Abstract

We consider algebraic and topological $K$-theory of complex locally convex algebras stabilized by harmonic operator ideals. A symmetrically normed Banach ideal $J \subset B(H)$ is harmonic if it contains a compact operator whose sequence of singular values is the harmonic sequence $\{1/n\}_n$; for example, the Schatten ideals $\mathcal{L}_p$ with $p > 1$ are harmonic. We prove that if $A$ is a locally convex algebra and $J$ a harmonic ideal then there is a long exact sequence

$$K_{n+1}^{top}(A \otimes J) \to HC^Q_{n-1}(A \otimes J) \to K_n(A \otimes J) \to K_n^{top}(A \otimes J)$$

Here $\otimes$ is the completed (projective) tensor product, $HC^Q$ is the algebraic cyclic homology of $Q$-algebras, $K_*$ is algebraic $K$-theory and $K_*^{top}(A) = kk_*(C, A)$ is the covariant version of Cuntz’ bivariant $kk$ for locally convex algebras.