A staggered space-time discontinuous Galerkin method for the three-dimensional incompressible Navier-Stokes equations on unstructured tetrahedral meshes

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We propose a novel arbitrary high order accurate semi-implicit space-time discontinuous Galerkin method for the solution of the three-dimensional incompressible Navier-Stokes equations on staggered unstructured curved tetrahedral meshes. The scheme is based on the general ideas proposed in [1] for the two dimensional incompressible Navier-Stokes equations and is then extended to three space dimensions in [2]. As typical for space-time DG schemes, the discrete solution is represented in terms of space-time basis functions. This allows to achieve very high order of accuracy also in time, which is not easy to obtain for the incompressible Navier-Stokes equations. Similar to staggered finite difference schemes, in our approach the discrete pressure is defined on the primary tetrahedral grid, while the discrete velocity is defined on a face-based staggered dual grid. While staggered meshes are state of the art in classical finite difference schemes for the incompressible Navier-Stokes equations, their use in high order DG schemes still quite rare. A very simple and efficient Picard iteration is used in order to derive a space-time pressure correction algorithm that achieves also high order of accuracy in time and that avoids the direct solution of global nonlinear systems. Formal substitution of the discrete momentum equation on the dual grid into the discrete continuity equation on the primary grid yields a very sparse five-point block system for the scalar pressure, which is conveniently solved with a matrix-free GMRES algorithm. From numerical experiments we find that the linear system seems to be reasonably well conditioned, since all simulations shown in this presentation could be run without the use of any preconditioner, even up to very high polynomial degrees. For a piecewise constant polynomial approximation in time and if pressure boundary conditions are specified at least in one point, the resulting system is, in addition, symmetric and positive definite. This allows us to use even faster iterative solvers, like the conjugate gradient method.

The flexibility and accuracy of high order space-time DG methods on curved unstructured meshes allows to discretize even complex physical domains with very coarse grids in both, space and time. The proposed method is verified for approximation polynomials of degree up to four in space and time by solving a series of typical 3D test problems and by comparing the obtained numerical results with available exact analytical solutions, or with other numerical or experimental reference data.

Keywords: high order schemes, space-time discontinuous Galerkin finite element schemes, staggered unstructured meshes, space-time pressure correction algorithm, incompressible Navier-Stokes equations in 3D

Acknowledgments. Partially funded by the European Research Council (ERC) under the European Union’s Seventh Framework Programme (FP7/2007-2013) within the research project STiMulUs, ERC Grant agreement no. 278267.

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