Hybrid topological derivative and gradient–based methods for electrical impedance tomography

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ABSTRACT

The impedance imaging problem consists in producing an image of the electromagnetic properties of a medium by applying electric currents to its exterior surface and measuring voltages on it. The electromagnetic properties of interest are typically the electric conductivity and the permittivity. Information about the internal electrical properties of a body can be used for non-destructive material testing, locating mineral deposits, or for medical imaging. The range of medical applications is wide, because different tissues have different electromagnetic properties. For example we can think of monitoring for clots or screening for breast cancer.

In this work we solve the impedance imaging problem by a combination of topological derivative–based schemes and gradient techniques. We distinguish two cases. Initially, we consider the presence of discontinuities corresponding to inclusions of a different material. The geometry of the set of inclusions and the values of the electromagnetic parameters in them have to be identified. There are no restrictions on their size or distribution, and their parameters may be space dependent. The admittivity of the background is assumed to be known and may also be space dependent. In the second framework, we do not consider the presence of sharp interfaces. The conductivity and permittivity of the whole medium are considered to be unknown and vary smoothly. We propose a technique to reproduce their spatial variations.

Both imaging problems can be reformulated as inverse scattering problems and then recast as constrained optimization problems. In the first one, we have a matrix with known electrical properties containing several domains with unknown properties. Computing the topological derivative of the shape functional to be minimized we find the first approximation of the number, size and location of the domains. A gradient method produces the first guess of the values of the parameters. This procedure can be iterated to improve the approximation. In the second one, we just minimize the functional with respect to the unknown variable parameters by a gradient method.

References
