Persistent Issues in Inflation Persistence

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Abstract

One of the criticisms routinely advanced against models of the business cycle with staggered contracts is their inability to generate inflation persistence. This paper finds that staggered contracts à la Taylor are, in fact, capable of reproducing the inflation persistence implied by U.S. data. Following Fuhrer and Moore, I capture the moments that the contract specification needs to replicate by using the correlograms from a small vector autoregression (VAR), that includes inflation among the endogenous variables. A simple structural model substitutes the inflation equation from the VAR with the contract specification. I estimate the contract parameters in the structural model by maximum likelihood. The correlogram for the endogenous variables from the estimated structural model, including that for inflation, are very close to the correlograms from the VAR (and are contained within their 90% confidence intervals)

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1 Introduction

The study of the Phillips curve has been one of the central issues of macroeconomics since Phillips (1958) identified a negative correlation between inflation and unemployment. King and Watson (1994) give a comprehensive discussion of the evolution of the traditional empirical literature.


The results of Fuhrer and Moore (1995) stand out in the growing empirical literature on the new Phillips curve. They showed that the staggered price mechanism of Taylor (1980) (henceforth, referred to as “standard”) is not capable of generating the inflation persistence that they observed in the U.S. data. Fuhrer and Moore showed that an alternative contracting specification (henceforth, referred to as “relative”), first introduced by Buiter and Jewitt (1981), fares much more favorably in fitting the U.S. data. This alternative specification postulates that, when choosing a contract wage, workers care about the relative remuneration with respect to other outstanding contracts. This has the practical effect of introducing an extra lag of inflation in the implied Phillips curve, which accounts for its ability to generate greater persistence. However, this comes at the cost of a further departure from theoretical foundations.

The evaluation procedure of Fuhrer and Moore (1995) has two steps. First, a simple statistical model captures the properties of the data that the contracting specification needs to reproduce. The statistical model takes the form of an unconstrained vector auto-
regression (VAR) with output per person, inflation and the short-term interest rate as the endogenous variables. Then the equation for inflation in the VAR is replaced with a contracting specification, thus generating a structural model where only the parameters in the contracting specification are unknown. Second, the structural parameters are estimated via maximum likelihood.

Coenen and Wieland (2000) followed the methodology of Fuhrer and Moore (1995) to calibrate a general equilibrium model of the Euro area. In line with Gali, Gertler, and Lopez-Salido (2001), they found that both the standard and the relative contracting specification “fit euro area data reasonably well.”

The sample in Fuhrer and Moore (1995) spans the period from 1965 to 1993, thus including the oil crises of the 70s, as well as the Volcker disinflation. Evans and Wachtel (1977), Taylor (2000), and Cogley and Sargent (2000) documented that a high degree of inflation persistence is a characteristic of the late 1960s and 1970s, but not necessarily of the remaining postwar period. Erceg and Levin (2001) develop a model with standard contracts where agents use optimal filtering to disentangle persistent and transitory shifts in monetary policy. They attributed the observed persistence in inflation, following the Volcker disinflation, to uncertainty over monetary policy.

The purpose of this paper is to test whether or not the lower persistence of inflation found by other authors in the U.S. data for the 1980s and 1990s translates into significantly different estimates of the parameters in the standard and relative contract model. As a byproduct, this is also a test of whether the results of Fuhrer and Moore (1995) still hold true when using the additional data that have become available since the original publication of their study. Not only do we have longer time series at our disposal, but the series have been revised. The data that Fuhrer and Moore (1995) used come from the productivity release of the Bureau of Labor Statistics. Duke and Usher (1998) document the latest improvements to these series.

Using the sample from 1960 to 2001, I find that relative contracts are able to reproduce the inflation persistence observed in the data. The results concerning relative contracts reported by Fuhrer and Moore (1995) still hold with updated data and longer series, and
are resilient to introducing breaks in the linear detrending of output, as well as reestimation over smaller subsamples.

More surprisingly, I also find that standard contracts perform very well. The metric that I use to make these claims is the distance between correlograms for inflation, the interest rate and output coming from the VAR and the structural models. I compare the correlograms from the unrestricted VAR with the correlograms from the estimated structural model with standard contracts and with relative contracts. I find that the correlogram from the VAR for each of the three endogenous variables is close to the two structural counterparts across all the subsamples I consider. I compute the Monte-Carlo 90% confidence interval for the correlograms from the VAR. The correlograms for the two structural models invariably lie within the confidence bands.

Focusing on the 1980s and 1990s, I estimate a change in the structural parameters of the two contracting models I consider. This shift, is consistent with a lower persistence of the inflation series, but is not statistically significant.

In previous work, Guerrieri (2001), I had found that staggered contracts set up following Calvo (1983), produced a better fit to the U.S. data than staggered contracts of one single fixed duration à la Taylor. The contracts in this paper, by allowing the coexistence of multiple contract durations, follow more closely the setup of Taylor (1980). Yun (1996) showed how to reconcile contracts à la Calvo with a first order condition coming from a profit maximization problem. Chari, Kehoe, and McGrattan (2000), transferred the setup of Yun (1996) to contracts of fixed duration. In this paper I show how to allow for multiple contracts of fixed duration à la Taylor, in a way that can be mapped into a profit maximization exercise, and that is still parsimonious in terms of the size of the implied state space. This reinterpretation, can then be mapped into the setup of Fuhrer and Moore (1995). Allowing for a distribution of contract durations makes staggered contracts à la Taylor closer to the Calvo counterparts. The single contract duration is rejected by the data, substantiating that this development has empirical relevance.

The plan for the rest of the paper is as follows: in Section 2, I build some intuition for the difference between standard contracts and relative contracts; in Section 3 I lay out the methodology I used in the VAR estimation. In Section 4, I describe the structural
estimation, and report the estimation results; Section 5 concludes.

2 Comparing Standard and Relative Contracts

Galí and Gertler (1999) gave a good review of the recent state of the literature. I will only attempt to summarize the salient points.

The structure behind the new Phillips curve is an environment of monopolistically competitive firms that are faced with a constraint on price adjustment. Following Taylor (1980), firms are allowed to reset their contract price every \( n \) periods. Firms are otherwise symmetric in every other respect. At any period, \( n \) overlapping contracts are in force.

Chari, Kehoe, and McGrattan (2000) showed that profit maximization implies a first order condition for a firm resetting its price at time \( t \), that, by log-linearizing, leads to:

\[
P_t = \sum_{i=0}^{n-1} \frac{1}{n} E_t (P_{t+i} + \gamma \bar{y}_{t+i})
\]

(1)

\( P_t \) is the log of the contract price set at time \( t \), \( \bar{y}_t \) is the output gap and \( E_t \) denotes expectations conditional on the information set available at time \( t \). This also happens to be the contracting specification chosen by Taylor (1980)\(^2\). The log of the aggregate price, \( \bar{P}_t \), is then given by:

\[
\bar{P}_t = \sum_{i=0}^{n-1} \frac{1}{n} P_{t-i}
\]

(2)

combining equation (1) and equation (2), setting \( n = 2 \), allowing for the fact that under rational expectations \( E_{t-1} P_t = P_t - \epsilon_t \) (where \( \epsilon \) is a forecast error), and finally reworking the price equation in terms of inflation, denoted by \( \pi_t \), one obtains the Phillips curve equation, which as shown in Appendix A, is given by

\[
\pi_t = E_t \pi_{t+1} + \gamma (\bar{y}_t + E_t \bar{y}_{t+1} + \bar{y}_{t-1} + E_{t-1} \bar{y}_t) - \frac{1}{4} \epsilon_t
\]

(3)

2.1 The relative contract model

Fuhrer and Moore (1995) argued that the persistence imparted to inflation by the standard contracting specification does not fit the U.S. inflation data as well as their relative

\(^2\)This is indeed a special case of that model, for a particular set of contract weights
specification. Their alternative model can be summarized by the following equations, where each variable is to be thought in log deviation from steady state. The contract equation is the following:

\[ P_t - \bar{P}_t = \sum_{i=0}^{n-1} \frac{1}{n} E_t (V_{t+i} + \gamma \tilde{y}_{t+1}) \quad (4) \]

where \( P_t \) is the price contract that starts in period \( t \), \( \bar{P}_t \) is the aggregate price level, \( \tilde{y}_t \) is the output gap. The aggregate price level is still governed by equation (2). \( V_t \) is a relative price index, that takes following form

\[ V_t = \sum_{i=0}^{n-1} \frac{1}{n} (P_{t+i} - \bar{P}_{t-i}) \quad (5) \]

Then, for \( n=2 \), the Phillips curve equation implied by this contracting specification takes the form:

\[ \pi_t = \frac{1}{2} (\pi_{t-1} + E_t \pi_{t+1}) + \gamma (\tilde{y}_t + E_t \tilde{y}_{t+1} + \tilde{y}_{t-1} + E_{t-1} \tilde{y}_t) - \frac{1}{4} \alpha \quad (6) \]

Comparing equations (3) and (6), one can immediately see that the relative contract specification, for any given contract length, appends an extra lag of inflation to the Phillips curve equation.

### 2.2 Allowing for multiple contract lengths

Rather than maintaining that all contracts last \( n \) periods, a more flexible setup would allow for a distribution of contract durations. Following Blinder (1994), one could also brand such a setup as more plausible. A simple way to model this distribution is to assume that when firms set a price, they face uncertainty over the contract duration. The price they set might be in force for any length of time between 1 and \( n \) periods. Firms do know, however, the relevant probabilities. Then, let \( \theta_1 \) be the probability that a contract will be in force only one period, let \( \theta_2 \) be the probability that a contract will be in force for two periods, and so on. Let the vector \( \theta \) summarize the relevant contract weights. The elements of \( \theta \) are all non-negative and sum to 1. Fixing the longest contract duration at four periods \( (n = 4) \), the aggregate price level becomes

\[ \bar{P}_t = \theta_1 P_t + \theta_2 \frac{1}{2} \sum_{i=0}^{1} P_{t-i} + \theta_3 \frac{1}{3} \sum_{i=0}^{2} P_{t-i} + \theta_4 \frac{1}{4} \sum_{i=0}^{3} P_{t-i} \quad (7) \]
The setup of Fuhrer and Moore (1995) can be reinterpreted to conform to this setup. One way to impose some structure on the distribution of contract lengths would be to pick a functional form for the weights on contract prices in equation (7). Letting \( f_i \) denote the weight on the contract price with lag \( i \), equation (7) would then be rewritten as

\[
P_t = \sum_{i=0}^{3} f_i P_{t-i}
\]

Fuhrer and Moore (1995) imposed that

\[
f_i = 0.25 + (1.5 - i)s
\]

where \( s \) is the only parameter governing the shape of the distribution of contract durations. To be able to match a choice for \( s \) into a vector \( \theta \), \( s \) needs to be contained in the interval \( \frac{1}{6} \) between 0 and \( \frac{1}{6} \). In Appendix B, I show how to map a choice for \( s \) into a set of contract weights \( \theta_1 \) to \( \theta_4 \). In Table 1, I perform this mapping for selected values of \( s \). As shown, as \( s \) decreases, the weight on the longer contracts increases. In this stochastic contract setup, the contract price rule for the standard model becomes

\[
P_t = \sum_{i=0}^{3} f_i E_t(\bar{P}_{t+i} + \gamma \bar{Y}_{t+i})
\]

while, for the relative contract setup, following Fuhrer and Moore (1995)

\[
P_t = \sum_{i=1}^{3} \beta_i x_{t-i} + \sum_{i=1}^{3} \beta_i E_t P_{t+i} + \gamma^* \sum_{i=0}^{3} f_i E_t \bar{y}_{t+i}
\]

where

\[
\beta_i = \frac{\sum_{j=0}^{3} f_j f_{i+j}}{1 - \sum_{j=0}^{3} f_j^2} \quad \text{and} \quad \gamma^* = \frac{\gamma}{1 - \sum_{j=0}^{3} f_j^2}
\]

### 2.3 Comparing Impulse Response Functions

Fuhrer and Moore (1995) closed their model by estimating a VAR in output, inflation and the interest rate. This is the way I proceed for the purposes of estimating the unknown parameters in the contract equations. To understand the differences in the standard

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\(^3\)This is equivalent to the condition imposed by Fuhrer and Moore (1995) that the polynomial in the lag operator used to rewrite the aggregate price equation be invertible.
and relative contracts, instead of pursuing this route, one could more simply complete the model by following Taylor (1980), specifying equations for the demand and supply of money. It is easier to examine the differences imparted by the choice of contracting specification when the response of money is kept constant. Thus, let the demand for nominal money balances, $M_t$ take the form

$$M_t = P_t + y_t$$  \hspace{1cm} (13)$$

And let money supply be described by

$$M_t = M_{t-1} + \mu_t$$  \hspace{1cm} (14)$$

where $\mu_t$, the rate of growth of money supply, is given by $\mu_t = \rho \mu_{t-1} + \epsilon_t$ and $\epsilon_t$ is an i.i.d. error term\(^4\). One is now in a position to simulate the effects of shocks in the two models so as to assess the persistence properties of each specification. An area where one would expect the difference between the two contracts to emerge is in the response to monetary shocks.

I have performed a battery of tests, using temporary and permanent, announced and unannounced shocks to the rate of growth of money supply as well as to the level of money. In Figure 1 and 2, I report the impulse response functions for an unannounced shock to the rate of growth of money supply. The intuition gained in this case holds true for all the other shocks I considered. Holding the distribution of contract durations

\(^4\)The money supply equation adopted here comes from Christiano, Eichenbaum, and Evans (1998) who argue that this is a good approximation to money supply for both M1 and M2 in the U.S., as long as $\rho$ is chosen to be close to 0.5.
constant, the choice of $\gamma$, the weight on the output gap in the contract equation, governs the difference in the persistence of inflation that the two contracting specifications can yield. For lower the value of $\gamma$, the greater the gain in persistence offered by the relative contract specification. As Figure 2 shows, for $\gamma = 0.3$ the inflation persistence of the two contracting specifications under study is of comparable magnitude.

Varying the value of $s$, not surprisingly, also affect the path for inflation. Lower values of $s$, by placing a greater weight on longer contracts, yield a more persistent response of inflation. In the light of this analysis, for the purposes of generating greater inflation persistence, given the choice of $n$, and $\gamma$, one would then replace standard contracts with relative contracts if lowering $s$ did not produce enough extra persistence.

In the section that follows I employ maximum likelihood estimation to pick values for $\gamma$ and $s$. Then, I investigate whether or not one contracting specification produces a better fit than the other.

3 VAR estimation

In order to assess the properties of the data that the contracting specification needs to reproduce I rely on a simple statistical model that takes the form of a VAR. Detrended output and inflation are the series of interest. Following Bernanke and Blinder (1992), Fuhrer and Moore (1995) and Coenen and Wieland (2000), I include the short term nominal interest rate in the VAR to help in the formation of output expectations. Thus the three endogenous variables in the VAR are detrended log of output, inflation and the short term interest rate.

Just as Fuhrer and Moore (1995) the series for the above variables come from the productivity release of the Bureau of Labor Statistics. While the interest rate series goes back to 1934, the output and price series start in the first quarter 1947. For the VAR estimation I discard the first part of the sample and take the first quarter of 1960 as the starting date for the analysis. I keep this first portion of the data as a presample, that I exploit later in the maximum-likelihood estimation of the structural parameters.

The measure of output that I consider is log of the nonfarm business output per
Figure 1: Response to an unannounced shock to the rate of growth of money supply.

$\text{GAMMA}=0.004, \text{S}=0.08, \text{RHO}=0.56$

Deviation from Baseline

Solid: Standard Contracts
Dotted: Relative Contracts
Figure 2: Response to an unannounced shock to the rate of growth of money supply.

GAMMA=0.3, S=0.08, RHO=0.56
Deviation from Baseline

Solid: Standard Contracts
Dotted: Relative Contracts
person. The measure of inflation comes from a quarterly difference in the log of the nonfarm business output deflator. Finally the interest rate series is the 3 month treasury bill rate from the secondary market quoted on a discount basis.

I linearly detrend the output measure. While one-sided filtering would be more rigorous, I follow this procedure to ensure comparability with the results of Fuhrer and Moore (1995). I reserve this refinement to possible extensions of this paper. In the detrending I have considered both single as well as multiple trends. When using multiple trends, I have imposed a break in the first quarter of 1980. This coincides with the beginning of Paul Volcker’s Fed chairmanship. I have also used 1983q1, which coincides with the end of Volcker’s disinflation program, as an alternative break date for the linear trend.

To decide the number of lags for the endogenous variables in the VAR equations I started with a specification that included eight lags. I reduced this number, until the parameters on the longest lag were jointly significant across equations, and the residuals were uncorrelated. To test for correlation, I used a Portmentau test on lag 12. I report the relevant Q(12) statistics in Table 5. I settled on a VAR specifications that included three lags of all the endogenous variables. I focused the specification tests on the measure of output with two trends, and kept the resulting lag structure fixed even when dropping the number of trends. For the specification tests, it made no difference whether or not I placed the break in the trend in 1980 or in 1983.

The VAR structure on which I settle has the form

\[ \tilde{y}_t = \sum_{i=1}^{3} y_{1,i} \tilde{y}_{t-i} + r_{1,i} \tilde{r}_{t-i} + \pi_{1,i} \pi_{t-i} + \epsilon_{y,t} \]  
\[ r_t = \sum_{i=1}^{3} y_{1,i} \tilde{y}_{t-i} + r_{1,i} \tilde{r}_{t-i} + \pi_{1,i} \pi_{t-i} + \epsilon_{r,t} \]  
\[ \pi_t = \sum_{i=1}^{3} y_{1,i} \tilde{y}_{t-i} + r_{1,i} \tilde{r}_{t-i} + \pi_{1,i} \pi_{t-i} + \epsilon_{\pi,t} \]

where \( r_t \) is the short term interest rate. The intercept term is excluded from the VAR structure to ensure a zero-inflation steady state, consistent with the two contracting specifications in this paper. For reasons of space, I do not report all the coefficient estimates over the various subsamples I consider. I show the correlograms for the endogenous variables, for a subset of the VAR regressions I ran, in Figures 3-5 (these figures also include
the correlograms from the structural estimation described below). The correlogram has
the advantage over impulse response functions of not requiring an identification scheme.
McCallum *********** found this a preferable instrument to assess the performance of
theoretical models. I also report a 90% confidence interval around the correlograms. This
is calculated using the Monte Carlo procedure described by Christiano, Eichenbaum, and
Evans (1999) ***** Add footnote to explain method **********.

4 Structural estimation

In order to estimate the structural parameters in the standard and in the relative contract-
ing specification, I replace the the inflation equation in the VAR described in equations
(15) to (17) with the relevant contract equations. I link prices to inflation by using
\[ \pi_t = \bar{P}_t - \bar{P}_{t-1} \]  

(18)

Therefore, in the case of standard contracts, I call structural model the system of equations
(10), (8) and (18), plus (15) and (16) from the VAR.

In the case of relative contracts, I call structural model the system of equations (11),
(8) and (18), plus (15) and (16) from the VAR. For the purposes of estimation, I augment
the contract price equation in both structural models with an observational error that I
call \( \epsilon_{P,t} \).

In both cases, the state space is given by \( X_t \equiv (P_t, \pi_t, P_t, y_t, r_t)' \). For any choice of
the parameters \( \gamma \) and \( s \), by standard methods, I can find the AR(1) representation for the
variables in the state space, which can then be rewritten as

\[ X_t = A_1 X_{t-1} + A_2 X_{t-2} + A_3 X_{t-3} + C \epsilon_t \]  

(19)

where \( \epsilon_t = (\epsilon_{y,t}, \epsilon_{r,t}, \epsilon_{P,t})' \), while \( A_1, A_2, A_3, \) and \( B \) are conformable matrices of coefficients
(which can be thought of as functions of \( s \) and \( \gamma \)). This system of equations, however, still
holds two identities. I then split the state space \( X_t \) into two parts \( S_t \) and \( Z_t \). \( S_t \) is defined
as \( S_t \equiv (\bar{P}_t, P_t)' \), while \( Z_t \) is defined as \( Z_t \equiv (\bar{y}_t, r_t, \pi_t)' \). I can then rewrite equation (19)
as

\[ Z_t = \tilde{A}_1 Z_{t-1} + \tilde{A}_2 Z_{t-2} + \tilde{A}_3 Z_{t-3} + \tilde{B}_1 S_{t-1} + \tilde{B}_2 S_{t-2} + \tilde{B}_3 S_{t-3} + \tilde{C} \epsilon_t \]  

(20)
To form the maximum likelihood function, I follow Harvey (1981), and condition on the first observation. I use the innovation representation of equation (20), assuming that $\epsilon_t$ is identically and independently distributed across time as normal. To form the likelihood, the last hurdle to overcome is that the contract price $P_t$ is unobserved. To remedy this, I adopt the following procedure. I assume that $P_t$, prior to 1947, is in steady state. Given a choice for $\gamma$ and $s$, I use equation (20) to back out $\epsilon_t$. Using equations (19) and (20) I can then dynamically generate a series for $P_t$ and $\epsilon_t$. In order to dilute the assumption that $P_t$ be in steady state prior to 1947, I use data for the period between 1947 and 1960 as a presample, with the sole purpose of initializing the value of $P_t$. I have used Monte Carlo experiments to confirm that after a period of 13 years, the initial value of $P_t$ becomes irrelevant. I maximize the likelihood using a Newton-Raphson based algorithm. To verify that the output of the algorithm maximizes the likelihood function I use a linear-search procedure.

### 4.1 Estimation Results

The estimation results are reported in Tables 2 to 5. Table 2 reports the sample dates and, where appropriate breaks in the trend of output. The start of the presample (described in the previous sections) is kept constant at the second quarter of 1947. The convention of Table 2 is that the end of the presample coincides with the quarter preceding the beginning of the actual estimation sample. The VAR equations, as seen, don’t use the presample, and are re-estimated if the sample dates for the structural model change. Regressions 1 to 3 use the sample from the fourth quarter of 1960 till the fourth quarter of 2001. Regression 1 has only one break in the trend of output four quarters before the beginning of the sample.

Using the structural model with relative contracts, as reported in Table 3, $s$ is estimated at 0.0831, with a standard error of 0.000777. $\gamma$ is estimated at 0.00540, with a standard error 0.00264. A Portmanteau test on the residuals of this regression (one equation at a time), whose Q(12) statistics are reported in Table 5, rejects the null hypothesis of white noise disturbances at the 95 % significance level. However, a plot of the fitted values for inflation against actual inflation, as shown in Figure 6 confirms that despite the
evidence of serial correlation in the residuals, the model fits the inflation data well.

Using the structural model with standard contracts, as reported in Table 4, $s$ is estimated at 0.0359, with a standard error of 0.00460. $\gamma$ is estimated at 0.0172, with a standard error of 0.00234. Portmentau tests on the residuals of each equation, whose Q(12) statistics are reported in Table 5, indicate serial correlation in the residuals. However, once again, Figure 6 confirms that despite the evidence of serial correlation in the residuals, the model with standard contracts also fits the inflation data well.

Further reassurance that the two structural models are doing a good job in capturing the properties of the data comes from inspection of the correlograms for the VAR and the two structural models. As Figure 3 shows, the correlograms for the two structural models lie close to the correlograms from the VAR for all endogenous variables. Furthermore, they are contained within the Monte Carlo 90% confidence interval for the correlogram of the VAR.

While the estimation results for the model with relative contracts are in line with the results reported by Fuhrer and Moore (1995), the estimation results for the model with standard contracts are diametrically opposite. I attribute this to the choice of sample that Fuhrer and Moore were constrained to make. I elaborate on this point when commenting on Regression 8 below.

Regressions 2 and 3 provide a robustness check on whether a break in the trend of output in 1980 or in 1983 affects the estimation results. I gather that including a second trend does not change the nature of the results. In the case of relative contracts, the estimates are not significantly different across regressions 1-3. For standard contracts, the estimates are statistically different, but the correlograms and a plot of fitted values against actual values of inflation confirm that the model is still doing well.

In Regressions 4 to 7, I explore whether limiting the sample to the 1980s and 1990s, the estimates of the structural parameters significantly change. I observe an upward shift in the value of $\gamma$ (from the discussion in the previous sections, I can interpret the shift as a reduction in the persistence of inflation in the data). However this upward shift is statistically significant, at the 95% confidence interval, only in the case of Regression 5. The correlograms and the fitted values for regression 4 are displayed, respectively, in
In regression 8, I attempt to replicate the results that Fuhrer and Moore (1995) reported. I choose to focus on the sample from 1965 to 1993. Despite refinements of the data released by the Bureau of Labor Statistics, the estimates that I obtain for the model with relative contracts are not statistically different from the ones reported by Fuhrer and Moore (as well as being numerically close). The correlograms and a plot of the fitted series for inflation, shown in Figures 5 and 8 show that the relative contract model does a good job in fitting the data over this subsample as well. In the case of standard contracts, however, my maximum likelihood estimation routine failed to converge within acceptable bounds for the structural parameters ($\gamma$ non-negative and $0 \leq s \leq \frac{1}{6}$).

\section{Conclusion}

I have used a simple VAR to capture the properties of the data that a contract model needs to reproduce. My estimation results indicate that the contract model of Taylor (1980), focusing on a sample from 1960 to 2001, performs as well as the relative contract model featured in Fuhrer and Moore (1995). Both types of contract specifications come close to replicating the second moments captured by a simple, non-structural VAR.

When limiting the estimation sample to the 1980s and 1990s, I found that parameters for both contract models shifted consistently with lower inflation persistence. However, this shift was not statistically significant.

Using the sample from 1965 to 1993 (as Fuhrer and Moore were constrained to do), I can obtain estimates of the parameters for the relative contract model, but not for the standard model. These estimates are in line with the ones originally reported by Fuhrer and Moore (1995).

I read the estimation results in this paper as purporting that the standard staggered contract model of Taylor (1980) is perfectly adequate to capture the inflation persistence in the U.S. data. To explain the inflation behavior observed in the late 1960s and 1970s, it seems more appropriate to build extra structure to the model, rather than requiring that the contract model be able to explain a higher degree of inflation persistence.
### Table 2: Sample Length and Breaks in Trend of Output

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<th>Regression #</th>
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<th>Sample End</th>
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### Table 3: Relative Contracts Maximum Likelihood Estimation Results

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<th>Estimates</th>
<th>SE</th>
<th>SE of Residuals</th>
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<tr>
<td></td>
<td>gamma</td>
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<tr>
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<td>s</td>
<td></td>
<td>r.err</td>
</tr>
<tr>
<td></td>
<td>p.err</td>
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<td>0.00277</td>
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<td>0.00209</td>
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<td>0.00829</td>
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### Table 4: Standard Contracts Maximum Likelihood Estimation Results

<table>
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<tr>
<th>Regression #</th>
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<th>SE</th>
<th>SE of Residuals</th>
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<tr>
<td></td>
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<td></td>
<td>y.err</td>
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<tr>
<td></td>
<td>s</td>
<td></td>
<td>r.err</td>
</tr>
<tr>
<td></td>
<td>p.err</td>
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<td>No convergence in bounds</td>
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### Table 5: Portmanteau Tests on residuals

<table>
<thead>
<tr>
<th>Regression #</th>
<th>Q(12) VAR y.err</th>
<th>r.err</th>
<th>p.err</th>
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<tr>
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<td>36.9</td>
<td>18.5</td>
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<td>2</td>
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<td>3</td>
<td>14.8</td>
<td>35.9</td>
<td>19.6</td>
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<td>4</td>
<td>13.4</td>
<td>16.3</td>
<td>5.95</td>
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<td>5</td>
<td>22.8</td>
<td>12.9</td>
<td>8.37</td>
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<td>6</td>
<td>9.74</td>
<td>12.7</td>
<td>7.22</td>
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<td>9.6</td>
<td>12.8</td>
<td>7.26</td>
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<td>8</td>
<td>14.2</td>
<td>23.4</td>
<td>17.3</td>
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</table>

<table>
<thead>
<tr>
<th>Q(12) Relative Contracts y.err</th>
<th>r.err</th>
<th>p.err</th>
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<tbody>
<tr>
<td>104</td>
<td>126</td>
<td>40.7</td>
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<td>40.7</td>
<td>152</td>
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<table>
<thead>
<tr>
<th>Q(12) Standard Contracts y.err</th>
<th>r.err</th>
<th>p.err</th>
<th>Q(12) Standard Contracts y.err</th>
<th>r.err</th>
<th>p.err</th>
</tr>
</thead>
<tbody>
<tr>
<td>79.9</td>
<td>79.9</td>
<td>109.4</td>
<td>No Convergence in bounds</td>
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<td></td>
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</table>
Figure 5: Correlogram for Regression 8

Correlogram for inflation

Correlogram for the output gap

Correlogram for the interest rate

VAR
VAR Conf. Int
Relative Contracts
Figure 6: Fitted Values for Inflation from Regression 1

Regression 1: 1960q4 to 2001q4

- Solid: Data
- Dotted: Fitted Values from Structural model with Relative Contracts
- Dashed: Fitted Values from Structural model with Taylor Contracts

Inflation (+/-)


0.16
0.14
0.12
0.10
0.08
0.06
0.04
0.02
0.00
-0.02
-0.04


Solid: Data
Dotted: Fitted Values from Structural model with Relative Contracts
Dashed: Fitted Values from Structural model with Taylor Contracts
Figure 7: Fitted Values for Inflation from Regression 4

Regression 4: 1980q1 to 2001q4

Solid: Data
Dotted: Fitted Values from Structural mode with Relative Contracts
Dashed: Fitted Values from Structural model with Taylor Contracts
Figure 8: Fitted Values for Inflation from Regression 8

Regression 8: 1965q4 to 1993q3

Solid: Data
Dotted: Fitted Values from Structural model with Relative Contracts
References


A The equation for inflation under the setup of Taylor (1980)

In a symmetric two-period setup, the log of the aggregate price level, $P_t$, is given by:

$$P_t = \frac{1}{2}(P_t + P_{t-1})$$  \hfill (21)

where $P_t$ is the contract price. Equation (4), that governs the contract price, for a two period setup, can be rewritten as

$$P_t = \frac{1}{2}P_{t-1} + \frac{1}{2}E_tP_{t+1} + \gamma(\tilde{y}_t + E_t\tilde{y}_{t+1})$$  \hfill (22)

where $\tilde{y}_t$ adjusts the contract for excess demand. Combining equation 21 and equation 22, one obtains:

$$P_t = \frac{1}{2}\left(P_{t-1} + \frac{1}{2}E_tP_{t+1} + \frac{1}{2}P_{t-2} + \frac{1}{2}E_{t-1}P_t\right) + \frac{\gamma}{2}(\tilde{y}_t + E_t\tilde{y}_{t+1} + \tilde{y}_{t-1} + E_{t-1}\tilde{y}_t)$$  \hfill (23)

Using equation 21, equation 23 can be rewritten as:

$$P_t = \frac{1}{2}(E_t\tilde{P}_{t+1} + \tilde{P}_{t-1}) + \frac{\gamma}{2}(\tilde{y}_t + E_t\tilde{y}_{t+1} + \tilde{y}_{t-1} + E_{t-1}\tilde{y}_t) - \frac{1}{4}\epsilon_t$$  \hfill (24)

where $\epsilon_t$ is a forecast error such that $E_{t-1}w_t = w_t - \epsilon_t$.

To reformulate equation 24 in terms of inflation, notice that since the price level, $\tilde{P}_t$, is in log form, the inflation at time $t$, $\pi_t$, is given by $\pi_t = \tilde{P}_t - \tilde{P}_{t-1}$. Therefore, using equation 24, subtracting $\tilde{P}_{t-1}$ from both sides:

$$\tilde{P}_t - \tilde{P}_{t-1} = \frac{1}{2}(E_t\tilde{P}_{t+1} - \tilde{P}_{t-1}) + \frac{\gamma}{2}(\tilde{y}_t + E_t\tilde{y}_{t+1} + \tilde{y}_{t-1} + E_{t-1}\tilde{y}_t) - \frac{1}{4}\epsilon_t$$  \hfill (25)

Rearranging the terms in the equation above, and adding and subtracting $\frac{1}{2}P_t$:

$$\pi_t = \frac{1}{2}(E_t\pi_{t+1} + \pi_t) + \frac{\gamma}{2}(\tilde{y}_t + E_t\tilde{y}_{t+1} + \tilde{y}_{t-1} + E_{t-1}\tilde{y}_t) - \frac{1}{4}(\epsilon_t)$$  \hfill (26)

which, in turn, can be rewritten as:

$$\pi_t = \frac{1}{2}(E_t\pi_{t+1} + \pi_t) + \frac{\gamma}{2}(\tilde{y}_t + E_t\tilde{y}_{t+1} - \tilde{y}_{t-1} + E_{t-1}\tilde{y}_t) - \frac{1}{4}(\epsilon_t)$$  \hfill (27)

Therefore, collecting terms in equation 26 yields:

$$\pi_t = E_t\pi_{t+1} + \gamma(\tilde{y}_t + E_t\tilde{y}_{t+1} + \tilde{y}_{t-1} + E_{t-1}\tilde{y}_t) - \frac{1}{2}\epsilon_t$$  \hfill (27)
B Mapping s into contract weights

Expanding equation (7), one obtains

\[ P_t = \theta_1 P_t \]
\[ + \frac{\theta_2}{2} (P_t + P_{t-1}) \]
\[ + \frac{\theta_3}{3} (P_t + P_{t-1} + P_{t-2}) \]
\[ + \frac{\theta_4}{4} (P_t + P_{t-1} + P_{t-2} + P_{t-3}) \]

But \( \theta_4 = 1 - \theta_1 - \theta_2 - \theta_3 \). Using equation (8), combined with the equation above, one can see that

\[ f_0 = \theta_1 + \frac{1}{2} \theta_2 + \frac{1}{3} \theta_3 + \frac{1}{4} (1 - \theta_1 - \theta_2 - \theta_3) \]
\[ f_1 = \frac{1}{2} \theta_2 + \frac{1}{3} \theta_3 + \frac{1}{4} (1 - \theta_1 - \theta_2 - \theta_3) \]
\[ f_2 = \frac{1}{3} \theta_3 + \frac{1}{4} (1 - \theta_1 - \theta_2 - \theta_3) \]

Which leads to

\[
\begin{pmatrix}
\theta_1 \\
\theta_2 \\
\theta_3
\end{pmatrix} = \begin{pmatrix}
\frac{3}{4} & \frac{1}{4} & \frac{1}{12} \\
-\frac{1}{4} & \frac{1}{4} & \frac{1}{12} \\
-\frac{1}{4} & -\frac{1}{4} & \frac{1}{12}
\end{pmatrix}^{-1} \begin{pmatrix}
f_0 \\
f_1 \\
f_2
\end{pmatrix} - \frac{1}{4} \begin{pmatrix}1 \\
1 \\
1\end{pmatrix}
\]

where \( f_i = 0.25 + (1.5 - i)s \), for \( 0 < s \leq \frac{1}{6} \).