Asymptotic analysis of a coffee extraction model for a fixed coffee bed

Kevin M. Moroney¹, William T. Lee², Stephen B.G. O’Brien¹, Freek Suijver³, Johan Marra³

¹ MACSI, Department of Mathematics, University of Limerick, Ireland.
² Department of Mathematics, University of Portsmouth, UK.

kevin.moroney@ul.ie, william.lee@port.ac.uk, stephen.obrien@ul.ie, freek.suijver@philips.com, johan.marra@philips.com

The extraction of soluble coffee from a porous bed of roast and ground coffee grains is a complex process. The properties of the resulting dilute solution of extracted coffee solubles and hot water depend on a large number of brewing parameters. Achieving accurate and repeatable extraction of coffee solubles is a key challenge to consistently brewing high quality coffee.

Here we consider a recently published, experimentally validated model of coffee extraction [1]. Extraction of coffee is described using a double porosity model, which includes the dissolution and transport of coffee. It is shown that the model can describe extraction curves from experiments in two situations: extraction from a dilute suspension of coffee grains and water, and extraction from a packed coffee bed. Previously, the motivation and derivation of this model via volume averaging was the subject of a contributed talk at ECMI 2014. In this study we consider asymptotic solutions to the model of coffee extraction in a packed bed.

At the scale of the coffee grain, extraction is modelled by two processes. Rapid dissolution of coffee from the broken cells on the grain surface dominates extraction initially. As coffee near the grain surface is depleted, extraction from the grain kernel begins to dominate. The grain kernel consists of a network of intact coffee cells. The much slower diffusion of coffee through the intragranular pore network to the grain surfaces, drives extraction for the rest of the brewing time. Extraction of coffee is also dependent on the speed of advection of coffee from the bed. To obtain asymptotic solutions for the coffee concentrations in and exiting the bed, we utilise the small parameter resulting from the ratio of the advection timescale to the grain diffusion timescale. The resulting initial layer and outer solutions are matched using the method of matched asymptotic expansions. The asymptotic solutions are compared with the numerical solutions of the model and available experimental data.

**Keywords:** Double porosity model, Coffee brewing process, Coffee extraction kinetics, Solid-liquid extraction, Asymptotic analysis, Matched asymptotic expansions.

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