Abstract
This paper investigates the empirical saving-investment relationships for Côte d’Ivoire and Ghana over the period 1960–1998, using a Markov Switching VAR model. We find regime-dependent causality from saving to investment in Côte d’Ivoire but not in Ghana. In terms of Feldstein and Horioka (1980) capital mobility hypothesis these findings suggest a more capital mobility in Ghana than in Côte d’Ivoire implying that foreign capital flows towards Côte d’Ivoire and not to Ghana during the studied period.

JEL Classification: I20, C22, C51, O54
Keywords: Saving, Investment, Markov Switching VAR, Causality.

1. Introduction
Following the seminal paper by Feldstein and Horioka (1980), the relationships between saving and investment have been studied by several authors including Baxter and Crucini (1993); Coaley and Kulazsi, (1997); Obstfeld and Rogoff (1995); Cadoret (2001). The suggested relationships between saving and investment by Feldstein and Horioka (1980) are modelled as follows:

\[
\frac{I}{Y}_{it} = \alpha + \beta \frac{S}{Y}_{it} + \varepsilon_{it}
\]  

(1)

where \((I/Y)\) is the ratio of gross domestic investment to domestic product, \((S/Y)\) is the ratio of gross domestic saving to domestic product; \(\beta\) is the saving-investment retention coefficient and an error term.

This equation (1) allows for the investigation of the full capital mobility hypothesis. In case of perfect capital mobility there should be no relation between domestic saving and investment and \(\beta\) is expected to be close to zero, which means that investment decision in one country should be independent of domestic level of saving and reciprocally. The interpretation is that saving in each country responds globally to worldwide opportunities for higher profit investment while domestic investment is financed by the worldwide capital.

While a lot of literature has extensively analysed the Feldstein and Horioka equation; the results obtained are mixed according to time period. These studies make use of various methods as cross-section or panel data model estimation (Feldstein and Horioka (1980), Feldstein (1983)); time series modeling (Obstfeld (1986), Sesaiah and Sriyval (2005)), to our knowledge none has used a non-linear model, particularly for developing countries. In this paper we aim at filling this gap by extending the existing literature on this matter, with a special emphasis on two African countries’ experiences.

This paper is an attempt to use an alternative methodology based on multivariate Markov switching model to test for causal link between saving and investment. Two reasons motivate the use of such a model: (i) the worldwide environment is fluctuating

* Associate Professor, Department of Economics, University of Bouaké, Côte d'Ivoire, & Researcher, University of Luxembourg, CREA, 162A, avenue de la Faïencerie, L-1511 Luxembourg. Email: francois.aka@uni.lu or akbdia@yahoo.fr
and thus one could expect the value of the $\beta$ parameter to be non-constant across time span. Cadoret (2001) has confirmed this observation and to overcome the variability of the coefficient, the equation (1) has been often estimated using sub-periods after detecting structural breaks on the entire period (Kanas, 2005). This procedure supposes a prior knowledge of the break dates, (ii) splitting the sample impoverishes the data and doesn’t allow to seizing the whole phenomenon. Moreover breaks are recurrent and require non-linear model as Markov switching model.

The main objective of the paper is to use a multivariate Markov switching model introduced by Krolzig (1997a; 1997b), Krolzig and Toro (1999) as a generalization of Hamilton (1989; 1990) univariate model to investigate the relationships between saving and investment. The MS-VAR approach has the advantage not only to avoid splitting the sample period under study into sub-periods but also the variability and structural change of the link between saving and investment over time is endogenously taken into account in the model.

Our major findings are as follows. We find regime-dependent causality from saving to investment in Côte d’Ivoire but not in Ghana.

Section 2 presents the methodology used in the paper, Section 3 the econometric results and finally Section 4 concludes.

2. Econometric Methodology

Suppose we intend to analyse Granger causality between the bivariate series $\{x_{t} = [x_{1,t} : x_{2,t}]\}$ Granger causality analysis is based on the following MS-VAR model:

\[
\begin{align*}
\dot{x}_{1,t} &= \mu_{x_{1}}(s_{t}) + \sum_{k=1}^{p} \Phi_{x_{1},x_{1},k}(s_{t})x_{1,t-k} + u_{x_{1,t}} \\
\dot{x}_{2,t} &= \mu_{x_{2}}(s_{t}) + \sum_{k=1}^{p} \Phi_{x_{2},x_{1},k}(s_{t})x_{1,t-k} + \sum_{k=1}^{p} \Phi_{x_{2},x_{2},k}(s_{t})x_{2,t-k} + Z(s_{t})u_{x_{2,t}}
\end{align*}
\]

where $s_{t}$ is an unobservable random variable indicating the state of regime at date $t$, and $Z(s_{t})$ is a regime dependent matrix. The transition probabilities $p_{ij}$ are assumed:

\[
p_{ij} = P(s_{t+1} = j | s_{t} = i) \sum_{j=1}^{2} p_{ij} = 1 \quad \forall i, j \in \{1,2\}
\]

To test for Granger causality from $x_{2}$ to $x_{1}$ in regime 1, the null hypothesis of non-causality is

\[
H_{0}: \Phi_{x_{1},x_{2},1}(s_{t} = 1) = \cdots = \Phi_{x_{1},x_{2},p}(s_{t} = 1) = 0
\]

This null hypothesis test is conducted using the MS-VAR model by imposing restriction on the values of the autoregressive parameters.

In fact the significance of the regime dependent autoregressive parameter $\Phi_{x_{1},x_{2}}$ in equation 1 infers causality from $x_{2}$ to $x_{1}$ in regime 1. On the other hand the significance of $\Phi_{x_{2},x_{1}}$ infers causality from $x_{1}$ to $x_{2}$ in the concerned regime.
3. Econometric Results

The yearly data for Côte d’Ivoire and Ghana are from World Development Indicators (WDI) and cover the period 1960–1998. We choose 2 neighboring countries Côte d’Ivoire, which belong to a monetary union (WAEMU) and Ghana, which has own money. Investment is measure by \( \frac{I}{Y} \) the ratio of gross domestic investment to domestic product, while saving \( \frac{S}{Y} \) is the ratio of gross domestic saving to domestic product. Figure 1 contains the log of the variable (top panel) and the growth rate (bottom panel). We observe that saving and investment grow from 1960 to the peak around 1979 in Côte d’Ivoire and decline to the trough around 1990 for investment and around 1993 for saving. In average saving has been higher than investment in Côte d’Ivoire In Ghana saving and investment decrease from 1960 to the trough around 1982-1983 and after 1982, investment grows faster than saving, investment being higher than saving in average.It could be observed from Figure 1 that saving could not finance investment in Ghana but the reverse holds in Côte d’Ivoire.

The variable \( x_t = [lnv_t = x_{1,t}; sav_t = x_{2,t}] \) and the estimations results are provided in Table 1\(^1\).

The likelihood ratio (LR) test suggests that the null hypothesis of no regime switching or equivalently to linear VAR model against the alternative of regime switching is rejected. The rejection is equivalent to the rejection of linear VAR in favour of the MS-VAR model. Moreover the log likelihood (LL) indicates that a 2-Lag VAR (against 1 Lag) is suitable to the MS-VAR in the Côte d’Ivoire while a 1-Lag VAR is suitable in Ghana, which is confirmed by the computed Bayes Factor (BF)\(^2\). These results indicate that the bivariate saving-investment relationships are characterized by volatility regime switching and subject to regime switching.

The standard deviation of variables across regime is 0.10 in Côte d’Ivoire for saving and 0.13 for invest in regime 1 while in regime 2, standard deviation is 0.10 for saving and 0.05 for investment. In Côte d’Ivoire Investment and saving are both more volatile in regime 1 than in regime 2.

In Ghana investment and saving are both more volatile in regime 1 than in regime 2 (standard deviation are 0.39 for saving in regime 1 and 0.22 in regime 2, while standard deviation of invest is 0.32 in regime 1 and only 0.10 in regime 2). Regime 2 is a low volatility regime in Ghana and in Côte d’Ivoire as well.

The transition probability from regime 1 to regime 1 is 0.82 and the transition probability from regime 2 to regime 2 is 0.35, which indicates that regime 1 is very persistent compare to regime 2 in Côte d’Ivoire. But in Ghana the two regimes are persistent (0.78 in regime 1 and 0.77 in regime 2).

The value of the contemporaneous correlation reflects the contemporaneous link between saving and investment. During regime 1 (the high volatility regime) the correlation is negative −1.167 and positive 0.904 in the low volatility regime (regime 2) for Côte d’Ivoire. In Ghana this correlation is 0.366 in regime 1 and 0.086 in regime 2 thus positive in both regimes.

\(^1\) Estimations are realized using MS-VAR package from PcGive 10 (Hendry and Doornik 2001).

\(^2\) The BF(1:2) interpretation (see Poirier 1995). B12 > 1, evidence supports H1.10e-1/2 < B12 < 1, very slight evidence against H1. 10e-1 < B12 < 10e-.1/2, slight evidence against H1.10e-2 < B12 < 10e-1, strong to very strong evidence against H1. B12 < 10e-2, decisive evidence against H1.
These results reveal a dynamic regime dependent link between saving and investment both in Côte d’Ivoire and Ghana. In Côte d’Ivoire the correlation is high during low volatility regime while in Ghana the correlation is high in the high volatility regime.

Table 1. MS-VAR Model Estimation

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Côte d’Ivoire</th>
<th>Ghana</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Regime 1</td>
<td>Regime 2</td>
</tr>
<tr>
<td>$\mu_{sav}$</td>
<td>-0.035 (-1.68)</td>
<td>0.277* (5.33)</td>
</tr>
<tr>
<td>$\phi_{sav,sav1}$</td>
<td>-0.146 (-0.93)</td>
<td>-1.745* (-7.64)</td>
</tr>
<tr>
<td>$\phi_{sav,sav2}$</td>
<td>0.378* (2.39)</td>
<td>-1.087 (*-6.06)</td>
</tr>
<tr>
<td>$\phi_{sav,inv1}$</td>
<td>0.110 (0.460)</td>
<td>-0.654* (-7.21)</td>
</tr>
<tr>
<td>$\phi_{inv,sav1}$</td>
<td>-0.080 (-0.59)</td>
<td>1.838* (6.15)</td>
</tr>
<tr>
<td>$\phi_{inv,sav2}$</td>
<td>-0.743* (-4.24)</td>
<td>0.624* (3.92)</td>
</tr>
<tr>
<td>$\phi_{inv,inv1}$</td>
<td>0.316 (1.82)</td>
<td>0.654* (4.45)</td>
</tr>
<tr>
<td>$\phi_{inv,inv2}$</td>
<td>-0.172 (-0.78)</td>
<td>0.280* (3.36)</td>
</tr>
<tr>
<td>$\sigma_{sav}$</td>
<td>0.1035</td>
<td>0.1029</td>
</tr>
<tr>
<td>$\sigma_{inv}$</td>
<td>0.1345</td>
<td>0.0549</td>
</tr>
<tr>
<td>C. Correlation</td>
<td>-1.167</td>
<td>0.904</td>
</tr>
<tr>
<td>$p_{11}$</td>
<td>0.820</td>
<td>0.786</td>
</tr>
<tr>
<td>$p_{22}$</td>
<td>0.355</td>
<td>0.0549</td>
</tr>
<tr>
<td>log Like.</td>
<td>50.003 (25.48)</td>
<td>50.000 (25.48)</td>
</tr>
<tr>
<td>LR</td>
<td>49.04</td>
<td>25.40</td>
</tr>
<tr>
<td>AIC</td>
<td>-1.22 (-0.69)</td>
<td>1.66 (1.76)</td>
</tr>
<tr>
<td>HQ</td>
<td>-0.79 (-0.49)</td>
<td>1.97 (1.89)</td>
</tr>
<tr>
<td>Bayes Factor (BF)</td>
<td>BF(1;2) = 37.10 / 50.00 = 0.60</td>
<td>BF(1;2) = -10.85 / -4.56 = 2.37</td>
</tr>
</tbody>
</table>

Notes: t-ratios in parentheses; * denotes significance at 0.05 level; BF(1;2) is Bayes factor 1-Lag model against 2-Lag model.

We notice that the regime dependent autoregressive parameters $\phi_{sav,inv1}$ and $\phi_{sav,inv2}$ are statistically significant in regime 2 but not in regime 1 for Côte d’Ivoire. These findings suggest that investment has cause saving only in regime 2 in Côte d’Ivoire. Similarly we can see that $\phi_{inv,sav2}$ is statistically significant in both regime while $\phi_{inv,sav1}$ is significant only on regime 1, meaning that causality runs from saving...
to investment in Côte d’Ivoire in regime 1. None of the autoregressive parameters are significant in Ghana meaning that causality from saving to investment is rejected in Ghana.

To formally test for causality from saving to investment we restrict the model and test the null of the autoregressive parameters being zero (the non causality hypothesis). The Likelihood Ratio of the unrestricted model is always greater than the restricted one. The LR-test results (Table 2) confirm the findings for Côte d’Ivoire and Ghana. We find causality from saving to investment in Côte d’Ivoire but not in Ghana.

<table>
<thead>
<tr>
<th>Model</th>
<th>LR Côte d’Ivoire</th>
<th>LR Ghana</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unrestricted</td>
<td>50.00</td>
<td>-10.85</td>
</tr>
<tr>
<td>Restricted</td>
<td>41.30</td>
<td>-12.62</td>
</tr>
<tr>
<td>LR-test</td>
<td>17.39</td>
<td>3.5</td>
</tr>
<tr>
<td>$\chi^2$</td>
<td>15.5</td>
<td>15.5</td>
</tr>
<tr>
<td>Result</td>
<td>Reject the null</td>
<td>Accept the null</td>
</tr>
</tbody>
</table>

In terms of capital mobility these findings could be interpreted as a more capital mobility in Ghana where saving and investment are independent than in Côte d’Ivoire where they are not. This implies that foreign capital flows towards Côte d’Ivoire and not to Ghana in the studied period, which could be explained in part by the fact that Côte d’Ivoire being embedded in monetary zone (WAEMU) with a supra national currency makes this country a more secure place for investors than Ghana which has its own
currency. In effect Côte d’Ivoire was a very stable country for investors before the political coup of December 1999 and war since September 2002, while Ghana has been instable from 1966 until recent years. Moreover the investment code (law) in Côte d’Ivoire is very favourable to investors, which are allowed to take a large part of earnings out of the country.

5. Conclusion
The variability and structural change of the link between saving and investment over time is endogenously taken into account in this paper. We investigate the saving-investment relationships for two African countries using a MS-VAR approach, which reveals the non-homogeneity of the relation between the 2 variables over the period of study. We find causality from saving to investment in Côte d’Ivoire but not in Ghana.

In terms of capital mobility these findings suggest a more capital mobility in Ghana than in Côte d’Ivoire implying that foreign capital flows towards Côte d’Ivoire and not to Ghana. With the history of both countries in mind, Côte d’Ivoire being very stable politically before the coup (December 1999) and war (September 2002) while Ghana was instable since 1966 up to recent years, these results confirm the prior need for a peaceful environment to attract foreign investment in the sub-Saharan African region.

References
Journal published by the EAAEDS: http://www.usc.es/economet/eaa.htm