TESTING THE VALIDITY OF ECONOMIC GROWTH THEORIES WITH SEEMINGLY UNRELATED REGRESSION MODELS: APPLICATION TO TURKEY IN 1980-2010
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Abstract:
Contrary to the Simultaneous Equation Systems, there is not any type of relation between equations in the Seemingly Unrelated Regression method. What is intended with the correlation state, indicating whether correlated or not, of regression models within the equation system, is the correlation state, indicating whether correlated or not, of error terms related to the models in question. With the help of different exogenous variables, economic growth theories are trying to explain the same endogenous variable, namely, economic growth. The primary objective of this study is to test the joint validity of the growth models, introduced by Solow, Harrod-Domar, Barro and Romer, for the Turkish economy by applying the Seemingly Unrelated Regression method.

Keywords: Growth, Endogenous growth, Neo classical growth, Solow, Harrod-Domar, Barro, Romer, R&D, SUR, Seemingly Unrelated Regression

JEL Codes:

1. Introduction.

By setting up separate regression equations for growth models, which are of the theories put forth in order to explain the reasons of economic growth and are developed by Solow, Harrod-Domar, Barro and Romer, the validity of the growth theories in question for the Turkish economy is examined. Due to the concept of economic growth being a phenomenon trying to be clarified in all mentioned models, it is an encountered fact that "economic growth" is being expressed as the explained variable in all regression models to be set up for economic growth models. The explained variable being economic growth in the model set up for each growth theory brings on an equation system in case of examination of the four theories in question. From this point forward, an equation system constituted of growth models developed by Solow, Harrod-Domar, Barro and Romer will be established for the econometric analysis of this study. "Seemingly Unrelated Regression Models" will be used subsequently for the resolution of the mentioned equation system.

Seemingly unrelated regression models
Econometric theory examines economic phenomena within equation systems with the help of two different methods, first of which is called "Simultaneous Equation Systems", while the second is named as "Seemingly Unrelated Regressions". (Erlat, 2004)

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Contrary to the Simultaneous Equation Systems, there is not any type of relation between equations in the Seemingly Unrelated Regression method. What is intended with the correlation state, indicating whether correlated or not, of regression models within the equation system, is the correlation state, indicating whether correlated or not, of error terms related to the models in question.

Initially introduced by Zellner in 1962, Seemingly Unrelated Regression models were subsequently studied by Telser (1964), Kmenta and Gilbert (1968), Avery (1977), Srivastava and Dwivedi (1979), Baltagi (1980), and Phillips (1977, 1985). (Aktaş, 2004)

Proposed by Zellner (1962), Seemingly Unrelated Regression models are constituted of classical linear regression models in which any variable within the equation system is not appearing in another equation, which means the equation system of which is not a simultaneous equation system. Seemingly Unrelated Regression models are forming an equation system, which is consisting of models fulfilling the assumptions of classical normal linear models of each regression equation appearing within the system.

The Seemingly Unrelated Regression model, consisting of \( i \ (i = 1, \ldots, m) \) set of multivariate linear regression equations is as follows:

\[
\begin{align*}
y_{1t} &= \alpha_{10} + \alpha_{11}x_{12} + \ldots + \alpha_{1j}x_{1j} + u_{1t} \\
y_{2t} &= \alpha_{20} + \alpha_{21}x_{22} + \ldots + \alpha_{2j}x_{2j} + u_{2t} \\
&\vdots \\
y_{it} &= \alpha_{i0} + \alpha_{i1}x_{i2} + \ldots + \alpha_{ij}x_{ij} + u_{it}
\end{align*}
\]

(1)

As illustrated above, the equation system (1) contains \( i \) regression equations. The number of independent variables considered for each equation is shown as \( j \ (j = 1, 2, \ldots, k_i) \). Considering the Seemingly Unrelated Regression model, the number of independent variables in regression models, which appear within the equation system, may differ from each other. In order to perform an accurate calculation of the covariance among error terms in all regression models appearing within the equation system, the number of observations \( (t = 1, 2, \ldots, T) \) of the variables of each model appearing within the equation system has to be constant.

The assumptions of Seemingly Unrelated Regression models are as follows:

- Considering that the Seemingly Unrelated Regression model is constituted of more than one regression model, each regression model is
expected to fulfill the assumptions of the classical normal linear regression model.

- \( X_{mj} \) is non-stochastic and \( X'_{mj} X_{mj} \) is non-singular, that is to say the matrix has a determinant which is non-zero. \( \lim_{T \to \infty} \frac{X'_{mj} X_{mj}}{T} \) exists.

Taking the assumptions in question into account, in order to apply the Seemingly Unrelated Regression model, some conditions need to be fulfilled:

- Estimating more than one regression equation with respect to the addressed economic phenomenon.

- A correlation between error terms of the estimated regression equations.

- These regression equations, which include correlated error terms, should not form a simultaneous system, in other words, there should be no seeming relation between the regression equations.

There are many methods used for the estimation of the seemingly unrelated regression model, which can be listed as follows:

- The Ordinary Least Squares (OLS) Method

- The Generalized Least Squares (GLS) Method

- The Feasible Generalized Least Squares (FGLS) Method

- The Maximum Likelihood (ML) Method

The variance-covariance matrix of error terms is jointly applied in these estimation methods of the Seemingly Unrelated Regression model. Without the definition of the variance-covariance matrix or its estimations, it is impossible to estimate the parameters of the Seemingly Unrelated Regression model. (Nemlioglu, 1990)

From the mentioned estimation methods, the one used in this study is the Feasible Generalized Least Squares (FGLS) method. The underlying reason is that it is considered as a method providing a more realistic approach because what is taken into account during the application of the mentioned method is the estimation of the variance-covariance matrix, rather than its definition.
The contemporaneous $\Omega$ covariance matrix is rarely known in practice. (Davidson, 2004) Rather than being defined, in most frequent cases in practice the $\Omega$ matrix is undefined or estimable. In case of the $\Omega$ matrix being estimable, the method, which can be used for the estimation of the Seemingly Unrelated Regression model, is the Feasible Generalized Least Squares (FGLS) method. The only difference between the GLS and FGLS method used for the estimation of the Seemingly Unrelated Regression model, is that the estimation is carried out by estimating the $\Omega$ matrix. As it is understood, the FGLS method is a two-staged estimation method. The initial step to be taken while applying this method is to estimate the error terms of the regression models by estimating each regression model within the equation system with the use of the Feasible Generalized Least Squares method and with the use of these error terms to estimate the $\Omega$ variance-covariance matrix as shown below:

$$s_{mp} = \frac{1}{T - k_m} \sum_{t=1}^{T} e_m e_p^{\top} \quad k_m > k_p \quad m, p = 1, 2, \ldots, M$$

(2)

$$\hat{\Omega} = \begin{pmatrix} s_{11}I_T & \cdots & s_{1m}I_T \\ \vdots & \ddots & \vdots \\ s_{m1}I_T & \cdots & s_{mm}I_T \end{pmatrix}$$

(3)

$$\hat{\beta} = (X'\hat{\Omega}^{-1}X)^{-1}(X'\hat{\Omega}^{-1}y)$$

(4)

Here, $e_m$ and $e_p$ are the error terms obtained from the equation system. $T$ is the total number of observations and $k_m$ represents the number of independent variables in each regression equation. Given in the calculation for the estimation of the variance-covariance matrix of error terms, $e_m e_p^{\top}$ represent the cross product matrices obtained after the estimation of error terms in the equation system. From this point forward, the estimated $s_{mp}$ variance or covariance values are calculated and multiplied by $I_T$ identity matrix by applying the Kronecker product, and thus $\hat{\Omega}$ is obtained.

Considering the Feasible Generalized Least Squares (FGLS) method, when the elements, which represent the covariance of error terms and are excluded from the diagonal elements representing the variance of error terms of the $\hat{\Omega}$ matrix, are equal to zero, it is ensured that the FGLS and OLS estimations are the same.

By applying the Generalized Least Squares method instead of the Ordinary Least Squares method in order to carry out the estimation of the equation system...
consisting of growth models developed by Solow, Harrod-Domar, Barro and Romer, much lower standard errors of parameter estimators are produced; thereby, more efficient estimators are obtained.

Due to the relevant models being constituted of an equation system, the classical coefficient of determination $R^2$ in Seemingly Unrelated Regression models is computed differently than the coefficient of determination $R^2$, as illustrated below:

$$R^2 = 1 - \frac{\bar{e}'\hat{\Omega}^{-1}\bar{e}}{y'N_{nt}\hat{\Omega}^{-1}N_{nt}y}$$

(5)

**ECONOMETRIC APPLICATION RESULTS BETWEEN THE YEARS 1980-2010 WITH TURKEY AS AN EXAMPLE**

In the econometric application part of this study, the equation system, which is constituted of growth models developed by Solow, Harrod-Domar, Barro and Romer, is estimated by the application of Seemingly Unrelated Regression models. The growth model developed by Solow points out that output and labour are determinant factors of economic growth, while the Harrod-Domar model indicates that the source of economic growth is investments. In the model developed by Barro, the source of economic growth is defined as public expenditures, whilst in the model introduced by Romer the source is claimed to be R&D (research and development) expenditures.

In accordance with the aim of this study the variables defined are GNP, total investment, total public expenditures, R&D expenditures, total input and labour. From the mentioned variables, the defined observation period for the R&D expenditures variable is between the years 1990-2010, as for the other variables this period is between the years 1980-2010. All variables (with the exception of the labour variable) are obtained from the electronic data delivery system of the Central Bank of the Republic of Turkey (CBRT/EDDS) as Thousand TRY (New Turkish Lira). All variables are expressed as real variables and in conclusion of the study the natural logarithms of variables are taken.

From this point forth, the equation system subject to analysis is as illustrated below:

$$\ln(GNP)_t = \alpha_{10} + \alpha_{11} \ln(INPUT)_t + \alpha_{12} \ln(LABOUR)_t + u_{1t}$$

$$\ln(GNP)_t = \alpha_{20} + \alpha_{21} \ln(INVESTMENTS)_t + u_{2t}$$

(6)
ln(GNP)ₜ = α₃₀ + α₃₁ ln(GOV.EXP)ₜ + u₃ₜ

ln(GNP)ₜ = α₄₀ + α₄₁ ln(R&D.EXP)ₜ + u₄ₜ

Equations included in the equation system (6) represent the Solow, Harrod-Domar, Barro and Romer models, respectively. While in the regression equations, expressed for the Sollow, Harrod-Domar and Barro models, the total number of observations for each equation was defined as 27, the total number of observations for the regression equation for the Romer model was defined as 17. Considered from this point of view, being one of the necessary assumptions for the Seemingly Unrelated Regression model, the assumption that the number of observations of variables included in each equation of the equation system are equal, is fulfilled. On the other hand, as a result of the examination, the assumptions of the classical normal regression model, which is necessary for each regression model included in the equation system, are also observed to be fulfilling.

Within the frame of the Seemingly Unrelated Regression model, the equation system (6) is estimated with the application of the Feasible Generalized Least Squares (FGLS) method and the help of EViews 5.0 software package. Accordingly, the estimation results for the Harrod-Domar, Barro and Romer models are as follows, respectively:

**Table 1: EViews Printout of the Seemingly Unrelated Regression Model**

Estimation Method: Seemingly Unrelated Regression
Sample: 1 31
Included observations: 27
Total system (unbalanced) observations 109
Linear estimation after one-step weighting matrix

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C(11)</td>
<td>0.497005</td>
<td>0.029095</td>
<td>17.08188</td>
</tr>
<tr>
<td>C(10)</td>
<td>5.309937</td>
<td>0.224578</td>
<td>23.64403</td>
</tr>
<tr>
<td>C(20)</td>
<td>-8.939363</td>
<td>2.439770</td>
<td>-3.664019</td>
</tr>
<tr>
<td>C(21)</td>
<td>0.393325</td>
<td>0.097322</td>
<td>4.041499</td>
</tr>
<tr>
<td>C(22)</td>
<td>1.509026</td>
<td>0.318170</td>
<td>4.742836</td>
</tr>
<tr>
<td>C(30)</td>
<td>7.714103</td>
<td>0.162010</td>
<td>47.61511</td>
</tr>
<tr>
<td>C(31)</td>
<td>0.396160</td>
<td>0.039211</td>
<td>10.10324</td>
</tr>
<tr>
<td>C(40)</td>
<td>2.060827</td>
<td>0.496930</td>
<td>4.147115</td>
</tr>
<tr>
<td>C(41)</td>
<td>0.929782</td>
<td>0.065095</td>
<td>14.28341</td>
</tr>
</tbody>
</table>

Determinant residual covariance 1.32E-09
Equation: $\text{LOG}(\text{GDP}) = C(11) \times \text{LOG} (\text{GOVEXP}) + C(10)$
Observations: 27

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>R-squared</td>
<td>0.89862</td>
</tr>
<tr>
<td>Mean dependent var</td>
<td>9.143700</td>
</tr>
<tr>
<td>Adjusted R-squared</td>
<td>0.8945</td>
</tr>
<tr>
<td>S.D. dependent var</td>
<td>0.321490</td>
</tr>
<tr>
<td>S.E. of regression</td>
<td>0.104420</td>
</tr>
<tr>
<td>Sum squared resid</td>
<td>0.272589</td>
</tr>
<tr>
<td>Durbin-Watson stat</td>
<td>0.499245</td>
</tr>
</tbody>
</table>

Equation: $\text{LOG}(\text{GDP}) = C(20) + C(21) \times \text{LOG} (\text{INPUT}) + C(22) \times \text{LOG} (\text{LABOUR})$
Observations: 22

<table>
<thead>
<tr>
<th>Parameter</th>
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</thead>
<tbody>
<tr>
<td>R-squared</td>
<td>0.955253</td>
</tr>
<tr>
<td>Mean dependent var</td>
<td>9.051558</td>
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<tr>
<td>Adjusted R-squared</td>
<td>0.950542</td>
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<tr>
<td>S.D. dependent var</td>
<td>0.277803</td>
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<tr>
<td>S.E. of regression</td>
<td>0.061781</td>
</tr>
<tr>
<td>Sum squared resid</td>
<td>0.072521</td>
</tr>
<tr>
<td>Durbin-Watson stat</td>
<td>1.283277</td>
</tr>
</tbody>
</table>

Equation: $\text{LOG}(\text{GDP}) = C(30) + C(31) \times \text{LOG} (\text{RD})$
Observations: 17

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>R-squared</td>
<td>0.834372</td>
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<tr>
<td>Mean dependent var</td>
<td>9.350267</td>
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<tr>
<td>Adjusted R-squared</td>
<td>0.823330</td>
</tr>
<tr>
<td>S.D. dependent var</td>
<td>0.178827</td>
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<tr>
<td>S.E. of regression</td>
<td>0.075165</td>
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<tr>
<td>Sum squared resid</td>
<td>0.084746</td>
</tr>
<tr>
<td>Durbin-Watson stat</td>
<td>1.195282</td>
</tr>
</tbody>
</table>

Equation: $\text{LOG}(\text{GDP}) = C(40) + C(41) \times \text{LOG} (\text{INVESTMENTS})$
Observations: 27

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>R-squared</td>
<td>0.850904</td>
</tr>
<tr>
<td>Mean dependent var</td>
<td>9.143700</td>
</tr>
<tr>
<td>Adjusted R-squared</td>
<td>0.844940</td>
</tr>
<tr>
<td>S.D. dependent var</td>
<td>0.321490</td>
</tr>
<tr>
<td>S.E. of regression</td>
<td>0.126595</td>
</tr>
<tr>
<td>Sum squared resid</td>
<td>0.400660</td>
</tr>
<tr>
<td>Durbin-Watson stat</td>
<td>0.370364</td>
</tr>
</tbody>
</table>

Estimation results of the **Solow** model:

$$\ln(\text{GNP})_t = -8.939 + 0.393 \ln(\text{INPUT})_t + 1.509 \ln(\text{LABOUR})_t$$

$(SE) \quad (2.439) \quad (0.097) \quad (0.318)$

$t \quad -3.664 \quad 4.041 \quad 4.742$

$R^2 = 0.955 \quad \bar{R}^2 = 0.950$

Estimation results of the **Harrod-Domar** model:
\[ \ln(GNP)_t = \alpha_{20} + \alpha_{21} \ln(INVESTMENTS)_t \]

\[
\begin{array}{l}
SE \hspace{1cm} 0.496 \hspace{1cm} 0.065 \\
t \hspace{1cm} 4.147 \hspace{1cm} 14.283
\end{array}
\]

\[ R^2 = 0.850 \]

Estimation results of the **Barro** model:

\[ \ln(GNP)_t = \alpha_{30} + \alpha_{31} \ln(GOV.EXP)_t + u_{3t} \]

\[
\begin{array}{l}
SE \hspace{1cm} 0.029 \hspace{1cm} 0.224 \\
t \hspace{1cm} 17.081 \hspace{1cm} 23.644
\end{array}
\]

\[ R^2 = 0.898 \]

Estimation results of the **Romer** model:

\[ \ln(GNP)_t = \alpha_{40} + \alpha_{41} \ln(R&D.EXP)_t + u_{4t} \]

\[
\begin{array}{l}
SE \hspace{1cm} 0.162 \hspace{1cm} 0.039 \\
t \hspace{1cm} 47.615 \hspace{1cm} 10.103
\end{array}
\]

\[ R^2 = 0.834 \]

Obtained from the equation system (6) estimated with the Seemingly Unrelated Regression method, with the aim of testing the validity of the growth models introduced by Solow, Harrod-Domar, Barro and Romer for the Turkish economy, the results of the study prove that the total input and labour variables for the Solow model, with the t-test carried out at the %5 level of significance, are significant variables. Similarly, within the frame of the Harrod-Domar model, the influential factor of economic growth is defined as investments; in the Barro model as public expenditures; finally in the Romer model as R&D expenditures. The coefficients of determination of the mentioned models are defined, respectively, as %95, %85, %89 and %83.

**According to these results;**

The estimated growth models introduced by Solow, Harrod-Domar, Barro and Romer, prove their validity for the Turkish economy. One benefit of “The Feasible Generalized Least Squares (FGLS) Method”, used for the resolution of the equation system (6), is that it leads to more efficient estimations.

The results acquired from the Solow model indicate that a %1 increase in labour
causes a %1.50 increase in national income. Furthermore, a 1% increase in input leads to a %0.40 increase in national income.

The results obtained from the Harrod-Domar model, on the other hand, show that a %1 increase in investment brings forth an approximate %0.92 increase in national income.

Also, the results of the Barro model demonstrate that a %1 increase in public expenditure gives rise to a %5.30 increase in national income.

Finally, the results obtained from the Romer model illustrate that a %1 increase in R&D causes a %0.40 increase in national income.

In line with all these results, considering that it provides a %5.30 increase in national income, public expenditure is the most important source of economic growth for the Turkish economy. Considering both increases of %0.40, the relatively least contributing factors of economic growth are input and R&D expenditures. Accordingly, public expenditure can be identified as the main source of economic growth for the Turkish economy, which in turn supports the idea that public demand revives production and confirms economic structures with the involvement of government intervention in production increase, as represented in the Barro model.

**Conclusion**

Economic growth is of vital importance for the development of countries. In all economies, the scale of production needs to be improved in order to produce output. Furthermore, inputs like labour, human capital, natural resources, real capital, etc. need to be brought together at different rates and within the scope of different technological information in order to obtain an increase in the per capita product.

The insufficiency of the classical factors of production in explaining modern economic phenomena during the process of economic growth led to new quests. As a matter of fact, internal growth theories emerged because of this demand. The key factor of economic development and growth is the human being; however, labour productivity certainly also necessitates the improvement of information competency skills. A. Smith was the first to contribute to addressing the human element in terms of investment. Afterwards, studies of economists like Lucas, Romer, Barro, Schultz pointed out the importance of the contribution of human capital to economic growth.

Economic growth necessitates the improvement of physical, human and social opportunities; a level which can be reached by whole community’s use of its opportunities at a certain degree.
A. Smith noted that the decrease in misgovernment and prodigality in the public sector would lead to the increase in growth and attached knife-edge balance conditions to the proposed growth defined by the Harrod-Domar growth model. As a growing power which is providing consistency, the government`s constant intervention in economy was thereby brought up. As for the neoclassical growth model, the government intervention was removed but it was pointed out that with time developed economies would move into a steady-state.

Internal growth models internalized factors like R&D, technological development, information, human capital, which in other models are external; pulled economies out of moving into a steady-state, and turned the government into a newly active power in economy.

In our econometric application, which is comprising the years between 1980-2010 with Turkey as an example, the contributions of public expenditure, R&D expenditures, labour, input added to production, investments and of other variables to growth have been observed to emerge as a result of the established models. Considering that it provides a % 5.30 increase in national income, public expenditure can be regarded as the most important source of economic growth for the Turkish economy. On the other hand, when considering both increases of %0.40, the relatively least contributing factors of economic growth are input and R&D expenditures. In this case, public expenditure can be identified as the main source of economic growth for the Turkish economy, which in turn supports the idea that public demand revives production and confirms economic structures with the involvement of government intervention in production increase, as represented in the Barro model.

References


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