is described by the equation

$$\gamma\Delta\rho = \mu(\rho) - \mu_0, \tag{1}$$

where γ and μ_0 are suitable constants. When searching for a solution with spherical symmetry which depends only on one variable r, equation (1) is reduced to the following ordinary differential equation

$$\gamma(\rho'' + \frac{N-1}{r}\rho') = \mu(\rho) - \mu_0, \quad r \in (0, \infty). \tag{2}$$

In fact, together with the boundary conditions

$$\rho'(0) = 0, \quad \lim_{r\to\infty} \rho(r) = \rho_\ell > 0, \tag{3}$$

equation (2) describes the formation of microscopical bubbles in a non-homogeneous fluid, in particular, vapor inside one liquid. The first condition in (3) follows from the central symmetry and it is necessary for the smoothness of solutions of the singular equation (2) at 0. The second condition in (3) means that the bubble is surrounded by an external liquid with the density ρ_ℓ. We prove the existence of a strictly increasing solution of problem (2),(3) having exactly one zero in $(0, \infty)$. If such solutions exist, many important physical properties of the bubbles depend on them. In particular, the gas density inside the bubble, the bubble radius and the surface tension. Numerical investigation of the problem can be found in [1-3].

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Two-Parameter Nonlinear Eigenvalue Problems

ARMANDS GRITSANS, FELIX SADYRBAEV AND NATALIJA SERGEJEVA

Keywords: *Eigenvalue problems, Fučík type spectra.*
MSC2000 Classification: 34B15.

Abstract

Equations of the form

$$x'' = -\lambda f(x^+) + \mu g(x^-)$$

are considered together with boundary conditions of different types. Functions f and g are nonlinear, $x^+ = \max\{x, 0\}$, $x^- = \max\{-x, 0\}$. We are looking for (λ, μ) such that the boundary value problem has nontrivial solutions. The structure of spectra is discussed.

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On the existence of heteroclinic trajectories for some non-autonomous equations

A. GAVIOLI AND L. SANCHEZ

Keywords: *non-autonomous equations, heteroclinics, homoclinics.*
MSC2000 Classification: 34B15, 34B16.

Abstract

We study conditions for the existence of heteroclinics connecting ± 1 for a nonautonomous equation of the form

$$\ddot{u} = a(t) f(u) \tag{1}$$

where $a(t)$ is a bounded positive function, $0 < a_1 \le a(t) \le a_2 \ \forall t \in \mathbf{R}$, and $f(\pm 1) = 0$. Here $f = F'$, where F is a C^1 non-negative function such that $F(0) = F(1) = 0$. We are mainly interested in the case where $\lim_{|t|\to\infty} a(t) = a_1$.

We find sufficient conditions in the form of inequalities that involve the L^∞ norm of a (via $\frac{a_2}{a_1}$) or the L^1 norm of $a - a_1$ (via $(\sup F) \int_{-\infty}^{+\infty} (a(t) - a_1)\, dt$) and the quantities $m_- := \sqrt{2a_1} \int_{-1}^{0} \sqrt{F(z)}\, dz \ \ m_+ := \sqrt{2a_1} \int_0^1 \sqrt{F(z)}\, dz$.

We also consider the case where $l_\pm := \lim_{t\to\pm\infty} a(t)$ are different, with $|l_- - l_+|$ small.

Variational methods are used in the proofs. Solutions are obtained in finite intervals $[-T, T]$ as critical points of the functional

$$\mathcal{J}_T(u) := \int_{-T}^{T} \left(\frac{\dot{u}^2}{2} + a(t) F(u) \right) dt \tag{2}$$

in the linear manifold

$$\mathcal{E}_T := \left\{ u \in H^1(-T, T) \mid \ u(\pm T) = \pm 1 \right\}$$

and then we take a limit as $T \to \infty$.

These results extend the research started in [1,2,3,4].

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Boundary value problems depending on parameters

SVATOSLAV STANĚK

Keywords: *Singular equations, singular systems, parameter, nonlinear boundary conditions.*
MSC2000 Classification: 34B15, 34B16, 34B18.

Abstract

If we have a second order differential equation of the type

$$u'' = h(t, u, u', \mu) \tag{1}$$

depending on the parameter μ and the three independent boundary conditions, we can look for a value of the parameter μ for which equation (1) has a solution satisfying the three boundary conditions. This problem for regular second-order differential equations is studied in the literature by the following methods: (i) shooting method, (ii) method of successive approximations (numerical-analytic method), (iii) linearization method, (iv) method based on surjective mapping in $\mathbf{R}^N$, (v) method based on the Leray-Schauder degree, (vi) 'gluing' method.

We present an existence principle for solving of the singular problem

$$(\phi(u'))' + \mu f(t, u, u') = 0, \tag{2}$$

$$u \in \mathcal{S} \tag{3}$$

in the set $AC^1[0,T]$ (see [1]). Here ϕ is an increasing homeomorphism from $\mathbf{R}$ onto $\mathbf{R}$, $f \in Car([0,T] \times \mathcal{D})$, $\mathcal{D} \subset \mathbf{R}^2$, $f(t,x,y)$ may be singular in its space variables x, y, and $\mathcal{S}$ is a closed subset of $C^1[0,T]$. The proof of our existence principle is based on a combination of the Leray-Schauder degree method with regulariation and sequential techniques. Applications of the existence principle are given for various sets $\mathcal{S}$ which are defined by three boundary conditions. The first two are the Dirichlet boundary conditions or the antiperiodic boundary conditions, the last one is a nonlinear and nonlocal boundary condition (see [1,3]).

Next, the system of second-order differential equations

$$(\phi(u'))' = \mu \star (p(t, u, u') + f(t, u)) \tag{4}$$

is considered, where $\mu \star a = (\mu_1 a_1, \dots, \mu_N a_N)$ for $\mu, a \in \mathbf{R}^N$. Here $\phi(x) = (\phi_1(x_1), \dots, \phi_N(x_N))$, where ϕ_j is one-dimensional Laplacian; $\mu \in \mathbf{R}^N$; $p \in Car([0,T] \times ([0,\infty)^N \times \mathbf{R}^N); \mathbf{R}^N)$; $f \in Car((0,T) \times (\mathbf{R} \setminus \{0\})^N); \mathbf{R}^N)$ and $f(t,x)$ may be singular at $t=0$ and/or $t=T$ of the time variable t and at the value 0 of the components $x_1, \dots, x_N$ of its space variable x. We give condition on p and f which guarantee that there exists a value of μ for which system (4) has a solution $u = (u_1, \dots, u_N) \in AC^1([0,T]; \mathbf{R}^N)$ satisfying the Dirichlet conditions $u(0) = 0$, $u(T) = 0$ and the extra condition

$$\max\{u_j(t) : t \in [0,T]\} = \alpha_j, \;\; \alpha_j > 0, \; 1 \le j \le N.$$

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On behavior of a class of difference equations with maximum

STEVO STEVIĆ

Keywords: *Max-type difference equation, convergence, semi-cycle.*
MSC2000 Classification: 39A11.

Abstract

Some results on the behavior of positive solutions to the following max-type difference equation

$$x_n = \max\left\{B_0, B_1\frac{x_{n-p_1}^{r_1}}{x_{n-q_1}^{s_1}}, B_2\frac{x_{n-p_2}^{r_2}}{x_{n-q_2}^{s_2}}, \dots, B_k\frac{x_{n-p_k}^{r_k}}{x_{n-q_k}^{s_k}}\right\}, \quad n \in \mathbb{N}_0,$$

where p_i, q_i, $i = 1, \dots, k$, are natural numbers such that $p_1 < p_2 < \cdots < p_k$, $q_1 < q_2 < \cdots < q_k$, $k \in \mathbb{N}$, $r_i, s_i \in [0, \infty)$, $i = 1, \dots, k$ and $B_i \in [0, \infty)$, $i = 0, 1, \dots, k$ are presented in this talk.

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Existence of solutions for fourth-order ordinary differential equations in water wave models

MELINE APRAHAMIAN AND STEPAN TERSIAN

Abstract

We study the existence of solutions of equation $\gamma u^{iv} = u'' + \mu(2uu'' + u'^2) + f(u)$ appearing in theory of water waves via variational method. Let $L > 0$.We consider the problems (P_1) and (P_2) for equations $\gamma u^{iv} = u'' + \mu(2uu'' + u'^2) + u - u^3$ and $\gamma u^{iv} = u'' + \mu(2uu'' + u'^2) - u - u^2$, respectively with boundary conditions $u(0) = u(L) = u'(0) = u'(L) = 0$. Both problems have variational structure and their weak solutions in the space $X = H_0^2(0, L)$ are critical points of the functionals

$$I(u; L) = \int_0^L \left(\frac{\gamma}{2} u''^2 + \frac{1}{2} u'^2 + \mu u\, u'^2 - \frac{1}{2} u^2 + \frac{1}{4} u^4 \right) dx$$

and

$$J(u; L) = \int_0^L \left(\frac{\gamma}{2} u''^2 + \frac{1}{2} u'^2 + \mu u\, u'^2 + \frac{1}{2} u^2 + \frac{1}{3} u^3 \right) dx.$$

Our main results are as follows:

Theorem 1. *Let $0 < \mu < \min(1, 2\gamma)$. Then problem (P_1) has a solution u which is a minimizer of the functional $I : X \to \mathbb{R}$. If L is sufficiently large, $L > L_0$, this solution is nontrivial. Suppose that u is a nonnegative minimizer of $I(., L)$ for $L > L_0$. Then $u(x) > 0$ for every $x \in]0, L[$.*

Theorem 2. *Let $0 < \mu < 2\gamma$. Then problem (P_2) has a nontrivial solution u which is a mountain pass point of the functional $J : X \to \mathbb{R}$.*

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On nonlinear second order boundary value problems arising in hydrodynamics

JAN TOMEČEK

Keywords: *Singular equations, linear boundary conditions on infinite interval, second-order conservative systems.*
MSC2000 Classification: 34B16, 34B40, 37C29.

Abstract

We investigate singular boundary value problem arising in study of nonhomogenous fluids. We study the problem

$$(t^2u')' = 4\lambda^2 t^2 (u+1)u(u-\xi), \qquad t \in (0,\infty), \tag{1}$$

$$u'(0) = 0, \quad u(\infty) = \xi, \tag{2}$$

where $\lambda > 0$ and $\xi \in (0,1)$ are real parameters. We are interested in strictly increasing solutions, because such solutions have reasonable physical meaning.
For this purpose, it turned out to be useful to investigate the autonomous equation

$$u'' = 4\lambda^2 (u+1)u(u-\xi). \tag{3}$$

We can understand the equation (3) as a second–order conservative system. The strictly increasing solution of problem (3), (2) is homoclinic (and it is unique). Equation (3) is a simple tool for investigation of behavior of solutions of the equation (1). Moreover, its appropriate perturbation generates lower and upper functions to equation (1). Using these functions, we are able to prove the existence result for the problem (1), (2).

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Existence and stability of periodic solutions of the relativistic oscillator

PEDRO J. TORRES

Keywords: *Periodic solution, relativistic oscillator.*
MSC2000 Classification: 34B15, 34B16.

Abstract

Some aspects on the dynamics of a periodically forced oscillator with relativistic effects are considered. If c is the speed of light in the vacuum, the equation under study is

$$\left(\frac{mx'}{\sqrt{1 - \frac{x'^2}{c^2}}} \right)' + kx' + g(x) = p(t), \tag{1}$$

where $m > 0$ is the mass at rest, p is a continuous and T-periodic forcing term and k is a possible viscous friction coefficient. Physically, we are assuming a basic principle of Special Relativity: the mass of a moving object is not constant but depends on its velocity (see for instance [1]). From a more mathematical perspective, the equation can be seen as a singular ϕ-laplacian oscillator. The recent publication of [2] has renewed the interest in the study of equations with singular ϕ-laplacian operators.
We present some new results on the existence and stability of periodic solutions. It turns out that important differences with the non-relativistic case arise.

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Multiplicity results for singular periodic problems

MILAN TVRDÝ

Keywords: *Second order nonlinear ordinary differential equation, periodic problem, singularity, multiple solutions, lower and upper functions.*
MSC2000 Classification: 34B15, 34C25.

Abstract

The aim of the contribution is to present some results on the existence of at least two solutions to singular periodic boundary value problems for nonlinear second order differential equations. A typical example is the Duffing equation $u'' = g(u) + e(t),$ where $e \in L[0,T]$ and $g \in C(0,\infty)$ is continuous and has a strong singularity at 0, i.e.

$$\lim_{x\to 0+} \int_x^1 g(s)\,\mathrm{d}s = \infty.$$

Up to now, such problems have been studied by many authors, starting from the celebrated paper [5] by Lazer and Solimini, cf. e.g. [2], [6], [9], [12] and the survey [8] and its bibliography. On the other hand, still there are not too many multiplicity results available. Let us mention here e.g. [1], [3], [4], [7] and [10]. We would like to recall and possibly extend some of the results of the joint paper [10] with Irena Rachůnková and Ivo Vrkoč.

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Positive solutions of a nonlocal boundary value problem of conjugate type

JEFF WEBB

Keywords: *higher order equations, nonlocal boundary conditions.*
MSC2000 Classification: 34B18, 34B16.

Abstract

We will study existence of positive solutions for the problem

$$u^{(n)}(t) + g(t)f(t, u(t)) = 0, \; t \in (0, 1),$$

with the nonlocal boundary conditions

$$u^{(k)}(0) = 0, \; 0 \leq k \leq n-2, \;\; u(1) = \alpha[u],$$

where $\alpha[u]$ is a linear functional on $C[0, 1]$, thus is given by a Riemann-Stieltjes integral

$$\alpha[u] = \int_0^1 u(s)dA(s). \tag{1}$$

When $\alpha[u] \equiv 0$ these are called $(n-1, 1)$ conjugate boundary conditions. We suppose that g, f are non-negative and α satisfies some positivity hypothesis but we do *not* need to assume that $\alpha[u] \geq 0$ for *all* $u \geq 0$. We show how some recent work of Webb and Infante [2],[3], which gave a unified method of tackling many nonlocal boundary value problems, can be applied to this type of boundary value problem. This allows us to improve some recent work on these problems by Eloe and Ahmad [1], and others.

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Complex dynamics in a Lotka-Volterra predator-prey model

M. PIREDDU AND F. ZANOLIN

Keywords: *Volterra predator–prey system, harvesting, periodic solutions, subharmonics, chaotic–like dynamics, topological horseshoes, linked twist maps.*
MSC2000 Classification: 34C25, 37E40, 92C20.

Abstract

We present an elementary topological approach in order to find periodic points and detect the presence of chaotic - like dynamics for discrete dynamical systems in the plane. An application is given to the proof of complex dynamics for a first order planar differential system of Lotka - Volterra type with periodic coefficients.

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Applications of coincidence equations to boundary value problems

MIROSŁAWA ZIMA

Keywords: *coincidence equation, cone, boundary value problem, positive solution.*
MSC2000 Classification: 34B10, 34B18.

Abstract

In [2] O'Regan and Zima obtained some results on the existence of solutions in cones for coincidence equation $Lx = Nx$. Here L is a Fredholm operator of index zero and N is a nonlinear mapping. We will show how these results can be applied for studying the existence of positive solutions for second order multipoint boundary value problems at resonance. We will also discuss periodic boundary value problems for first order differential equations.

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Contributed Talks

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New Method for Solving Fuzzy Partial Differential Equations

M. AFSHAR KERMANI AND T. ALLAHVIRANLOO

Keywords: *fuzzy partial differential equation, difference method.*
MSC2000 Classification: 35K05, 35L05.

Abstract

We propose a method for computing approximate solution for a fuzzy partial differential equation using numerical methods. Since finding this set of solutions analytically does only work with trivial examples, a numerical approach seems to be the only way of "solving" such problems.
J. Buckley and T. Feuring proposed a method to solutions of elementary fuzzy partial differential equations. T. Allahviranloo used a numerical method to solve FPDE, that was based on the Seikala derivative. In this paper a new method for solving "fuzzy partial differential equation" (FPDE) is considered. This numerical method based on the definition of derivative that considered by *Y. Chalco-Cano, H. Roman-Flores* . We present a difference method to solve the FPDEs such as fuzzy hyperbolic equation and fuzzy parabolic equation , then see whether stability of this method exist, and conditions for stability are given. Examples are presented showing the Hausdorff distance between exact solution and approximate solution is small.

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A new method for solving fuzzy linear differential equations

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Keywords: *Eigenvalue, Eigenvector, Seikkala derivative, Fuzzy differential equation.*
MSC2000 Classification: 65L06, 35E15.

Abstract

In this paper, an analytic method for presenting n-th order fuzzy differential equations with fuzzy initial values is presented. The fuzzy differential equation is converted to a fuzzy system. To solve the system, three cases are considered for the eigenvalues, namely when all eigenvalues are real and distinct, when some eigenvalues are complex, and some eigenvalues are multiple. In each case, it is shown that the solution of the differential equation is a fuzzy number. In addition, the method is illustrated by presenting several numerical examples. Also a comparative example is provided to compare our method with the Buckley-Feuring method.

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Numerical methods for hybrid fuzzy differential equations

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Keywords: *Hybrid system; Fuzzy differential Equations; Multi-step methods.*
MSC2000 Classification: 65L06, 35E15.

Abstract

Hybrid systems are devoted to modeling, design, and validation of interactive systems of computer programs and continuous systems. That is, control systems that are capable of controlling complex systems which have discrete event dynamics as well as continuous time dynamics can be modeled by hybrid systems. The differential systems containing fuzzy-valued functions and interaction with a discrete time controller are called hybrid fuzzy differential systems.
The numerical methods for solving hybrid fuzzy differential equations are introduced in [4,5] by Pederson and Sambandham. They developed numerical methods for addressing hybrid fuzzy differential equations by an application of the Euler and Runge-Kutta method using the Seikkala derivative. In this article, we use Adams-Bashforth and Predictor Corrector method in [2] for solving hybrid fuzzy differential equations.

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Population Growth Model Proportional to Beta Densities

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Keywords: *Beta Densities and Allee Effect.*
MSC2000 Classification: 34B15, 34B16.

Abstract

We present populational growth models proportional to beta densities with shape parameters p and 2, with $p > 1$. Our results give explicit methods to investigate the chaotic behaviour of populational growth models, when the malthusean parameter increases. The Allee effect is analized in these models.

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Blow-up topologically equivalent singularities

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Keywords: *Singularities, topological equivalence, blowing-up.*
MSC2000 Classification: 35B38.

Abstract

As a consequence of the Hartman-Grobman Theorem, we know that any pair of hyperbolic singularities with invariant stable varieties of the same dimension are equivalent from the topological point of view. A natural question in this context is to know in what conditions they are *blow-up topologically equivalent*, that is, if it possible to find a topological equivalence "downstairs" that lifts by any finite sequence of quadratic transformations. We show that in dimension three, two hyperbolic singularities under non-resonance conditions are blow-up topologically equivalent if and only if the eigenvalues are proportional.

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Improved Predictor Corrector Method for solving fuzzy initial value problem

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Keywords: *Fuzzy Differential Equations, explicit method, implicit method, Predictor Corrector Method.*
MSC2000 Classification: 34B15, 34B16.

Abstract

In this paper an Improved Predictor Corrector (IPC) method to solve the "fuzzy initial value problem" is discussed. The IPCM is obtained by combining an explicit three-step method and an implicit two-step method. These methods are compared with the methods of [1] and they have more accuracy. Convergence and stability of the proposed methods are also presented in detail. In addition, these methods are illustrated by solving a fuzzy Cauchy Problem. Three numerical methods to solve "The Fuzzy Ordinary Differential Equations" are discussed. These methods are Adams Bashforth, Adams Moulton and Predictor-Corrector. Predictor-Corrector is obtained by combining Adams Bashforth and Adams Moulton methods.

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Protein Complex and Functional Module Prediction Using PCA Method

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Keywords: *Protein protein interaction; Functional module; complex.*
MSC2000 Classification: 92B05, 92B15, 92D20.

Abstract

Let H be the graph obtained from protein- protein interaction network (PPI) reported in MIPS and G be the graph which is refined of H regarding to the location and type of interaction between vertices of H. We define a weighted function ω on the edges of G based on location and type of interaction of proteins too. Suppose A is the adjacent matrix of the graph (G, ω) which the entry in v'th row and u'th column of A is ω (vu). Now by using PCA method, A is translated to a matrix B, and the maximum eigenvalue λ of B is calculated. Let α be the normal eigenvector corresponding to λ. The maximum and minimum value of column vector of A.α are corresponded to two vertices (proteins) of G. We know that these vertices are in the dense region of graph G. So, such vertices together with its neighbors seem to be a good candidate for a complex or functional module in PPI. A candidate is called good if the summation of its edges' weights is at least a threshold ε. We merge two good candidates if the induce sub graph generated by them is good too. Finally, we extend the goods sub graphs using 2-hop procedure which is introduced by Limsoon et al. The obtained sets are considered as complexes or functional modules. In comparison with the other algorithms it has seen that the result produced from our method is better than the other algorithms.

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On exponential stability for a linear delay differential equation

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Keywords: *Delay differential equations; exponential stability.*
MSC2000 Classification: 34K20.

Abstract

For a scalar linear delay equation

$$\dot{x}(t) = -\sum_{k=1}^{m} a_k(t)x(h_k(t)), \quad h_k(t) \leq t,$$

we discuss some methods and results on exponential stability.
In particular we consider a method based on Bohl-Perron type theorem, positiveness of the fundamental function and comparison with known exponentially stable delay differential equations.
New explicit stability conditions were obtained, including equations with positive and negative coefficients and equations with oscillating coefficients.

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Dichotomous and normally resolvable operator in Banach space

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Keywords: *Bounded solution, dichotomous, boundary value problems with conditions at infinity, generalized inverse operator, normally resolvable and Fredholm operators.*
MSC2000 Classification: 34L05, 47A53, 47A55.

Abstract

It's well-known [1] that if we consider equation $(Lx)(t) := \dot{x}(t) - A(t)x(t) = f(t)$ in $\mathbb{R}^n$ under the assumption that the corresponding linear homogeneous equation is exponentially dichotomous on both semi-axes $\mathbb{R}_+$ and $\mathbb{R}_-$ with projectors P and Q, respectively, then the operator $(Lx)(t)$ must be only Fredholm operator . In the case, when we consider this equation in the Banach space $\mathbf{B}$ under the same conditions we have much more variants. We proved , that if the operator $D = P - (E - Q) : \mathbf{B} \to \mathbf{B}$ be invertible in the generalized sense[2], then, using the classification by S.G.Krein [3], the operator $(Lx)(t)$ maybe

- or **a normally resolvable operator :** $\overline{\operatorname{Im} L} = \operatorname{Im} L;\ dim\ ker(L) = dim\ coker(L) = \infty$;
- or **a** $d-$ **normally resolvable operator :** $dim\ coker(L) = d < \infty,\ dim\ ker(L) = \infty$;
- or **an** $n-$ **normally resolvable operator :** $dim\ ker(L) = n < \infty,\ dim\ coker(L) = \infty$;
- or, at last, **a Fredholm operator :** $dim\ ker(L) < \infty$, $dim\ coker(L) < \infty$.

Conditions for the appearance of these cases are considered [4]. Similarly results are discussed for perturbed operator $(L_\varepsilon x)(t) := \dot{x}(t) - A(t)x(t) - \varepsilon A_1(t)x(t)$. Examples of the existence of bounded solutions for countable systems of differential equations are considered [5]. Research partially supported by the grants VEGA1/3238/06 and VEGA1/0771/08 of Slovak Grant Agency.

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Boundary Weak Solution of the Semicoercive 2D Contact Problem

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Keywords: *Steklov-Poincaré operator, variational inequality, domain decomposition, BETI.*
MSC2000 Classification: 65N38, 65N55.

Abstract

We consider the following model 2D contact semicoercive problem: find a function (u^1, u^2) satisfying

$$-\Delta u^m = f \quad \text{in } \Omega^m, \quad u^1 = 0 \quad \text{on } \Gamma_u^1, \quad \frac{\partial u^m}{\partial n} = 0 \quad \text{on } \Gamma_f^m, \quad m = 1, 2,$$

together with the conditions given on $\Gamma_c := \Gamma_c^1 = \Gamma_c^2$:

$$u^2 - u^1 \geq 0, \quad \frac{\partial u^2}{\partial n} \geq 0, \quad \frac{\partial u^2}{\partial n}(u^2 - u^1) = 0, \quad \frac{\partial u^1}{\partial n} + \frac{\partial u^2}{\partial n} = 0.$$

The solution (u^1, u^2) of our model problem may be interpreted as a vertical displacement of two membranes stretched by normalized horizontal forces and pressed down by forces with the density f. The left membrane Ω^1 is fixed on the left edge. The left edge of Ω^2 is not allowed to penetrate below the right edge of Ω^1.
We firstly decompose the domain into the disjunct subdomains and then we use the symmetric representation of the local Steklov - Poincaré operator to get the boundary weak formulation of our problem in the form of variational inequality.

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On oscillation and stability of equations with a distributed delay

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Keywords: *Distributed delay, stability, oscillation, equations of population dynamics.*
MSC2000 Classification: 34K20, 34K11, 92D25.

Abstract

Distributed delays model integrated dependence of the growth rate of the system on its previous states. Historically, a logistic equation with a distributed delay goes back to the works of Volterra published more than 20 years before the logistic equation with a constant delay was formulated by Hutchinson. In [1] we have studied oscillation of the logistic equation with a distributed delay which incorporates the integral equation of Volterra and Hutchnson's equation, as well as nonautonomous equations with delay and integral terms. Further, we applied the linearization theory [2] to equations with a distributed delay [3]: logistic, Lasota-Wazewska and Nicholson's blowflies models. The version of the Mean Value Theorem which reduces equations with a distributed delay to equations with a concentrated delay [3] allows to employ known results for nonlinear equations with a variable concentrated delay to deduce oscillation and stability conditions for equations with a distributed delay. The method applied to study the stability of Nicholson's blowflies equation [4] allows to consider more general models with a unimodal growth rate function and a distributed delay.

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Existence of limit cycles for rigid planar vector fields

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Keywords: *Periodic solutions, Abel equation, planar rigid systems.*
MSC2000 Classification: 34D05.

Abstract

For any rigid planar vector field, $x' = y + xP(x, y)$, $y' = -x + yP(x, y)$, where $P(x, y) = \sum_{k=1}^{m} a_k x^{i_k} y^{j_k}$, we characterize the existence of limit cycles surrounding the origin only in terms of the parity of the degrees of the monomials i_k, j_k.
In particular, when the set $\{i_k + j_k : k = 1, \ldots, m\}$ has at least three elements, we prove that the rigid planar vector field has no limit cycles surrounding the origin for any choice of $a_1, \ldots, a_m$, if and only if there exists at most one k_0 such that i_{k_0} and j_{k_0} are even, and every i_k, $k \neq k_0$, or every j_k, $k \neq k_0$ are odd.

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Limit cycles of a quadratic polynomial planar system: the third order Melnikov function

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Keywords: *Quadratic systems, limit cycles, bifurcation, high order Melnikov functions.*
MSC2000 Classification: 34C07, 34C23, 34C25.

Abstract

The Hilbert sixteenth problem [3] asks for the number and distribution of limit cycles inside the family of planar vector fields of given degree. In relation to this difficult problem, we study how many limit cycles can appear in the following quadratic system

$$\begin{array}{l} \dot{x} = -y(1+x) - \varepsilon P(x,y), \\ \dot{y} = x(1+x) + \varepsilon Q(x,y), \end{array} \tag{1}$$

where $\varepsilon > 0$ is a small parameter and P and Q are arbitrary polynomials of degree two. The unperturbed system (i.e. for $\varepsilon = 0$) has a center at the origin and the first integral $H = (x^2 + y^2)/2$ in the region $x^2 + y^2 < 1$. Using the energy level $H = h$ as a parameter, we can express the Poincaré map $\mathcal{P}$ of (1) in terms of h and ε. For the corresponding displacement function $d(h, \varepsilon) = \mathcal{P}(h, \varepsilon) - h$ we obtain the following representation as a power series in ε:

$$d(h, \varepsilon) = \varepsilon M_1(h) + \varepsilon^2 M_2(h) + \varepsilon^3 M_3(h) + ..., \tag{2}$$

which is convergent for small ε. The Melnikov functions $M_k(h)$ are defined for $h \in (0, 1/2)$. Each simple zero $h_0 \in (0, 1/2)$ of the first non-vanishing coefficient in (2) corresponds to a limit cycle of (1) emerging from the circle $x^2 + y^2 = 2h_0$. We compute these functions by using the algorithm developed in [2,4]. Our main result [1] states that, for system (1) at most three limit cycles can bifurcate from the set of periodic orbits of the unperturbed system, when considering the expansion of the displacement map (2) up to third order in ε. Furthermore this upper bound is reached.

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A general approach for front-propagation in functional reaction-diffusion equations

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Keywords: *Delayed reaction-diffusion equations, non-local reaction-diffusion equations, traveling waves, upper and lower-solutions method, fixed point.*
MSC2000 Classification: 35K57, 34K10.

Abstract

The study of the existence and qualitative properties of traveling fronts for reaction-diffusion equations is a widely investigated field of research, due to several applications in various biological phenomena. We refer e.g. to the monographs [3,5].
Recently, some models of non-local reaction-diffusion equations have been proposed by S.A. Gourley (see [4]), in which the reaction term contains a convolution integral. Another field having an increasing interest, is that of reaction-diffusion equations or systems with time-delay (see [2]).
Our purpose is to propose a general unifying approach for dealing with functional reaction-diffusion equations. By using topological methods suitably combined with upper and lower-solutions techniques, in [1] we establish sufficient conditions for the existence of traveling waves. Our results apply to equations having reaction terms with delay as well as depending on convolution integrals.

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Hilbert's Sixteenth Problem for Liénard equations

MAGDALENA CAUBERGH AND FREDDY DUMORTIER

Keywords: *Hilbert's Sixteenth Problem, Liénard equation, limit cycles, Poincaré compactification, semi-hyperbolic saddles.*
MSC2000 Classification: 34C07, 34C23, 34C25.

Abstract

In this talk I present recent progress in the understanding of Hilbert's Sixteenth Problem, that is joint work with F. Dumortier. Recall that Hilbert's Sixteenth Problem essentially asks for the maximal number of limit cycles of a planar polynomial vector field for a given degree. We consider the restricted version of this problem to polynomial vector fields that are associated to Liénard equations: $x''+f(x)x'+g(x)=0$, where f and g are polynomials of degree n and m respectively. Only for some low degrees n and m, upperbounds $N(m,n)$ for the number of limit cycles are known. In [1] we give a general global finiteness result for 'compact' classical Liénard equations, i.e., when $g(x)=x$ and f belongs to an arbitrary compact set of polynomials of degree n. In this talk I will sketch the ideas to obtain this result and discuss some related local cyclicity problems.

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A mathematical model of cancer as a competition

JOANNA CHROBAK AND HENAR HERRERO

Keywords: *Model of cancer.*
MSC2000 Classification: 92C50.

Abstract

Cancer is one of the main problems of humanity nowadays. In Spain it is already the first cause of death of men and the second of women, after cardiovascular illnesses [1].
Because of the size of the problem and the level of its complexity, mathematical models of cancer were created to serve as a tool of modelling and prediction. During the last 30 years a big number of models of tumor growth have been developed, most of them based on the use of partial differential equations, just to mention a few: N. Bellomo and L. Preziosi [2], I. Ramis-Conde, M. Chaplain and A. Anderson [3], T. Roose, S.J. Chapman and P.K. Maini [4], A. Bru and M.A. Herrero [5]. These models are not very practical for their lack of representation of real biological and medical variables and because of the complexity of taking the significant variables or the main mathematical effort necessary to work with them.
In a different approach a tumor can be understood as a kind of ecosystem, in which different species fight for food and space to live. This kind of model supporting the notion that heterogeneous tumors are more like ecological systems than integrated tissues was proposed by J.D. Nagy [6]. Our model is similar. We treat healthy cells and malignant ones as two competing populations, while the response of the immune system is like a presence of a predator which hunts only one of the species (in this case tumor's cells). This response consist of increment of the concentration of macrophages in the area of the tumor. Healthy cells compete with the cancerous ones for nutrients (glucose and oxygen) and for space.

As this model approaches to the problem of cancer from a tissue point of view, it has not the inconveniences mentioned before. We collaborate with oncologists who provide samples of biopsy, so we can compare our theoretical results with the real situation.

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Local Controllability of Nonlinear Dynamical Systems

Radek Cibulka

Keywords: *Open-mapping principle, nonlinear systems, constrained controls, exact controllability.*
MSC2000 Classification: 34H05, 93B05.

Abstract

We consider an infinite-dimensional dynamical system described by a nonlinear abstract differential equation. The controls are subject to constraints given by a closed convex subset of $L^\infty([0,T],U)$, where U is a real Banach space and $T > 0$. Using a generalized Graves theorem, sufficient conditions for constrained exact local controllability in time T are proved.

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A PDE model applied to Conditional Image Filtering

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Keywords: *digital images, diffusion, anisotropic filtering*
AMS Subject Classification: 68U10, 35K65.

Abstract

In this work, a theoretical framework for the conditional filtering of digital images is presented. The most simple diffusion model is the heat equation which is linear and isotropic. However, isotropic diffusion is not well suited for natural images since it does not preserve the location and direction of edges which convey the main perceptual information of the image. In [1] and [3], a first models based on a non linear PDE equation were introduced in order to favor diffusion directions adapted to the local geometry of the image. Different approaches have been proposed to solve this problem for color images by extrapolating the idea of the anisotropic diffusion for a grey level images to vector-valued images (see [4] for instance). Then, the diffusion or filtering of each channel is conditioned to a direction which normally takes into account information from all channels. In [2] we propose the PDE

$$u_t = D^2u\left(\frac{W}{||W||}, \frac{W}{||W||}\right) + f(||W||)D^2u\left(\frac{W^\perp}{||W||}, \frac{W^\perp}{||W||}\right),$$

where $W(x,t)$ is a vector field giving the direction of the diffusion process and f is a positive smooth decreasing function. In our approach, the diffusion model assumes the *a priori* knowledge of the diffusion direction during all the process.
The consistency of the model is shown by proving the existence and uniqueness of solution for the proposed equation from the viscosity solutions theory. Also a numerical scheme adapted to this equation based on the neighborhood filter is proposed. Finally, we discuss several applications and we compare the corresponding numerical schemes for the proposed model.

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The stability of a unique symmetric and periodic solution of the ordinary differential equation

NATALIYA DILNA AND MICHAL FEČKAN

Keywords: *periodic solution, symmetric systems, stability of solution, k-hyperbolicity.*
MSC2000 Classification: 34C14, 34C15, 34C25.

Abstract

We consider equations of the form

$$x'(t) = \varepsilon f(x,t),\ t \in [0,T], \tag{1}$$

subjected to the periodic and symmetric condition

$$x(T) = Ax(0),$$

where ε is a small parameter, A is a matrix fulfiling $A^p = \mathbb{I}$ for some $p \in \mathbb{N}$, f is a continuous function, symmetric in x for arbitrary t and pT-periodic in t for arbitrary x in the following sense

$$Af(x,t) = f(Ax, t+T).$$

We establish conditions under which equation (1) has a unique symmetric and periodic solution, i.e. $Ax(t) = x(t+T)$. Moreover, we study its stability. These theorems are continuations of [1,3,4] and are generalizations of the antiperiodic case $A = -\mathbb{I}$ of [2].

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Existence of solutions for some quasilinear elliptic systems

Zakaria El Allali , El Bekkaye Mermri and Najib Tsouli

Keywords: *Boundary value problem, truncation,* L^1*, Laplacian, spectrum.*
MSC2000 Classification: 35J15, 35J70, 35J85.

Abstract

We are concerned here with the existence of solutions for quasilinear elliptic systems such as

$$\begin{cases} -\Delta u = au + bv + f_1(x, u, v, \nabla u, \nabla v) + \mu_1 & \text{in } \Omega, \\ -\Delta v = bu + dv + f_2(x, u, v, \nabla u, \nabla v) + \mu_2 & \text{in } \Omega, \\ u = v = 0 \quad \text{on } \partial\Omega, \end{cases} \tag{1}$$

where Ω is a bounded open set in $\mathbb{R}^N$, $N \geq 2$, $1 < p < +\infty$, a, b, d are given real numbers $\mu_i (i = 1, 2)$ is a Radon measure on Ω and $f_1, f_2 : \Omega \times \mathbb{R} \times \mathbb{R}^N \to \mathbb{R}$ are assumed to be Carathéodory functions. We will prove the existence of a solution $(u, v) \in (H_0^1(\Omega))^2$, if and only if the signed measure $\mu_i (i = 1, 2)$ is zero on sets of capacity zero in Ω. (i.e $\mu_i(E) = 0$ $(i = 1, 2)$ for every set E such that $\text{cap}_p(E, \Omega) = 0$).

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Positive Solutions for Boundary-Value Problems of Nonlinear Fractional Differential Equations

MOUSTAFA EL-SHAHED

Keywords: *Fractional differential equations, Krasnoeselskii's fixed point.*
MSC2000 Classification: 34B15, 34B16.

Abstract

In this paper, we investigate the problem of existence and nonexistence of positive solutions for the nonlinear boundary value problem of fractional order:

$$D^{\alpha}u(t) + \lambda\, a(t)f(u(t)) = 0,\;\; 0 < t < 1, \quad n-1 < \alpha \leq n,$$
$$u(0) = u''(0) = u'''(0) = ... = u^{(n-1)}(0) = 0,\;\; ,\nu\; u'(1) + \delta\; u''(1) = 0,$$

where D^{α} is the Caputo's fractional derivative [1, 2] and λ is a positive parameter. By using Krasnoesel'skii's fixed-point theorem of cone preserving operators[3, 4, 5, 6], we establish various results on the existence of positive solutions of the boundary value problem. Under various assumptions on $a(t)$ and $f(u(t))$, we give the intervals of the parameter λ which yield the existence of the positive solutions. An example is also given to illustrate the main results.

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Permanence for n-dimensional nonautonomous Lotka-Volterra cooperative systems with loop structure

YOICHI ENATSU

Keywords: *Lotka-Volterra cooperative system; Permanence; Nonautonomous; Population dynamics; Loop structure.*
MSC2000 Classification: : 34A34.

Abstract

In this paper, we study a class of non-autonomous n species Lotka-Volterra cooperative population systems with time delays as follows.

$$\begin{aligned}\frac{dx_1(t)}{dt} &= x_1(t)\left[r_1(t) - \sum_{j=1}^{n-1}\sum_{l=0}^{m} a_{1j}^l(t)x_j(t-l) + \sum_{l=0}^{m} a_{1n}^l(t)x_n(t-l)\right],\\ \frac{dx_i(t)}{dt} &= x_i(t)\left[r_i(t) - \sum_{j=1, j\neq i-1}^{n}\sum_{l=0}^{m} a_{ij}^l(t)x_j(t-l) + \sum_{l=0}^{m} a_{i\,i-1}^l(t)x_{i-1}(t-l)\right],\ i = 2,\ldots,n.\end{aligned}$$

x_i denotes the number of individuals in the ith population. $r_i(t)$ and $a_{ij}^l(t)$ for $1 \leq i, j \leq n$ and $0 \leq l \leq m$ are continuous, bounded and strictly positive functions on $[-m, +\infty]$. The model illustrates loop structure which is commonly seen in marine system. We establish sufficient conditions under which the system is permanent. Our results take advantage of the fact that the conditions need no restriction of the time delays l. In recent work, the 2-dimensional Lotka-Volterra cooperative systems are well understood by [1]. This research is of interest in that all species does not extinct under biologically natural conditions regardless the time delay.

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Secondary structure assignment using entropy

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Keywords: *secondary structure assignment; entropy; carbon alpha distance; torsion angle.*
MSC2000 Classification: 92B05, 92B15, 92D20.

Abstract

The automatic assignment of the protein secondary structure from three dimensional coordinates is an essential step in the characterization of protein structure. Although the recognition of secondary structure elements as alpha helices and beta sheets seem straightforward, but there are many different definitions, each regarding different criteria. We introduce a new algorithm for the protein secondary structure assignment based on a number of geometric parameters and by using the entropy, the sequence of protein is partitioned to segments. Then the secondary structure elements are assigned to each of these segments. It is shown that if the entropy of a segment increases then the regularity in the structure decreases. So it is concluded that the concept of entropy could be used as a measure of regularity of the secondary structure.

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Dirichlet problems in graphs

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Keywords: *Simple and connected graphs, mean value property, harmonic functions.*
MSC2000 Classification: 34B45.

Abstract

Let $G(V, E)$ be a simple graph, B be a subset of V and $T : V \to \mathbb{R}$ be a function. Let $A(u)$ be the set of adjacent vertices of u and $|A(u)|$ be its cardinal. We say that T has the mean value property in $V \setminus B$ if

$$T(u) = \frac{\sum_{v \in A(u)} T(v)}{|A(u)|} \quad \forall u \in V \setminus B.$$

If $G(V, E)$ is connected, then for all $B \subset V$ there is one and only one $T : V \to \mathbb{R}$ taking prefixed values in B and having the mean value property in $V \setminus B$. When $G(V, E)$ is a square or triangular grid we present some MATLAB programs to obtain functions with prefixed values in the vertices of the border with the mean value property in the remainder vertices. We also prove that the solutions in square and triangular grids are discrete forms of the solutions of classical Dirichlet problems respectively in square and triangular plates.

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Mathematical Models in Engineering, Biology and Medicine.
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Boundary value problems for strongly nonlinear multivalued equations involving different Φ-Laplacians

LAURA FERRACUTI AND FRANCESCA PAPALINI

Keywords: *Φ-Laplacian, nonlinear boundary conditions, lower and upper solutions, fixed point techniques, nonlinear differential operators.*
MSC2000 Classification: Primary: 34B15; Secondary: 34A60, 34C25.

Abstract

We investigate the existence of solutions to the scalar differential inclusion

$$(D(x(t))\Phi(x'(t)))' \in G(t, x(t), x'(t)) \text{ a.e. } t \in I = [0, T],$$

where $D(x)$ is a positive and continuous function, $G(t, x, x')$ is a Carathéodory multifunction and the increasing homeomorphism Φ can have a bounded domain of the type $(-a, a)$ or it can be the p-Laplacian operator. Using fixed point techniques combined, in some cases, to the method of lower and upper solutions, we prove the existence of solutions satisfying various boundary conditions.

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Discontinuous first-order functional boundary value problems

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Keywords: *Functional differential equations; discontinuous differential equations; boundary value problems.*
MSC2000 Classification: 34B15, 34B16.

Abstract

We consider the existence of extremal (a least and a greatest) absolutely continuous solutions to functional problems of the type

$$\begin{cases} x'(t) = f(t, x(t), x) \text{ for almost all (a.a.) } t \in I = [t_0, t_0 + L], \\ B(x(t_0), x) = 0, \end{cases} \tag{1}$$

where $t_0 \in \mathbb{R}$, $L > 0$, and f and B may be discontinuous with respect to all of their arguments. Interesting particular cases of (1), which may serve as a motivation for it, are periodic integro-differential problems of the form

$$\begin{cases} x'(t) = f\left(t, x(t), \int_{t_0}^{t_0+L} k(t,s)x(s)ds\right) \text{ for a.a. } t \in I, \\ x(t_0) = x(t_0 + L), \end{cases}$$

where k is a nonnegative kernel. Similar problems have been recently considered by many authors, see for instance [1], [2], [3], [4], [5]. Other types of functional equations, such as delay differential equations, differential equations with maxima, and so on, are also particular cases of the differential equation in (1). Similarly, the functional boundary condition in (1), which already featured in [6], [7], allows us to study in an unified way the usual conditions, such as initial or periodic, together with some other more sofisticated types of them.

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Nodal and multiple constant sign solutions for equations concerning p-Laplacian

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Keywords: *Scalar p-Laplacian, eigenvalues, $(S)_+$-operator, local minimizer, positive solution, nodal solution.*
MSC2000 Classification:

Abstract

We consider nonlinear elliptic equations driven by the p-Laplacian with a nonsmooth potential (hemivariational inequalities). We obtain the existence of multiple nontrivial solutions and we determine their sign (one positive, one negative and the third nodal). Our approach uses nonsmooth critical point theory coupled with the method of upper-lower solutions.

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Asymptotic analysis in the Prandtl' problem

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Keywords: *Nonlinear boundary value problems, thin layer, asymptotics.*
MSC2000 Classification: 35G30, 74C10.

Abstract

The non-linear system of five equations in $\Omega = \{(x_1, x_2) : -l_1 < x_1 < l_1,\ -l_2 < x_2 < l_2\}$; $\alpha = l_2/l_1 \ll 1$:

$$-\frac{\partial p}{\partial x_1} + \frac{\partial s_{11}}{\partial x_1} + \frac{\partial s_{12}}{\partial x_2} = 0, \quad -\frac{\partial p}{\partial x_2} - \frac{\partial s_{11}}{\partial x_2} + \frac{\partial s_{12}}{\partial x_1} = 0, \quad s_{11}^2 + s_{12}^2 = \tau^2 = \text{const},$$
$$s_{11}\left(\frac{\partial v_1}{\partial x_2} + \frac{\partial v_2}{\partial x_1}\right) = 2s_{12}\frac{\partial v_1}{\partial x_1}, \quad \frac{\partial v_1}{\partial x_1} + \frac{\partial v_2}{\partial x_2} = 0$$

coupled with two boundary conditions $v_2|_{x_2=\mp l_2} = \pm V = \text{const}$, simulates an engineering process of quasistatic compression between two absolutely rigid rough plates $x_2 = \mp l_2$ of thin perfectly plastic layer with shear yield stress τ and is called the Prandtl' problem (see, for example, [2, 3]). Here p is the pressure, s_{11}, s_{12} are the components of stress deviator tensor, v_1, v_2 are the components of velocity vector. The generalized solution of the Prandtl' problem

$$p = p_0 - \frac{\tau}{l_2}\left(sm_0x_1 + \sqrt{l_2^2 - m_0^2x_2^2}\right), \quad s_{11} = \frac{\tau}{l_2}\sqrt{l_2^2 - m_0^2x_2^2}, \quad s_{12} = -\frac{\tau}{l_2}sm_0x_2,$$
$$v_1 = \frac{V}{l_2}\left(x_1 + \frac{2s}{m_0}\sqrt{l_2^2 - m_0^2x_2^2}\right), \quad v_2 = -\frac{V}{l_2}x_2, \quad s = \text{sign}\, x_1, \quad p_0, m_0 = \text{const}$$

where m_0 is the coefficient of plates roughness, was constructed under the static hypothesis that the function s_{12} (shear stress) linearly depends on x_2. This solution is valid far from the sections $x_1 = \pm l_1$ (boundary effect domains) and $x_1 = 0$.
In this work, on the basis of asymptotic analysis with the geometric low parameter α, the exact solution (in sense of finiteness of asymptotic terms) coinciding with the generalized Prandtl' solution is obtained by unique way without some kind of hypotheses [1]. An illegality of these asymptotics near the section $x_1 = 0$ is accurately shown as well as the other, internal expansion, is constructed. Two possible variants of joining of the mentioned expansions in the section $x_1 = \pm l_2$ are realized.

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On the *clamped grid* problem in polygonal domains

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Keywords: *Orthotropic plate, fourth order elliptic, clamped boundary conditions, domain with corner, spectrum, regularity.*
MSC2000 Classification: 35J35, 35J40, 34L16, 74E10, 74G15, 74G40, 74G65, 74K10, 74K20.

Abstract

A model for small deformations of a thin isotropic elastic plate is $u_{xxxx} + 2u_{xxyy} + u_{yyyy} = f$. Here f is a force density and u is the vertical displacement of a plate; the model neglects the influence of horizontal deviations. Non-isotropic elastic plates are still modeled by fourth order differential equations but the coefficients in front of the derivatives of u may vary. Two interesting extreme cases are

$$L_1 = \frac{\partial^4}{\partial x^4} + \frac{\partial^4}{\partial y^4} \quad \text{and} \quad L_2 = \frac{1}{2}\left(\frac{\partial^4}{\partial x^4} + 6\frac{\partial^4}{\partial x^2 \partial y^2} + \frac{\partial^4}{\partial y^4}\right). \tag{1}$$

One may think of these operators as of the operators appearing in the model of an elastic medium consisting of two sets of intertwined (not glued) perpendicular fibers: L_1 for fibers running in cartesian directions and L_2 for the diagonalized ones. We call such medium *a grid*.
Let $\Omega \subset \mathbb{R}^2$ be open and bounded domain and have a corner in $0 \in \partial\Omega$ with the opening angle $\omega \in (0, 2\pi]$. The boundary value problem we pose for the operators L_1 and L_2 defined by (1) is a homogeneous Dirichlet problem:

$$\begin{cases} L_i u = f & \text{in} \quad \Omega, \\ u = \frac{\partial}{\partial \nu} u = 0 & \text{on} \quad \partial\Omega \backslash \{0\}, \quad i = 1, 2. \end{cases} \tag{2}$$

Here ν is the unit outward normal vector on $\partial\Omega$ and the boundary conditions in (2) correspond to the clamped situation meaning that the vertical position and the angle are fixed to be 0 at the boundary. This problem we call *a clamped grid model*.
A recent paper of Kawohl and Sweers [3] concerned the positivity question for L_1 and L_2 in a rectangular domain for hinged boundary conditions. We did not find any other references to these special operators in literature.
We use the Kondratiev theory (see the seminal paper [4] or the monographs [2], [5]) to study the regularity of solution to (2) depending on the opening angle ω of the corner. We compare our results with those known for the Dirichlet problem (2) for the biharmonic operator Δ^2 (see e.g. [1], [2]).
We end up with the numerical solutions to (2) obtained by FreeFem++ package [6] when $\Omega \subset \mathbb{R}^2$ is a square, a rectangle, an L-shaped domain and a pentagonal domain of a special form and f is assumed to be the point load.

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Multiple Positive Solutions in the Sense of Distributions of Singular BVPs on Time Scales.

ANA GÓMEZ GONZÁLEZ AND VICTORIA OTERO–ESPINAR

Keywords: *Dynamic Boundary Value Problem, Emden–Fowler equation, Solutions in the sense of distributions, Variational Methods.*
MSC2000 Classification: 39A10.

Abstract

The Emden–Fowler equation in time scale

$$u^{\Delta\Delta}(t) + q(t)\, u^{\alpha}(\sigma(t)) = 0,\ t \in (0,1)_T, \tag{1}$$

arises in the study of gas dynamics and fluids mechanics, and in the study of relativistic mechanics, nuclear physics and chemically reacting system; see, e.g. [3] and the references therein, for the continuous model. The negative exponent Emden–Fowler equation ($\alpha < 0$) has been used in modelling non–Newtonian fluids such as coal slurries [2]. The physical interest lies in the existence of positive solutions. We are interested in a broad class of singular problem that includes those related with equation (1) and the more general

$$u^{\Delta\Delta}(t) + q(t)\, u^{\alpha}(\sigma(t)) = g(t, u^{\sigma}(t)),\ t \in (0,1)_T, \tag{2}$$

subject to homogeneous Dirichlet boundary conditions.
We will use the results obtained in [1] to prove the existence of multiple positive solutions in the sense of distributions to a singular second order dynamic equation with homogeneous Dirichlet boundary conditions, which includes those problems related with the negative exponent Emden–Fowler.

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Numerical integration of nonlinear boundary value problems arising in nucleation theory

JOSÉ C. GRAÑA AND IGNACIO E. PARRA

Keywords: *nonlinear BVP, BVP on infinite intervals, functional equations.*
MSC2000 Classification: 34B15, 34B40, 39B22.

Abstract

In nucleation theory, the rate of formation of new nuclei inside a metastable fluid takes the general form $\bar{J} = \bar{J}_0 \mathrm{e}^{-\Omega^*}$. Thus, the main features of the different nucleation approaches are determined by the definition of the so-called nucleation barrier, Ω^*. In the approaches developed in the framework of the density functional theory of capillarity, such a nucleation barrier is obtained as a saddle-noddle stationary point of a free-energy functional of the number density, $\rho(\mathbf{r})$, of an inhomogeneous system. In some models, the corresponding critical density profiles, $\rho^*(r)$, are obtained as non trivial solutions of BVP's of the form, [1], [2] and [3]:

$$\left.\begin{array}{l} \dfrac{1}{r}\dfrac{\mathrm{d}^2(r\, g(\rho^*))}{\mathrm{d}r^2} = (\mu(\rho^*) - \mu(\rho_e)) \\ \dfrac{\mathrm{d}\rho^*(0)}{\mathrm{d}r} = 0, \qquad \rho^*(\infty) = \rho_e \end{array}\right\}.$$

The function μ is the substance chemical potential and is such that the second member of the above equation is null for three different values of constant density $\rho_e \neq \rho_m \neq \rho_i$. The function g is a twice differentiable monotone function of density, which depends on the considered density functional model. In this work, a shooting procedure to obtain the nontrivial solutions of these BVP's and the corresponding nucleation barriers is proposed. This method is more robust and efficient than the usually employed successive approximation schemes, that are generally unstable owing to the saddle-noddle character of critical solutions, [2]. The work also includes some needed results about the asymptotic behavior at zero and infinity of the solutions of the above problem, [4].
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Boundary Value Problems for Abstract Equilibrium Transport Equations

William Greenberg

Keywords: *Transport equations, transport theory, bisemigroups.*
MSC2000 Classification: 35Q72, 82B40.

Abstract

Boundary value problems for stationary transport equations describing a number of different transport processes in half space or slab geometry may be modeled as bisemigroup evolution equations. [1] Such processes include neutron transport, kinetic theory of gases, radiative transfer and electron transport, among others. [2] We investigate the unique solvability of the abstract transport equation $T\psi'(x) = -A\psi(x)$ where A and T are operators on a Hilbert or Banach space, $T^{-1}A$ is the generator of a bisemigroup, and the left (resp. right) side of the equation describes free streaming (resp. collisions). When the operator A is unbounded, there is a natural imbedding in a Krein space structure. [3] The limitation in the analysis is the dearth of perturbation results in bisemigroup theory. We will present two such perturbation results, demonstrate application, and point to a number of open problems.

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The Analysis of Protein Structures Based on Graph Theory

MAHNAZ HABIBI, CHANGIZ ESLAHCHI, HAMID PEZESHK AND MEHDI SADEGHI

Keywords: *graph theory; accessible surface area; molecular surface area; hydrophobic core.*
MSC2000 Classification: 92B05, 92B15, 92D20.

Abstract

The analysis of protein structure is a main challenging problem in bioinformatics allowing detailed exploration of the biological function. There are several features of protein structure which help to predict the protein function. In this work; we present a method based on graph theory for analysis of protein structure. Corresponding to a protein, P, and using the carbon alpha coordinates of each amino acids of P, we consider the Delaunay triangulation of P as the graph G(P). Using the properties of G(P), we design a fast and precise algorithm which calculate main characteristics of P, such as packing density, hydrophobic core and molecular surface area of P. There are different methods to obtain solvent accessible surface area using a probe ball and geometrical criteria. For finding the molecular surface area of P, the radius of the probe ball should be decreased to zero. But decreasing the radius of the probe ball causes an increase in running time of algorithm. We compare our algorithm with GETAREA reported at http://pauli.utmb.edu. We observe that by decreasing the radius of the probe ball in this algorithm, the agreement between the results of our algorithm and the above mentioned one is around 100%.

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Periodic boundary value problem for third order functional differential equations

ROBERT HAKL

Keywords: *Periodic boundary value problem, third order functional differential equations*
MSC2000 Classification: 34K10.

Abstract

Consider the third order functional differential equation

$$u'''(t) = F(u)(t) \qquad \text{for } 0 \leq t \leq \omega, \tag{1}$$

where $F : C([0, \omega]; R) \to L([0, \omega]; R)$ is a continuous operator satisfying Carathéodory condition. For a special class of operators F, we will establish efficient conditions sufficient for the existence of a solution to (1) satisfying the periodic boundary conditions

$$u^{(i)}(0) = u^{(i)}(\omega) \qquad (i = 0, 1, 2). \tag{2}$$

The uniqueness of a solution to (1), (2) will be discussed, as well. The results obtained will be specified for the nonlinear differential equations with deviations.

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Solvability of four-point boundary value problem with jumping nonlinearities

GABRIELA HOLUBOVÁ AND PETR NEČESAL

Keywords: *Four-point problem, Fučík spectrum.*
MSC2000 Classification: 34B10, 34B15, 34L05.

Abstract

We study the four-point boundary value problem

$$\begin{cases} u'' + \alpha u^+ - \beta u^- + g(u) = f, & x \in (0,1), \\ u'(0) = u'(\xi),\ u(1) = u(\eta), \end{cases}$$

with jumping nonlinearities $\alpha u^+ - \beta u^-$ and sublinear term $g(u)$. We discuss the solvability with respect to the Fučík spectrum $\Sigma(L)$ of the corresponding linear differential operator. Namely, we consider the non-resonance case $(\alpha, \beta) \notin \Sigma(L)$, where the situation differs in regions of type I and in regions of type II, and the resonance case $(\alpha, \beta) \in \Sigma(L)$. Further, we mention some bifurcation results for special case $g(u) \equiv 0$, $f(x) \equiv 1$.

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Fuzzy optimization model for land use change

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Keywords: *fuzzy optimization models, land use change,Full fuzzy system.*

MSC2000 Classification: 34B15, 34B16.

Abstract

Land is the stage on which all human activity is being conducted and the source of the materials needed for this conduct. Land use being shaped under the influence of two broad sets of force- human needs and environmental features and processes. The magnitude of land use change varies with the time period being examined as well as geographical area. There are some important questions in Land use change literature, for instance "How much land to allocate to each of a number of land use type in order to maximize of (household or individual) rent -paying ability, minimization of environmental impacts or maximization of population income". In this paper, we investigate that and propose mathematical models to find an answer for these questions. Since most of the parameters in this process are linguistics and fuzzy logic is a powerful tools to handle them, a fuzzy linear programming model is used in model building. To this end, Fuzzy Linear Programming (FLP) with fuzzy related system of constraint and fuzzy coefficient vector in the objective function, that is a full fuzzy system of simultaneous equations with fuzzy objective function is discussed. The related production operations in the objective function and in the constrains are performed in the basis of standard production between fuzzy numbers. The constraint which can be take into account depend on the case but representative objective include : Lower and upper limits on land use, availability of Labour and so on.

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Boundary value problems for second order differential equations

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Keywords: *Nonlinear boundary conditions, second order differential equations, causal operators, differential inequalities with positive linear operators, monotone iterative method, extremal solutions, unique solution.*
MSC2000 Classification: 34A45, 34B15.

Abstract

Let $J = [0, T],\ \ E = C(J, \mathbb{R})$ and $Q \in C(E, E)$. In this paper, we investigate nonlinear four–point boundary value problems for second order differential equations with a causal operator Q of the form

$$\begin{cases} x''(t) &= (Qx)(t), \quad t \in J, \\ 0 &= g_1(x(0), x(\delta)), \quad 0 < \delta < T, \\ 0 &= g_2(x(T), x(\gamma)), \quad 0 < \gamma < T, \end{cases} \tag{1}$$

where $g_1, g_2 \in C(\mathbb{R}\times\mathbb{R}, \mathbb{R})$. Functional equations with causal operators are discussed in book [1], see also the references therein. To obtain approximate solutions of nonlinear differential problems we can apply the monotone iterative technique. This technique can be used both initial and boundary problems. Recently, it is also applied to first order differential equations with causal operators, see [4] (nonlinear boundary conditions). This paper extends the application of this method to nonlinear four-point boundary problems for second order differential equations with causal operators. In this paper we also discuss differential inequalities with positive linear operators. We formulate sufficient conditions which guarantee that problem (1) has extremal solutions assuming an one–sided Lipschitz condition (with corresponding linear operators) on the causal operator Q. The problem when (1) has the unique solution is also investigated. An example is added to illustrate the theoretical results.

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Toward fuzzy differential inclusions

N.A. KIANI AND L.JAHANSHAHLOO

Keywords: *Fuzzy differential Equations, Differential transformation method,Generalized differentiability.*

MSC2000 Classification: 34B15, 34B16.

Abstract

There are some situations in the real word which can not be modeled by known methods. To get rid of this difficulty, the concept of Fuzzy Differential Inclusion (FDI) is introduced. An extension of Differential Transformation Method (DTM) which is an analytical-numerical method for solving the Fuzzy Differential Equation (FDE), is proposed. For implementing the method ,the concept of generalized H-differentiability is considered. Also a method to obtain fuzzy partition is presented. Proposed algorithm is illustrated by numerical examples.

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Bifurcation Conditions for Perturbed Fredholm Impulsive Boundary Value Problems

MARTINA LANGEROVA

Keywords: *Ordinary differential equation, impulsive action, boundary value problem, Laurent series, Fredholm operator.*
MSC2000 Classification: 34B05, 34B37.

Abstract

Perturbed boundary value problems for linear ordinary differential equations [1] with finite number of impulses [2, 3] in solutions are considered. Under the assumption that the unperturbed boundary value problem has no solution, it is proved that the problem possesses a ρ-parameter family of linear independent solutions in the form of Laurent series in powers of small parameter ε ($\rho = m - n$ where n is the dimension of the differential system and m is the number of boundary conditions). Bifurcation conditions of such solutions are established and an algorithm for construction of these solutions using the Vishik-Lyusternik method is suggested (see [4]).
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A general fixed point method for the study of dynamical systems

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Keywords: *Periodic solutions, Delay equations, PDEs, ODEs, Equilibria, Fixed Points.*
MSC2000 Classification:

Abstract

In this talk, we introduce a general fixed point method to prove the existence of dynamical system objects such as periodic solutions of delay equations, equilibria of PDEs and periodic solutions of ODEs. The central idea is to construct the so-called radii polynomials, which are used to verify the hypotheses of a contraction mapping fixed point theorem in a computationally efficient way. The resulting fixed point yields the wanted dynamical system object.

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Numerical methods for singular boundary value problems involving the one-dimensional p-laplacian

PEDRO LIMA AND LUISA MORGADO

Keywords: *Singular boundary value problems, p-laplacian*
MSC2000 Classification: 65L05.

Abstract

In this work our concern is to find the positive solution of the generalized Emden-Fowler equation

$$\left(|g'(u)|^{p-2}\, g'(u)\right)' = a u^{\sigma} g^{n}\,(u)\,, \quad 0 < u < u_0, \tag{1}$$

where $n < 0$, $a < 0$, $p > 1$, $u_0 > 0$ and $\sigma \in \mathbb{R}$, that satisfies the boundary conditions

$$\begin{array}{c} g'(0) = 0 \\ g(u_0) = \lim\limits_{u \to u_0^-} \left[(u_0 - u)\, g'(u) \right] = 0. \end{array} \tag{2}$$

The differential operator on the left-hand side of (1) is the one-dimensional p-laplacian, $\triangle_p g$, which reduces to the classical laplacian when $p = 2$ and, for $p \neq 2$, is used in nonlinear models of physical phenomena, as for example, the deformation of a nonlinear elastic membrane and problems arising in non-newtonian fluid mechanics: the case $1 < p < 2$ corresponds to pseudoplastic fluids and the case $p > 2$ to dilatant fluids.
In [1] and [2], asymptotic expansions of the one-parameter families of solutions satisfying the boundary conditions at the singular endpoints were obtained and stable shooting algorithms were constructed.
In this work, we will present alternative computational methods for the numerical solution of this problem and we compare the numerical results with the previous ones.

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Observation and control in a model of a cell population affected by radiation

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Keywords: *Observation, control, mathematical model, radiation.*
MSC2000 Classification: 34H05, 93B07.

Abstract

The effect of radiation on a cell population is described by a two-dimensional nonlinear system of differential equations. If the radiation rate is not too high, the system is known to have an asymptotically stable equilibrium (Freedman, 2008). First, for the monitoring of this effect the concept of observability is applied. For the case when the total number of cells is observed, without distinction between healthy and affected cells, a so-called observer system is constructed, which, at least near the equilibrium state, makes it possible to recover the dynamics of both the healthy and the affected cells, from the observation of the total number of cells without distinction. For former applications of the concept of observability in population systems, and the construction of an observer system see Varga et al. 2003, López et al. 2007.

If we want to control the system into a required new equilibrium state, and maintain this new equilibrium by a constant control, we can apply a technique of theory of optimal control (see Rafikof et al. 2007) to construct a feedback control system.

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Bifurcation of limit cycles from a 2-dimensional center in control systems

JAUME LLIBRE AND AMAR MAKHLOUF

Keywords: *Limit cycle, bifurcation, control system, averaging method.*
MSC2000 Classification: 34C05, 34A34, 34C14.

Abstract

We study the bifurcation of limit cycles from the periodic orbits of linear differential system in $\mathbb{R}^n$ perturbed inside a class of piecewise linear differential system, which appear in a natural way in control theory. Our main result shows that at most one limit cycle can bifurcate up to first order expansion of displacement function with respect to small parameter. This upper bound is reached. For proving this result, we use the averaging theory in the form when the differentiability of the system is not needed.

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Weak solutions of doubly degenerate diffusion equations

ALEŠ MATAS AND JOCHEN MERKER

Keywords: *p-Laplacian, doubly nonlinear PDE, weak solution, Faedo–Galerkin method.*
MSC2000 Classification: 35B05, 35K55, 35K65.

Abstract

Recently, there are two widely studied evolution partial differential equations which could describe the nonlinear diffusion process. It is porous media equation

$$\dot{u} = \Delta u^{m-1} + f$$

and evolution p-Laplacian equation

$$\dot{u} = \Delta_p u + f.$$

The generalized doubly nonlinear diffusion equation has the following form

$$\dot{u} = \Delta_p u^{m-1} + f. \tag{1}$$

One of the common strategies how to proof the existence of a weak solution is based on the projection of the equation on a finite dimensional space together with the compactness method provided in [1] which uses the monotonicity of the nonlinear operator. In such case, the compactness over special set of approximated solutions can be proved.
We prove in [2] the existence of a weak solution. We use Faedo–Galerkin method together with the mentioned compactness argument. Instead of the original problem we consider the implicit integral version of the equation (1). Moreover, we study the problem on general unbounded domain.

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The behavior of a population under the effects of the propagation of an epidemie with fixed infection length

K. NIRI, I. IGGIDR AND E. MOULAY ELY

Keywords: Compartment model, epidemie, equilibrium, stability, oscillation.

Abstract

Many mathematical models in epidemiology, describe the propagation of the disease by a model of compartments by supposing that the population studied is divided into two or three principal categories: susceptible, infected, or removed. We often assume that the rates of transfer between compartments are instantaneous. When these transfers require a period of time, the epidemic models are governed by differential equations with delay, We can say for example an epidemic with infection period of fixed length.
A generally accepted idea on this type of modelling, is that the consideration of the delay in the modelling leads to the apportion of the fluctuations of solution (oscillations) around an equilibrium state.
We prove in this work that the existence of these fluctuations is not always automatic. We study separately a certain number of models.
We study models while going from simplest when the population is closed until most complex when the death rates and birthrate are different, or the epidemic is fatal for a proportion of the population in spite of the vaccination of some ones. We illustrate this work by graphs witch will translate our theory.

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Contractivity and global asymptotic stability for nonautonomous Lotka-Volterra systems with piecewise constant arguments

YUKIHIKO NAKATA

Keywords: *Lotka-Volterra systems, Logistic equation, Contractivity, Global attractivity, Piecewise constant arguments, Discrete population systems, Periodic solutions,*
MSC2000 Classification: : 34A34.

Abstract

We consider the following nonautonomous multi-species Lotka-Volterra systems with piecewise constant arguments

$$\frac{dN_i(t)}{dt} = N_i(t)r_i(t)\left(1 - a_i(t)N_i(t) - \sum_{j=1}^{n}\sum_{l=0}^{m} b_{ij}^{l}(t)N_j\left([t-l]\right)\right), n \leq t < n+1. \tag{1}$$

where l and m are integers such that $0 \leq l \leq m$ and $1 \leq j \leq n$. Logistic equation with a piecewise constant argument which models the dynamics of a population has been studied by a number of papers (see, for example [1-5] and the references therein). These differential systems has the property of continuous dynamical systems within intervals of time unit length and describe hybrid dynamical systems (a combination of continuous and discrete time). We obtain a set of sufficient conditions for global asymptotic stability of solution even if the term of piecewise constant delays dominate over the term of instantaneous feedback and our results generalize and extend previous results [Y. Muroya, New contractivity condition in a population model with piecewise constant arguments, *J. Math. Anal. Appl.* **346** (2008) 65–81] to the nonautonomous n-species systems and really improves known earlier results with using a suitable Lyapunov-function technique. Moreover, we introduce the specific case that nonautonomous Lotka-Volterra systems with piecewise constant arguments has the periodic solution and discuss the global stability of the solution.

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On the Variety of the Fučík Spectrum Structure

GABRIELA HOLUBOVÁ AND PETR NEČESAL

Keywords: *Fučík spectrum, multi-point boundary value problem, non-selfadjoint operator.*
MSC2000 Classification: 34B10, 34B15, 34L05.

Abstract

We study the structure of the Fučík spectrum

$$\Sigma(L) = \{(\alpha, \beta) \in R^2 : \ Lu = \alpha u^+ - \beta u^- \text{ has a nontrivial solution}\}$$

for linear differential operators L, where $u^+ := \max\{u, 0\}$, $u^- := \max\{-u, 0\}$. We introduce the Fučík spectra in the case of non-selfadjoint ordinary differential operators of the second order which correspond to the four-point boundary value problem

$$\begin{cases} -u''(x) = \alpha u^+(x) - \beta u^-(x), & x \in (0, \pi), \\ u'(0) = u'(\xi), \quad u(\pi) = u(\eta), & \xi \in (0, \pi), \ \eta \in (0, \pi). \end{cases} \tag{1}$$

The non-selfadjointness results in new interesting patterns of the Fučík spectrum structure. Moreover, the complete analytical description of the non-trivial Fučík spectra provides us useful background to formulate new hypotheses.

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Mathematical Models in Engineering, Biology and Medicine.
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Exponential stability for a nonautonomous stage-structured population growth model

CARMEN NÚÑEZ, RAFAEL OBAYA AND ANA M. SANZ

Keywords: *Nonautonomous dynamical systems, infinite-delay differential equations, population models.*
MSC2000 Classification: 34B15, 34B16.

Abstract

We study the dynamical behavior of the trajectories defined by a recurrent family of monotone functional differential equations with infinite delay and concave nonlinearities. We analyze different sceneries which require the existence of a lower solution and of a bounded trajectory ordered in an appropriate way, for which we prove the existence of a globally asymptotically stable minimal set given by a 1-cover of the base flow. The properties we prove refine some previous analysis appearing in Zhao [6], Jiang and Zhao [2], Novo, Obaya and Sanz [4], and Novo, Núñez and Obaya [3].
We apply these results to the description of the long term dynamics of a nonautonomous model representing a stage-structured population growth without irreducibility assumptions on the coefficient matrices. This description extends to the nonautonomous setting some previous works of Freedman and Wu [1] and Wu, Freedman and Miller [5].

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Global stability for delayed neural network models

José J. Oliveira and Teresa Faria

Keywords: *delayed neural network models, global asymptotic stability, global exponential stability.*
MSC2000 Classification: 34K20, 34K25, 92B20.

Abstract

In this talk, we establish sufficient conditions for the existence and global asymptotic stability of an equilibrium point of the following neural network model with distributed delays

$$\dot{x}_i(t) = -\rho_i(x_i(t)) \left[b_i(x_i(t)) + \sum_{j=1}^{n} f_{ij}(x_{j,t}) \right], \ i = 1, \dots, n, \tag{1}$$

by assuming the existence of instantaneous negative feedbacks witch dominate the delay effect. The global exponential stability of the equilibrium is also addressed.
Some examples of neural network models, such as, Hopfield, Cohen-Grossberg, bidirectional associative memory, and static with S-type distributed delays are presented.

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Nonpositive solutions of a certain functional differential inequality

ZDENĚK OPLUŠTIL

Keywords: *Functional differential equation, boundary value problem, differential inequality.*
MSC2000 Classification: 34K06, 34K10.

Abstract

Some new effective conditions are found guaranteeing that every solution of the problem

$$u'(t) \geq \ell(u)(t), \quad u(a) \geq h(u)$$

is nonpositive, where $\ell : C([a,b];\mathbb{R}) \to L([a,b];\mathbb{R})$ is a linear bounded operator and $h : C([a,b];\mathbb{R}) \to \mathbb{R}$ is a linear bounded functional.
Further we concretize some results for the case when the operator ℓ have the following form

$$\ell(v)(t) \overset{def}{=} p(t)v(\tau(t)) - g(t)v(\mu(t)),$$

where $p, g \in L([a,b];\mathbb{R}_+)$ and $\tau, \mu : [a,b] \to [a,b]$ are measurable functions. So we obtain some new effective conditions for the solvability of the differential inequalities with deviating argument in this case.

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Boundary value problem for stationary case of generalized bi-stable equation

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Keywords: *bi-stable equation, boundary value problem, first integral.*
MSC2000 Classification: 34B15, 34B60.

Abstract

Our work is devoted to the stationary solutions of generalized bi-stable equation in the form

$$\begin{cases} u_t = (\varepsilon^p |u_x|^{p-2} u_x)_x - W'_\alpha(u), & x \in (0,1), \\ u_x(0) = u_x(1) = 0, & \text{for } t > 0. \end{cases}$$

with $C^{1,\gamma}$, $(0 < \gamma \leq 1)$ potential W. We aim to classical solution, its existence, multiplicity and bifurcations in dependence on parameters p, ε and various types of W. This analysis is supported by numerical experiments.

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Strongly nonlinear multivalued systems involving a singular Φ-Laplacian

FRANCESCA PAPALINI

Keywords: *Differential inclusion, Dirichlet problem, maximal monotone map, usc and lsc multifunction, Yosida approximation, variational inequalities.*
MSC2000 Classification: 34B15, 34A60.

Abstract

We study two vector problems with Dirichlet boundary conditions for second order strongly nonlinear differential inclusions involving a maximal monotone term. The first is governed by a nonlinear differential operator of the form $x \mapsto (\Phi(x'))'$, where Φ is an increasing homeomorphism defined on a bounded domain. In this problem the maximal monotone term need not be defined everywhere in the state space $I\!R^N$, incorporating into our framework differential variational inequalities. The second problem is governed by the more general differential operator of the type $x \mapsto (a(x)\Phi(x'))'$, where $a(x)$ is a positive and continuous scalar function. In this case the maximal monotone term is required to be defined everywhere. Using techniques from multivalued analysis and from nonlinear analysis, we prove the existence of solutions for both problems under convexity and nonconvexity conditions on the multivalued right-hand side.

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A Heuristic Algorithm for Haplotype Inference by Pure Parsimony

HADI POORMOHAMMADI, CHANGIZ ESLAHCHI, MEHDI KARGAR, LEILA PIRHAJI, MEHDI SADEGHI AND HAMID PEZESHK

Keywords: *genotype; haplotype inference; HIPP problem; heuristic algorithm.*
MSC2000 Classification: 92B05, 92B15, 92D20.

Abstract

Motivation: Haplotype are important informations in the study of complex diseases and drug design. However, due to technological limitations, genotype data rather than haplotype are usually obtained. Thus, haplotype inference from genotype data using computational methods is of interest for many researchers.
Results: There are several models for inferring haplotypes. One of the most important models is haplotype inference by pure parsimony (HIPP), consisting of finding the minimum number of haplotypes that can resolve all given genotypes. HIPP is an NP-hard problem. In this paper we propose a new heuristic algorithm for this problem. The heuristic algorithm accurately predicts an efficient Haplotype for inferring the remaining genotypes in each step. Results of applying our algorithm on a variety of biological and simulated data show that it is very effective with a high accuracy compared to other algorithms. Also a new measure for evaluating the effectiveness of the algorithms is introduced. This measure is based on the pure parsimony approach which seeks to find the minimum number of haplotypes for resolving the input genotypes.

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Periodic-Dirichlet Problems for dissipative hyperbolic systems and laser modeling

LUTZ RECKE

Keywords: *Fredholm property, anisotropic Sobolev spaces, well-posedness, no small denominators.*
MSC2000 Classification: 35B10, 35L50.

Abstract

This paper concerns linear hyperbolic systems of first order PDEs in one space dimension of the type

$$\left.\begin{array}{l}\partial_t u + \partial_x u + a(x)u + b(x)v = f(x,t)\\ \partial_t v - \partial_x v + c(x)u + d(x)v = g(x,t)\end{array}\right\} \; 0 \le x \le 1,\; t \in \mathbb{R}$$

with reflection boundary conditions $v(0,t) = r_0 u(0,t)$, $u(1,t) = r_1 v(1,t)$ and time-periodicity conditions. Here $r_0, r_1 \in \mathbb{R}$ are fixed numbers, $a, b, c, d : [0,1] \to \mathbb{R}$ are fixed (possibly discontinuous) coefficient functions, and the right-hand sides $f, g : [0,1] \times \mathbb{R} \to \mathbb{R}$ are supposed to be periodic with respect to t. In [1] it is proved the following: If

$$|r_0 r_1| \neq e^{\int_0^1 (a(x)+d(x))\,dx},$$

then the problem can be modeled by means of Fredholm operators of index zero between corresponding Sobolev spaces.
In the talk we will explain how to use this result for the description of Hopf bifurcation (for related numerical results see [2]) and forced frequency locking (for related finite dimensional results see [3]) in general semilinear hyperbolic systems and for semiconductor laser models (see e.g., [4]).

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Exact and approximate solutions of reaction-diffusion-convection equations

E. M. Rocha and M. M. Rodrigues

Keywords: *Nonlinear-reaction-diffusion-convection-equation, Nonuniform media, Lie symmetry, Nonclassical symmetry, Invariant surface equation.*
MSC2000 Classification: 35K55, 76M60, 22E15, 22E20, 54H15.

Abstract

A class of (1+1)-dimensional PDEs of reaction-diffusion-convection type in nonuniform media (i.e. with nonconstant rates) is studied from the Lie symmetry point of view, by considering a variant of the Bluman-Cole method (e.g., see [2]). Explicit solutions for such differential equations, with several types of data, are obtained.
Here, we consider a nonlinear reaction-diffusion equation with convection term of the form

$$u_t = [D(x)u_x]_x + C(x)u_x + R(x)f(u), \quad \text{for } (t,x) \in [0,T] \times \mathbb{R}, \tag{1}$$

where $u \equiv u(t,x)$ is the unknown function, $D(x)$, $R(x)$, $f(u)$ and $C(x)$ are arbitrary smooth functions, and the indices t and x denote differentiating with respect to these variables. The equation (1) generalizes a great number of well-known nonlinear second-order evolution equations, describing various processes in physics, chemistry and biology (e.g., see [5]). The construction of exact solutions for these equations is then of great importance. When the equations are invariant with respect to a (nontrivial) Lie group of transformations, exact solutions can be determined by exploiting this invariance. In general, the application of these groups methods involves tedious but rather mechanical computations, so symbolic packages are well suited for such computations (see [3] and [4]). Unfortunately, in many cases, the symmetry group obtained by the classical Lie method is rather trivial, which has pushed efforts to generalize Lie's original concept of symmetry (e.g., see [1],[2], [6], [7] and [8]).
In this work, we present a variant of Bluman-Cole method which seems more suitable for computing exact solutions of R-D-C equations in nonuniform media (x-dependence), as it is the case of equation (1).
We apply the proposed method to concrete examples, where the function rates are polynomial, exponential or Gaussian functions.

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Periodicity of solutions for fuzzy differential equations

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Keywords: *Fuzzy differential equations, Periodic solutions.*
MSC2000 Classification: 26E50, 03E72, 34B37.

Abstract

On many occasions, to predict future events, one has to take into account some factors which can be imprecise due to inexact physical measurements or unknown data. One of the approaches to the study of this type of models which considers the uncertainty inherent to certain processes is fuzzy modelling. The formulation of a fuzzy model, given, for instance, by a fuzzy differential equation, can be appropriated to analyze the behavior of a process subject to uncertainty (see [2–6]). Fuzzy models have been used in applications to economics, engineering, or information sciences, for instance. On the other hand, the importance of periodic phenomena in the study of a wide range of real processes is clear.

There exist several different points of view in relation with the concept of differentiable fuzzy-valued function. One of the most extended procedures is to consider differentiability in the sense of Hukuhara. However, this approach brings some limitations: the solutions of a fuzzy differential equation from this point of view have level sets whose diameter is necessarily a non-decreasing function with respect to the time variable. As a consequence, the study of periodic phenomena through fuzzy differential models is difficult to deal with.

In relation with this issue, the theory of almost periodic functions of real variable and fuzzy real values is developed in [1], where an application to the existence of almost periodic solutions of fuzzy (partial) differential equations is shown. Besides, [8] includes a study of the existence of almost periodic solutions and asymptotically almost periodic solutions for fuzzy functional-differential equations.

Other approaches to the problem of existence of periodic solutions to fuzzy differential equations consider the introduction of impulses. We present some results on the existence of solution to fuzzy differential equations subject to boundary value conditions considering differentiability in the sense of Hukuhara.

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Seasonal fluctuations and harvesting in the Pearl-Verhulst population model

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Keywords: *Population dynamics, modified logistic equation, periodic solutions, harvesting.*
MSC2000 Classification: 92D25, 34C25.

Abstract

Modeling biological populations in a deteriorating environment (that is, under the assumption that $\lim_{t\to\infty} K(t) = 0$), Hallam and Clark [1] mentioned that in this case analysis of a standard nonautonomous logistic equation

$$\frac{dx(t)}{dt} = r(t)x(t)\left[1 - \frac{x(t)}{K(t)}\right] \tag{1}$$

yields nonrealistic conclusions, namely, "a population which (because of a very small intrinsic growth rate) is barely able to persist under the best of conditions is able to do as well or better in an intolerable environment." Pointing out unfavorable role of the domination of the intrinsic growth $r(t)$ in Eq. (1), they suggested a modified nonautonomous logistic equation

$$\frac{dx(t)}{dt} = x(t)\left[r(t) - c\frac{x(t)}{B(t)}\right], \tag{2}$$

where r is the intrinsic growth rate, B is the maximum population the environment can support, and c is a positive parameter which measures the population response to environmental stress. This modification eliminates the problem with "a completely bizarre behavior" of solutions of Eq. (1) as $r < 0$ and yields the expected decay of solutions to zero as $t \to +\infty$ for Eq. (2), see [1]. Complementing the studies in [1], we establish existence of a unique positive asymptotically stable solution of Eq. (2) when both $B(t)$ and $r(t)$ are ω-periodic continuous functions, the carrying capacity $B(t)$ is positive, whereas the intrinsic growth rate $r(t)$ is allowed to take on nonpositive values [3]. The effects of proportional and constant yield harvesting on the dynamics of Eq. (2) are also discussed.

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On linear singular functional differential equations

ANDRÁS RONTÓ

Keywords: *On singular functional differential equations.*
MSC2000 Classification: 34B15, 34B16.

Abstract

The problem on the existence and localisation of solutions of singular linear functional differential equations is considered. The class of equations under examination contains, in particular, the differential equation with argument deviations

$$u'(t) = \sum_{k=0}^{n} p_k(t)u(\omega_k(t)) + f(t), \qquad t \in (a, b], \tag{1}$$

considered together with the additional condition

$$\sup_{t \in (a,b]} h(t)\,|u(t)| < +\infty, \tag{2}$$

where $h : (a, b] \to \mathbb{R}$ is a certain given non-negative continuous non-decreasing function such that

$$h(a) = 0. \tag{3}$$

We are interested in conditions guaranteeing the existence of solutions with property (2) in the case where the coefficients of equation (1) are non-positive.

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On the global dynamics of the Nicholson blowflies and the Mackey-Glass equations

GERGELY RÖST

Keywords: *delay differential equations, global attractors, heteroclinic orbits*
MSC2000 Classification: 34K10.

Abstract

After many decades of intensive research, some seemingly simple nonlinear delay differential equations of the form

$$x'(t) = -\mu x(t) + f(x(t-\tau))$$

still pose massive problems to their understanding, even in some situations where the feedback is monotone and a comprehensive general theory is available. Furthermore, non-monotone delayed feedback may generate chaotic behaviour. In the talk some recent results will be presented for two celebrated model equations: the Nicholson blowflies equation arisen in population dynamics, and the Mackey-Glass equation which was proposed to model blood cell production and haematological diseases. In particular, we give sufficient conditions that ensure that all solutions eventually enter the domain where the feedback is monotone, thus chaotic behaviour can be excluded. We give sharp bounds for the global attractor and construct heteroclinic orbits from the trivial equilibrium to a slowly oscillating periodic orbit around the positive equilibrium. The main results are illustrated by many numerical examples.
Joint work with Eduardo Liz (Vigo, Spain), Jianhong Wu (Toronto, Canada) and Tibor Krisztin (Szeged, Hungary)

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Mathematical Models in Engineering, Biology and Medicine.
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Solvability of an elastic beam fully nonlinear equation in presence of a sign-type Nagumo control

MARIA DO ROSÁRIO GROSSINHO, FELIZ MINHÓS AND ANA ISABEL SANTOS

Keywords: *Two point fourth order nonlinear BVP, lower and upper solutions, sign-type Nagumo condition, a priori estimates, Leray-Schauder degree.*

MSC2000 Classification: 34B15, 34L30.

Abstract

The aim of this work is to establish existence and location results for the fourth order fully differential equation

$$u^{(iv)} = f\left(t, u, u', u'', u'''\right), \quad 0 < t < 1,$$

with various types of boundary conditions as, for example,

$$u\left(0\right) = u''\left(0\right) = u'\left(1\right) = u'''\left(1\right) = 0.$$

It is assumed that $f : [0,1] \times \mathbb{R}^4 \to \mathbb{R}$ is a continuous function satisfying a sign-type Nagumo growth condition which allows an asymmetric unbounded behaviour on the nonlinearity. The arguments make use of the lower and upper solution method and degree theory.

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Multiphase systems for medical image region classification

J.F. GARAMENDI, N. MALPICA AND E. SCHIAVI

Keywords: *Multiphase systems, Elliptic degenerate equations, Image segmentation and classification.*
MSC2000 Classification: 34B15, 34B16.

Abstract

Digital medical image segmentation and classification is a basic yet challenging problem in mathematics and bio-engineering. In this talk we consider the multiphase approach due to Chan and Vese ([1]) which leads to the consideration of a parabolic degenerate multiphase weakly coupled system of equations. For 2-phases systems, say ϕ_1, ϕ_2 this can be written in form

$$\begin{cases} \dfrac{\partial \phi_1}{\partial t} = \delta_\epsilon(\phi_1)\left[\nu\nabla\cdot\left(\dfrac{\nabla\phi_1}{|\nabla\phi_1|}\right) - \left((I-c_{11})^2-(I-c_{01})^2\right)H(\phi_2) + \left((I-c_{10})^2-(I-c_{00})^2\right)(1-H(\phi_2))\right] \\ \dfrac{\partial \phi_2}{\partial t} = \delta_\epsilon(\phi_2)\left[\nu\nabla\cdot\left(\dfrac{\nabla\phi_2}{|\nabla\phi_2|}\right) - \left((I-c_{11})^2-(I-c_{10})^2\right)H(\phi_1) + \left((I-c_{01})^2-(I-c_{00})^2\right)(1-H(\phi_1))\right] \end{cases}$$

where H_ϵ, δ_ϵ are smooth approximations to the Heaviside function and the Dirac delta distribution. The constants c_{ij}, $i, j = 0, 1$ are given and represent an initial estimation of the mean value of each class in the original digital image I. Typically, using a gradient descent, the system is solved by an iterative decoupling scheme until stabilization to the steady state of the system. This is computationally expensive and strongly dependent on the parameters of the model as well as the initial conditions. Following [2], we propose a novel method for solving such multiphase system which amounts to consider a unique scalar degenerate elliptic equation proposed originally by Rudin, Osher and Fatemi, [3] for image denoising. This originates a well-posed problem ([4], [5]) which avoids the above mentioned difficulties. Once the solution is numerically computed (see [6] for an interesting, recent analysis of the underlying numerical problem even if different from our approach) we threshold it by means of a genetic algorithm in order to produce the phase (class) partition of the digital image. As an application we show some recent results obtained for liver and brain minimal partition problems.

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Modelling the evolution in time of two languages in competition

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Keywords: *Computer simulations, language competition.*
MSC2000 Classification: 34A34, 92D25.

Abstract

Starting from a model by Abrams and Strogatz [1] we have undertaken the analysis of a system of two languages that are competing for the same speakers along time. The original model is basically based on a parameter that indicates the social status of both languages. Such an approach leads to the extinction of one of the languages. We have modified this model by considering the nature of the languages in competition, namely, defining a parameter of interlinguistic similarity [2], which allows the onset of a new group of bilingual speakers. We have studied the stability of such bilingual system by performing computer simulations of the system of differential equations given by the model upon variations of the parameters of status and interlinguistic similarity. These simulations point to the existence of stable points of the model that involve neither the extinction of the two competing languages nor the bilingual group.

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Nonlinear abstract boundary value problems modelling atmospheric dispersion of pollutants

AIDA SHAHMUROVA AND VELI SHAKHMUROV

Keywords: *Degenerate-operator equations, nonlinear problems, interpolation of Banach spaces, UMD spaces, semigroup of operators.*
MSC2000 Classification: 34G10, 34B10, 35J25.

Abstract

The boundary value problems for linear and non linear degenerate elliptic differential-operator equations of second order are studied. The principal part of differential operators generated by these problems possess varying coefficients and are non self-adjoint. Several conditions for the separability, R-positivity and the fredholmness in abstract L_p-spaces are given. By using these results the existence, uniqueness and the maximal regularity of boundary value problems for nonlinear degenerate parabolic differential-operator equations are established. In applications mixed boundary value problems for degenerate-diffusion systems, appearing on atmospheric dispersion of pollutants [1] are studied. Elliptic differential operator equations are studied e.g. in $[2-4]$.

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Maximal regular elliptic-convolution equations

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Keywords: *Differential-operator equations, operator-valued multipliers, interpolation of Banach spaces, semigroup of operators.*
MSC2000 Classification: 34G10, 45J05, 45K05.

Abstract

In this paper, by using the Fourier multiplier theorems in Banach valued L_p spaces, the maximal regularity for elliptic convolution-operator equations are investigated. Operator-valued multipliers have been studied e.g. in $[1-3]$. Convolution-differential equations have been treated e.g. in $[1]$, $[4]$, $[5]$. In this work we find the conditions that guarantee the separability of this problem. We obtain that the correspondıng convolution-elliptic operator is positive and is a generator of an analytic semigroup. Finally, these results applied to establish maximal regularity for the parabolic Cauchy problem, boundary value problems for anisotropic integro-differential equations and infinite systems of elliptic integro-differential equations.

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On a nonlinear parabolic equation with implicit degeneration not in divergence form

KAMAL N. SOLTANOV

Keywords: .
MSC2000 Classification: .

Abstract

In this talk we consider on the cylinder the homogeneous problem for the equation

$$\frac{\partial u}{\partial t} - |u|^{\rho} \Delta u + b_0 |u|^{\mu+1} = h(t,x), \quad (t,x) \in Q_T \equiv (0,T) \times \Omega.$$

We study the solvability of this problem and the behavior its solutions under various parameters $\rho > 0$, $\mu \geq 0$ and $b_0 \in R$, here $\Omega \subset R^n$ $(n \geq 2)$ be a bounded domain.
It is known that this equation describes the behavior of the flow on the boundary layer and is also called the Prandtl-von Mises type equation (see, for example, [4]). It should be noted that the solvability of the problems with the equation of such type and the behavior of their solutions were considered earlier in many works (see, for example [1, 2, 3] and their references).
Since the considered equation is an equation with implicit degeneracy, therefore we need study it on the corresponding space. Consequently in beginning we investigate here the space determined by the considered problem, which is a nonlinear space such as following (see also, [5])

$$\overset{0}{S}_{\Delta,\rho,2}(\Omega) \equiv \left\{ u \in L_1(\Omega) \,\middle|\, \int_{\Omega} |u|^{\rho} |\Delta u|^2 \, dx < +\infty, \quad u(x) \mid_{\partial\Omega} = 0 \right\}.$$

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Basic properties of partial dynamic operators

PETR STEHLIK AND BEVAN THOMPSON

Keywords: *Maximum principles, partial dynamic operators, time scales.*
MSC2000 Classification: 34B15, 39A10, 34B10.

Abstract

Motivated by the importance of maximum principles in the theory of partial differential equations and in the numerical analysis, we establish simple maximum principles for basic partial dynamic operators on multidimensional time scales. As in the case of ordinary dynamic operators we reveal a set of results and counterexamples which illustrate the distinct behaviour in continuous and discrete case. Finally, we provide some immediate consequences and prove uniqueness results to problems involving partial dynamic operators.

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Multivalued boundary valued problems in Banach spaces

VALENTINA TADDEI

Keywords: *Differential inclusions, degree arguments, condensing operators, bound sets, Floquet problems.*
MSC2000 Classification: 34A60, 34B15.

Abstract

The talk deals with the existence of a strong solution of a Carathéodory semilinear differential inclusion in a Banach space E with Floquet boundary conditions

$$\begin{cases} x' \in A(t)x + F(t,x), & \text{for a.a. } t \in [a,b], x \in E \\ x(b) = Mx(a). \end{cases} \tag{1}$$

where $A : [a,b] \to \mathcal{L}(E,E)$ is Bochner integrable, $F : [a,b] \times E \multimap E$ is a Carathèodory multivalued map, $M \in \mathcal{L}(E,E)$ is an invertible operator. This class includes periodic and anti-periodic problems.
To solve (1) we apply a continuation principle. The proof relies on degree arguments combined with a bound sets approach for checking the behavior of trajectories in a suitable convex parametric set of candidate solutions. In a general Banach space, we are only able to guarantee the positive invariance of set of candidate solutions (see [1]). In the special case of an Hilbert space, using an approximation argument, the bound sets approach enables to consider additional situations when some trajectories can escape from the set of parameters (see [2]). Our method allows also the localization of the solution in a given set. An application is also given.

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Multiscale model of auditory selective attention and habituation neural correlates

CARLOS TRENADO AND DANIEL J. STRAUSS

Keywords:
MSC2000 Classification:

Abstract

Auditory evoked cortical potentials (AECPs) have been consolidated as an important tool in experimental neuropsychology, neuroscience and psychiatry; e.g. for the treatment and diagnosis of disorders among them schizophrenia and attention deficit disorder, as well as the study of cognitive processes such as selective attention and habituation. With respect to the latest, numerous experimental and theoretical studies at the behavioral and physiological level emphasize the role of a cortico subcortical dynamics. However, the effect of such vertical bidirectional dynamics to neural correlates of selective attention and habituation, and their corresponding large-scale effects reflected in AECPs is still far from being understood. To address such issues, we propose a multiscale probabilistic approach to model neural correlates of auditory selective attention and habituation reflected in AECPs. The results of our simulations are compared with human experimental AECPs. It is concluded that our approach represents a useful methodology to gain deeper understanding of the neurodynamics of auditory selective attention and habituation neural correlates.

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Global stability of difference equations satisfying Yorke condition

VIKTOR TKACHENKO

Keywords: *Difference equations, global stability.*
MSC2000 Classification: 39A11.

Abstract

We give sharp global stability conditions for zero solution of difference equation with delay which arises in many contexts in mathematical biology

$$x_{n+1} = qx_n + f_n(x_n, ..., x_{n-k}), \quad n \in \mathbf{Z}, \tag{1}$$

where $q \in (0, 1]$ and functions $f_n : \mathbf{R^{n+1}} \to \mathbf{R}$ satisfy Yorke condition $a\mathcal{M}(\mathbf{z}) \leq f_n(\mathbf{z}) \leq -a\mathcal{M}(-\mathbf{z})$, where $a < 0$, $\mathcal{M}(\mathbf{z}) = \max_i\{0, z_i\}$.
For $a < 0, q \in (0, 1)$, positive integer k and integer $s, 0 \leq s \leq k$, we define the following functions

$$\Omega(s, k, q, a) = q^{k+s+1} + aq^s(s + \frac{1 - q^{k+1}}{1 - q}) + a^2 \frac{1 + q^s(sq - s - 1)}{(1 - q)^2}, \tag{2}$$

$$\omega(s, k, q) = \frac{q^{2k+2} - q^{2k+s+3} - q^{k+s+2}(1 - q)(s + 1)}{(s + 1)^2(1 - q)^2}. \tag{3}$$

If $q = 1$, we set $\omega(s, k, 1) = (s-2k)/(2s+2)$ that coincides with the limit of the right-hand side of (3) when $q \to 1$. Equation $\omega(s, k, q) = -1$ has the unique solution $q = q^*_s(k) \in (0, 1]$ once $0 \leq s \leq s_k = [(2k - 2)/3]$ ($[x]$ denote the integer part of a real number x). If $s > [(2k - 2)/3]$, then $\omega(s, k, q) > -1$.
Let $a_j(k, q), j = 1, ..., k$, be the biggest root of the quadratic (with respect to a) equation $\Omega(j, k, q, a) = -1$. And let $a_0(k, q)$ be the unique root of the linear equation $\Omega(0, k, q, a) = -1$.
Theorem. *Assume that* $q \in (0, 1)$ *and one of the following* $s_k + 2$ *conditions is true:*
0) $a \in (a_0(k, q), 0)$ *if* $q \in (0, q^*_0(k)]$;
j) $a \in (a_j(k, q), 0)$ *if* $q \in [q^*_{j-1}(k), q^*_j(k)]$, $j \leq s_k$;
$s_k + 1$*)* $a \in (a_{s_k+1}(k, q), 0)$ *if* $q \in [q^*_{s_k}(k), 1)$.
Then Eq. (1) is globally asymptotically stable, i.e. there exist $\lambda \in (0, 1)$ *and* $\Gamma > 0$ *such that*

$$\max_{j=n-k,n} |x_j| \leq \Gamma\lambda^{n-s} \max_{j=s-k,s} |x_j|, \; n \geq s,$$

for every solution $\{x_n\}$ *of Eq. (1).*
Let $q = 1$ *and* $\sum_j f_j(z_j, ..., z_{j-k}) = \infty$ *for each sequence* $\{z_n\}$ *converging to* $z^* \neq 0$*. Then every solution* $\{x_n\}$ *of (1) tends to zero if* $a \in (\bar{a}_k, 0)$*, where* $\bar{a}_k$ *is the right root of the quadratic equation* $a^2(s_k+1)(s_k+2)+2a(s_k+k+2)+4 = 0$.
The above conditions are sharp for the family of Eqs.(1) satisfying Yorke condition.

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Control and observation in population models

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Keywords: *controllability, observability, observer system, population biology.*
MSC2000 Classification: 93B05, 93C10, 93B07, 92D10, 92D40.

Abstract

In the paper we summarize some recent developments of a research line proposed in [1] and [2]. Most of the results concern basic qualitative properties of nonlinear models of population biology, such as controllability and observability. Controllability analysis is motivated by the idea of conservation ecology, where a relevant issue is to steer a population system to an equilibrium state, applying an abiotic human intervention as control function. Control models can also describe artificial selection in population genetics.
State monitoring of population systems near equilibrium, in many cases, can be reduced to the observation of certain indicator species, provided the corresponding system is locally observable, see e.g. [3]. In population genetics observability problems arise in a very natural way. Observer design makes it possible to recover the underlying genetic process, based on the observation of phenotype frequencies. For application of observers in evolutionary game models see [4].

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Mathematical Models in Engineering, Biology and Medicine.
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Parametrices of regular hypoelliptic boundary-value problems associated to the linear Schrödinger equation

NELSON VIEIRA

Keywords: *Hypoelliptic operator, Schrödinger operator, Clifford Analysis, Time-dependent problems.*
MSC2000 Classification: 35H10, 30G35, 35A08.

Abstract

Regular hypoelliptic problems were first studied by Hörmander in [3]. In it, Hörmander defined the characteristic function of a hypoelliptic boundary-value problem. This allow him to completely characterized regular hypoelliptic boundary value problems based on the behavior near the infinity of the zeros of its characteristic functions. His ideas were further developed by Barros-Neto in [1] and [2], where parametrices were described and their regularity studied for general hypoelliptic problems.
The time-dependent Schrödinger operator $-\Delta + i\partial_t$ has fundamental solutions with non-removable singularities in the hyperplane $t = 0$. This fact implies that the treatment of such operator is a non-trivial problem. To overcame this drawback, one uses a regularization procedure (see [4], or more recently [5].).
In this talk we present a regularization procedure which generates a family of hypoelliptic operators converging in the weak sense to the Schrödinger operator. For the elements of this family, we construct the associated parametrices and solve the correspondent boundary value problem.

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On solvability of a three-point boundary value problem for second order nonlinear functional differential equations

PETR VODSTRČIL

Keywords: *Second order nonlinear functional differential equation, nonlocal BVP.*
MSC2000 Classification: 34K06, 34K10.

Abstract

Sufficient conditions on solvability of the problem

$$u''(t) = \ell(u)(t) + F(u)(t),$$
$$u(a) = 0, \quad u(b) = u(t_0)$$

will be established, where $t_0 \in]a,b[$, $\ell, F : C([a,b];\mathbb{R}) \to L([a,b];\mathbb{R})$ are continuous operators which transforming the set of continuous functions into the set of integrable functions and, moreover, operator ℓ is linear.

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Analytic Normalization of Analytic Integrable Systems

XIANG ZHANG

Keywords: *Analytic system, analytic integrable, normal form, analytic normalization.*
MSC2000 Classification: 34A25, 34A34, 34C20, 37C15.

Abstract

In this talk we first summarize some results on the existence of analytically equivalent Birkhoff normal form for analytically integrable symplectic Hamiltonian systems (e.g. [1,2,3,4]). Then we present some of our recent results on the existence of analytically equivalent normal forms of analytic integrable differential systems and diffeomorphisms via analytic normalizations in $\mathbb{R}^n$ (e.g. [5,6]). Finally we consider the existence of embedding flows of an analytic integrable diffeomorphism.

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