Abstract
This paper shows that the Solow model’s predictions are consistent with the data. The standard of living is correlated positively with saving rates and negatively with population growth rates, while just these two variables explain jointly 67% to 73% of the sample’s cross-country variation. The empirical findings clearly reject absolute convergence in income per capita but are very strongly supportive of conditional convergence at an estimated average annual rate of 0.8% to 1.2% a year. It is also shown that the speed of convergence is far from constant over time: it has been mostly increasing during 1960-1990, but it has been falling since the early 1990s.
JEL classification: O40.
Keywords: Solow Model, Economic Growth, Convergence.

1. Introduction

The Solow (1956) growth model is one of the most widely used models in economics. Its usefulness is easily demonstrated by the extremely wide range of economic applications which employ it as a building block.¹ Not surprisingly, a substantial amount of empirical research has been devoted to the investigation of the validity of the Solow model’s predictions. The most influential of these studies is the contribution by Mankiw, Romer, and Weil (1992), who concluded that the empirical evidence is strongly consistent with (a somewhat modified) Solow model. These results, however, have been challenged by Bernanke and Gurkaynak (2001), who argue that an alternative class of growth models, the so-called endogenous growth models,² are more consistent with the data.

This paper updates the Mankiw, Romer, and Weil (1992) empirical study, using a data set that adds almost twenty years of additional observations for each of the countries.³

First, the paper tests the standard Solow model, using two data sets: one of 58 countries covering the period 1950-2003, and another of 99 countries over 1960-2003. The empirical results suggest that Solow’s predictions are consistent with the data: the

¹ Consider, for example, the following three recent working papers in areas as diverse as business-cycles (Arias, Hansen, and Ohanian, 2006), environmental economics (Brock and Taylor, 2004), and health and development (Acemoglu and Johnson, 2006).


³ Karras (2007) provides a similar study, but uses an earlier data set (PWT6.1) with a shorter time period.
standard of living is correlated positively with saving rates and negatively with population growth rates. Furthermore, these two variables explain two-thirds to three-quarters (67% to 73%) of the sample’s cross-country variation in income per capita.

Next, the paper tests several augmented variants of the Solow model, which allow for additional determinants of an economy’s steady-state standard of living. There is some evidence that income per capita is related positively to trade openness and negatively to government size, but the magnitudes and statistical significance of these effects are smaller and more fragile than those of investment and population growth.

Third, the paper’s empirical evidence clearly rejects absolute convergence in income per capita and, in fact, provides some evidence of (statistically insignificant) divergence. However, the results are very strongly supportive of conditional convergence. This implies that countries may be generally approaching different steady states, but when saving and population growth rates are taken into account, there is convergence at an estimated rate of 0.8% to 1.2% a year.

Finally, the paper estimates time-varying convergence rates for several sets of countries. The main finding is that the speed of convergence, which has been mostly increasing during 1960-1990, has been falling since the early 1990s.

The rest of the paper is organized as follows. The empirical methodology is outlined in section 2, while section 3 discusses the data sources and definitions. The empirical results are presented and discussed in section 4. Section 5 concludes.

2. Empirical Methodology

The methodology follows the approach of Mankiw, Romer, and Weil (1992). Assume that the production function is given by the Cobb-Douglas specification

\[ Y_i = K_i^\beta [A_iN_i]^{1-\beta}, \]

where \( Y \) is output, \( K \) is the capital stock, \( A \) captures the level of technology, \( N \) is employment, and \( 0 < \beta < 1 \). Exogenous growth rates for \( N \) and \( A \) are given by:

\[ \dot{N}_i/N_i = n \quad \text{and} \quad \dot{A}_i/A_i = a, \]

where a dot indicates a time derivative. A standard assumption of the Solow (1956) model is that a constant fraction of income, \( s \), is saved (0 < \( s < 1 \)). Mankiw, Romer and Weil (1992) show that this implies that the level of income per capita at the steady state will be given by:

\[ \ln(\frac{Y}{N}) = a + \frac{\beta}{1-\beta} \ln(s) - \frac{\beta}{1-\beta} \ln(n) + \epsilon, \]

where \( \epsilon \) is an error term. This will form the basis of our first cross-sectional estimated equation:

\[ \ln(\frac{Y}{N}) = \gamma_0 + \gamma_1 \ln(s_i) + \gamma_2 \ln(n_i) + \epsilon_i, \]
where $i$ is indexing over countries and a bar will indicate country-specific average values over a certain time period. Thus, $\bar{s}$ is the average saving rate, $\bar{n}$ the average population growth rate, and the $\gamma$’s are the parameters to be estimated. Simple inspection of (2) and (3) establishes the Solow model’s predictions: $\gamma_0 > 0$, $\gamma_1 > 0$ (so that a higher saving rate raises the steady-state level of per capita income), and $\gamma_2 < 0$ (so that a higher population growth rate reduces the steady-state level of income per capita).

To allow for the possibility of additional factors determining the steady state, “augmented” versions of the Solow model can be estimated. Here we focus on two additional variables: the government size and the degree of the economy’s trade openness. Letting $G$ denote government purchases, $EX$ exports, and $IM$ imports, we define government size as $G/Y$, the government’s share of GDP, and trade openness as $OPEN = (EX + IM)/Y$, total trade as a fraction of GDP. Then, the augmented version of equation (3) is:

$$\ln\left(\frac{Y}{N}\right)_i = \delta_0 + \delta_1 \ln(\bar{s}_i) + \delta_2 \ln(\bar{n}_i) + \delta_3 \ln(G/Y)_i + \delta_4 \ln OPEN_i + \omega_i,$$

where $G/Y$ is average government size, $OPEN$ is average trade openness, $\omega$ is the error term, and the $\delta$’s are the parameters to be estimated. Once more, the theoretical implication is that $\delta_1 > 0$ and $\delta_2 < 0$. In terms of the two new variables, $\delta_3$ is ambiguous in sign, as the distortionary effects of taxes may or may not be offset by the productive effects of government activities; while $\delta_4$ is expected to be positive.

The Solow framework can also be used to investigate the speed of convergence to the steady state. Letting $y_i = Y_i/N_i$ denote per capita income, and $\lambda$ be the convergence rate, the model also implies:

$$\ln(y_T) - \ln(y_0) = (1 - e^{-\lambda t})\ln(y^{ss}) - (1 - e^{-\lambda T})\ln(y_0).$$

(5)

Testing for unconditional (or, “absolute”) convergence, we start by assuming that the steady-state values are the same for each country. Then equation (5) can be written in regression format as

$$[\ln(y_T) - \ln(y_0)]_i = \theta_0 + \theta_1 \ln(y_0)_i + \nu_i,$$

(6)

where $\theta_0$ is a constant, the slope coefficient is $\theta_1 = -(1 - e^{-\lambda T})$, and $\nu$ is the error term. Note that a positive (negative) $\lambda$ implies a negative (positive) $\theta_1$. Absolute convergence ($\theta_1 < 0$) then means that the higher an economy’s income per capita is at the

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4 See Barro (1997) for several examples.
beginning of the period, the lower its growth rate will be over the subsequent time period. In other words, poor countries will grow faster than rich ones, closing the gap at the annual rate $\lambda$. Of course, if $\theta_1$ is positive, the implied $\lambda$ is negative, so that the poor are growing more slowly than the rich: there is divergence.

More realistically, however, countries do not all converge to the same income per capita, because the fundamental determinants of their steady states are not identical. Conditional convergence allows these steady-state values in equation (5) to differ. Substituting from (2), equation (5) can now be written in regression format:

$$\left[\ln(y_T) - \ln(y_0)\right] = \phi_0 + \phi_1 \ln(y_0) + \phi_2 \ln(s_i) + \phi_3 \ln(\bar{n}_i) + \nu_i,$$

(7)

where $\phi_0$ is a constant, $\phi_1 = -\left(1 - e^{-\lambda T}\right)$, $\nu$ an error term, and the Solow model still predicts $\phi_2 > 0$ and $\phi_3 < 0$. Once again, $\phi_1$ is negative (positive) if $\lambda$ is positive (negative). But a negative $\phi_1$ implies conditional convergence: a country that is further away from its steady state will experience faster growth than a country that is closer to its steady state, but there is no guarantee that the two countries are converging to the same steady state (the steady states will be the same only if $\phi_2 = \phi_3 = 0$). Note that the Solow model actually predicts conditional (but not necessarily absolute) convergence.

Finally, we will allow for a time-varying conditional convergence parameter, $\lambda_t$, by estimating equation (7) for a number of rolling, overlapping windows of length $k$. This way we can investigate how the speed of convergence has changed over time.6

3. The Data

All data are obtained from the Penn World Table (PWT, Mark 6.2), documented in Heston, Summers, and Aden (2006; see also Summers and Heston, 1991). Two data sets have been constructed, depending on the length of the period for which the series defined above are available in PWT 6.2 for the various economies.

Data Set I consists of the 58 economies for which data on all series exist for each year of the 1950-2003 period. Figures 1 and 2 present scatterplots for Data Set I. Figure 1 plots the 2003 value of GDP per capita (in logarithmic scale) against the investment rate, averaged over the 1950-2003 period. As can be seen from the graph, GDP per capita in 2003 has ranged from $687 in Ethiopia to $49,000 in Luxembourg, while average investment rates vary from 2.6% of GDP in Uganda to more than 29% of GDP in Finland. Consistent with the Solow model’s predictions, Figure 1 shows a clear positive relationship between GDP per capita and the investment rate.

Figure 2 plots again the 2003 value of GDP per capita (also in logarithmic scale) against the population growth rate, averaged over the same 1950-2003 period. As can be seen from the graph, average population growth rates have ranged from 0.3% in Austria.

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6 One can also estimate time-varying absolute convergence parameters following the same technique on regression (6). As our objective here is to evaluate the Solow model, however, we skip this exercise and focus on conditional convergence.
Karras, G.  
*Growth, Convergence and the Solow Model, 1950-2003*

to 3.2% in Kenya. Again consistent with the Solow model’s predictions, Figure 2 shows a clear negative relationship between per capita GDP and population growth.

*Data Set II* consists of the 99 economies for which data on the series are available for each year of the 1960-2003 period. The trade off between the two data sets is obvious: *Data Set I* covers a longer time period (by ten years, roughly one fourth of the shorter period) for each country, but *Data Set II* contains almost twice the number of economies, including a much larger number of developing countries. Figures 3 and 4 present scatter plots for Data Set II. Figure 3 plots the 2003 value of GDP per capita (in logarithmic scale) against the investment rate, averaged over the 1960-2003 period. As the graph shows, GDP per capita in this Data Set has ranged from $584 in Guinea-Bissau to (again) $49,000 in Luxembourg, while investment rates vary from 2.5% of GDP in Rwanda to 44% of GDP in Singapore. Consistent with the Solow model’s predictions, Figure 3 shows a clear positive relationship between GDP per capita and the investment rate.

Figure 4 plots for the countries of Data Set II the 2003 value of GDP per capita (also in logarithmic scale) against the population growth rate, averaged over the 1960-2000 period. As can be seen from this plot, average population growth rates have ranged from 0.3% in Great Britain to more than 4% in Jordan. Again consistent with the Solow model’s predictions, Figure 4 shows a clear negative relationship between GDP per capita and the population growth rate.

4. Empirical Results and Discussion

Table 1 reports two estimated versions of equation (3), one for each of the two Data Sets. Beginning with the first column of Table 1 for Data Set I, the estimated coefficients of the investment rate and the population growth rate are \( \gamma_1 = 1.074 \) and \( \gamma_2 = -0.625 \), respectively. They both have the expected signs and they are highly statistically significant. In addition, just these two variables account for an impressive three quarters of the sample’s variability in per capita income \( (R^2 = 0.734) \). The second column of Table 1 repeats the estimation for Data Set II. The estimated coefficients are now \( \gamma_1 = 0.936 \) and \( \gamma_2 = -0.918 \) (so they continue to have the expected signs) and are again highly statistically significant. As expected, because of the greater diversity of Data Set II, the joint explanatory power of the two variables in this specification is lower than the previous one’s, but the \( R^2 \) is still a substantial 66%.

The estimation is, therefore, statistically successful. But how meaningful is it economically? Are its implications about the model realistic? Comparing equations (2) and (3), note that the Solow model clearly predicts \( \gamma_1 = -\gamma_2 = \beta/(1 - \beta) \). How consistent is this with our evidence? The F-statistics for the null hypothesis \( \gamma_1 = -\gamma_2 \) are 2.999 for Data Set I and 0.013 for Data Set II, none of which is significant at the 5% level. This means that the Solow model’s prediction cannot be rejected by either of the two Data Sets. If the restriction \( \gamma_1 = -\gamma_2 \) is then imposed on the regression equation (3), the estimated \( \gamma_1 \) is 0.806 for Data Set I and 0.927 for Data Set II. These values can be
used to solve for the implied $\beta$’s, which are 0.446 for Data Set I and 0.481 for Data Set II.\footnote{Note that these are much more plausible than Mankiw, Romer, and Weil’s (1992) value of 0.6.}

It appears that the standard Solow model is quite consistent with the cross-country data. Nevertheless, it is worth trying a number of augmented models in order to see whether additional determinants of the steady state can be identified empirically. One of these attempts is reported in Table 2, which estimates equation (4), adding government size and trade openness as explanatory variables. For both Data Sets I and II, the coefficients of government size are negative (but statistically significant only for Data Set II), while those of trade openness are positive, but statistically insignificant. It is important to point out, however, that the estimated coefficients of the saving rate and population growth remain virtually unaffected in terms of sign, magnitude, and statistical significance, so that the data’s support for the Solow model is very robust.

Next we investigate convergence.\footnote{This also allows us to relax the implicit assumption that the 2003 value of $y$ is a good proxy for the steady state value.} Table 3 reports evidence on absolute convergence, estimating equation (6). For both Data Sets, the estimated $\theta_1$ is positive (though not statistically significant). This means that there has been no absolute convergence. The richer a county was in 1950 (or 1960), the faster it grew on average over 1950-2003 (or 1960-2003): there was actually divergence, as illustrated by the negative values for the implied $\lambda$’s. This is a very interesting finding (and robust). But it provides no information on the validity of the Solow model, because that model’s prediction concerns conditional, and not absolute, convergence.

Conditional convergence is tested in Table 4, which reports the results of estimating equation (7). Note first that the signs (and magnitudes) of the investment rate and population growth are as expected: $\phi_2 > 0$ and $\phi_3 < 0$, in both Data Sets. In addition, they are highly statistically significant in both models. The role of the two variables, therefore, remains as predicted by the Solow model. But now, unlike the evidence of Table 3, the coefficient of starting income is negative ($\phi_1 < 0$) and highly statistically significant. This means that there has been conditional convergence: controlling for the determinants of the steady state ($\bar{s}$ and $\bar{n}$ ), the poorer the economy was in 1950 (or 1960), the faster it grew in 1950-2003 (or 1960-2003). In other words, the further away the economy is from its steady state, the faster it grows towards it – but all economies do not converge to the same steady state. This of course is also reflected in the positive values of the implied $\lambda$’s, which suggest that (conditional) convergence has been taking place at the annual rate of 1.2% in Data Set I, but only at the slower annual rate of 0.8% in Data Set II.

Finally, we allow for time-varying (conditional) convergence rates, estimating rolling versions of equation (7) for windows of length $k$ years, as described above. We consider values of $k = 10, 15, \text{ and } 20$ years. Figures 5 and 6 report the estimated convergence rates for Data Sets I and II, respectively, for each of the three values of $k$. 

The $\lambda_i$'s follow a hump-shaped pattern, which is very robust to the choice of $k$, but differs between the two Data Sets. For Data Set I, the rate of convergence has been increasing until around 1990, but has declined rapidly in the 1990s. For Data Set II, convergence has been also increasing at first, but only until the mid 1980s, and then declined as well. A very important observation, however, is that the time variation of $\lambda$ is very sizable. This means that ignoring it, as in the pure cross-sectional approach of equation (7) and Table 4, can lead to oversimplified results. For example, the $\lambda = 1.2\%$ “average” value for Data Set I in Table 4 masks the very interesting evolution of $\lambda_i$ from 0.5% in 1960 to 2% in 1990, and down to 0.6% in 2003 (Figure 5, $k = 10$). Note also that the convergence rate for Data Set I has been generally higher than Data Set II’s.

To pursue this more, Data Set I has been divided into OECD and non-OECD subsamples. As Figures 7 and 8 reveal, the evolution of convergence has been very different in the two subsets of countries. In particular, the annual convergence rate among OECD countries has almost monotonically declined from a (very) high peak value of more than 3% in 1970 to a virtual zero since the mid 1980s (Figure 7, $k = 10$). On the contrary, the non-OECD subset’s annual convergence rate was increasing from a virtual zero in 1960 to a remarkable 2.5% in 1990, but then declined to 1.6% in 2003 (Figure 8, $k = 10$). Note that the overall pattern of Data Set I (Figure 5) is really dominated by the non-OECD subset (Figure 8). We also point out that the results are quite robust to the choice of the window $k$.

Figure 7: Convergence Rates over time – Data Set I (OECD)
6. Conclusions

This paper investigated the validity of the Solow model’s predictions, updating the Mankiw, Romer, and Weil (1992) empirical study, using two data sets that add almost twenty years of additional observations for each of the countries: Data Set I consists of 58 countries covering the period 1950-2003, and Data Set II of 99 countries over 1960-2003. The empirical results support a number of conclusions, which are qualitatively somewhat similar to, though quantitatively different from, those of Mankiw, Romer and Weil (1992). In particular:

(i) The standard Solow model’s predictions are consistent with the data: the standard of living is correlated positively with investment rates and negatively with population growth rates. Furthermore, just these two variables explain jointly two-thirds to three-quarters (67% to 73%) of the sample’s cross-country variation in income per capita.

(ii) The evidence cannot reject the Solow model’s prediction that the elasticities of income per capita with respect to saving and population growth are equal in absolute value (between 0.8 and 0.9). Furthermore, the implied output elasticity with respect to capital is around 0.45, which is higher than the usually assumed 0.3, but much more plausible than Mankiw, Romer, and Weil’s value of 0.6).

(iii) Testing several augmented versions of the Solow model, there is some evidence that income per capita is related positively to trade openness and negatively to government size, but the magnitudes and statistical significance of these effects are much smaller and more fragile than those of investment and population growth.

(iv) The empirical findings clearly reject absolute convergence in income per capita and, in fact, provide some evidence of divergence. However, the results are very strongly supportive of conditional convergence. This implies that countries may be generally approaching different steady states, but when saving and population growth rates are taken into account, there has been convergence at an estimated average annual rate of 0.8% to 1.2% a year.
Allowing for time-varying convergence rates, it is shown that the speed of convergence is far from constant over the postwar period. In particular, convergence rates, which have been mostly increasing during 1960-1990, have been falling since the early 1990s.

References

**Table 1:** The *Standard* Solow Model

<table>
<thead>
<tr>
<th></th>
<th>Data Set I</th>
<th>Data Set II</th>
</tr>
</thead>
<tbody>
<tr>
<td>CONSTANT</td>
<td>8.501**(0.716)</td>
<td>6.747**(0.716)</td>
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<tr>
<td>ln(I/GDP)</td>
<td>1.074**(0.164)</td>
<td>0.936**(0.142)</td>
</tr>
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<td>ln(n)</td>
<td>-0.625**(0.106)</td>
<td>-0.918**(0.113)</td>
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<tr>
<td>$R^2$</td>
<td>0.734</td>
<td>0.665</td>
</tr>
<tr>
<td>$N$</td>
<td>58</td>
<td>99</td>
</tr>
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</table>

Notes. Dependent variable: logarithm of GDP per capita in 2000. Estimated heteroskedasticity-consistent (White, 1980) standard errors in parentheses. **: significant at 1%, *: significant at 5%.

**Table 2:** An *Augmented* Solow Model

<table>
<thead>
<tr>
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<td>CONSTANT</td>
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<td>5.918** (0.787)</td>
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<td>ln(I/GDP)</td>
<td>1.059** (0.159)</td>
<td>0.881** (0.135)</td>
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<td>ln(n)</td>
<td>-0.599** (0.111)</td>
<td>-0.895** (0.102)</td>
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<tr>
<td>ln(G/GDP)</td>
<td>-0.104 (0.220)</td>
<td>-0.534** (0.190)</td>
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<td>ln(OPEN)</td>
<td>0.219 (0.127)</td>
<td>0.147 (0.120)</td>
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<td>$R^2$</td>
<td>0.750</td>
<td>0.704</td>
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<td>$N$</td>
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</table>

Notes: See Table 1.
Table 3: Testing *Unconditional* Convergence

<table>
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<td>CONSTANT</td>
<td>1.019</td>
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<tr>
<td></td>
<td>(0.790)</td>
<td>(0.506)</td>
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<tr>
<td>ln($y_0$)</td>
<td>0.011</td>
<td>0.055</td>
</tr>
<tr>
<td></td>
<td>(0.093)</td>
<td>(0.060)</td>
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<tr>
<td>implied $\lambda$</td>
<td>-0.0002</td>
<td>-0.0012</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.001</td>
<td>0.007</td>
</tr>
<tr>
<td>$N$</td>
<td>58</td>
<td>99</td>
</tr>
</tbody>
</table>

Notes: Dependent variable: log difference of GDP per capita 1951-2000 (Data Set I) or 1961-2000 (Data Set II). Estimated heteroskedasticity-consistent (White, 1980) standard errors in parentheses. $y_0$ represents GDP per capita in 1951 for Data Set I, and GDP per capita in 1961 for Data Set II. **: significant at 1%, *: significant at 5%.

Table 4: Testing *Conditional* Convergence

<table>
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<td>CONSTANT</td>
<td>4.629**</td>
<td>2.763**</td>
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<tr>
<td></td>
<td>(1.044)</td>
<td>(0.795)</td>
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<tr>
<td>ln($y_0$)</td>
<td>-0.473**</td>
<td>-0.304**</td>
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<tr>
<td></td>
<td>(0.097)</td>
<td>(0.080)</td>
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<tr>
<td>ln($I/GDP$)</td>
<td>0.755**</td>
<td>0.601**</td>
</tr>
<tr>
<td></td>
<td>(0.132)</td>
<td>(0.121)</td>
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<tr>
<td>ln($n$)</td>
<td>-0.401**</td>
<td>-0.394**</td>
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<tr>
<td></td>
<td>(0.079)</td>
<td>(0.082)</td>
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<tr>
<td>implied $\lambda$</td>
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<td>0.392</td>
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<tr>
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Notes: See Table 3.
Figure 1: GDP per capita and Investment Rate – Data Set I

Figure 2: GDP per capita and Population Growth Rate – Data Set I
Figure 3: GDP per capita and Investment Rate – Data Set II

Figure 4: GDP per capita and Population Growth Rate – Data Set II
Figure 5: Convergence Rates over time – Data Set I

Figure 6: Convergence Rates over time – Data Set II

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