On an efficient one and multiple time-step Monte Carlo simulation of the SABR model

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In the works [3] and [4], we propose an efficient Monte Carlo simulation for the SABR model [2]. The technique is based on an efficient simulation of SABR’s integrated variance process. The integrated variance process appears in the SABR model simulation since it is part of the conditional cumulative distribution of the SABR forward asset dynamics.

We base our approach on the derivation of the cumulative distribution function of the integrated variance and the use of a copula to approximate the conditional distribution (integrated variance conditional on the SABR volatility process). For that, a recursive procedure based on the COS method [1] is employed here. This Fourier-based methodology recovers the probability density function given the corresponding characteristic function. Several improvements in terms of efficiency are proposed.

Different types of copulas are compared like Gaussian, Student t and Archimedean copulas. In order to determine the most suitable one, a goodness-of-fit analysis of each one is carried out.

Resulting is a fast and accurate simulation algorithm. The one time-step version can be employed to price European options under the SABR dynamics. This converts this approach into an alternative to Hagan analytic formula for short maturities and calibration procedures, where some known issues of the implied volatility expression for small strike values are overcome. On the other hand, the multiple time-step extension of our technique is specially useful for long-term options and for exotic options.

We present a variety of experiments where the accuracy and the performance of our method is shown.

Keywords: SABR model, Monte Carlo simulation, Integrated variance, Copula.

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REFERENCES