FORECASTING MARKET CRASHES: DOES DENSITY SPECIFICATION MATTER?
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Abstract
The current research examines the capacity of the Edgeworth-Sargan density on forecasting market crashes. Focusing on the 1987 stock market crash the performance of this distribution is compared to the Student’s t concluding that the latter overestimates the risk. In contrast, and due to its flexible parametric structure, the Edgeworth-Sargan density is capable of more accurately forecasting the risk of highly volatile scenarios, especially when intraday data is available. We use daily and hourly data from the FTSE and Dow Jones indices.

Keywords: Market crash, Confidence intervals, Edgeworth-Sargan, Student’s t.
JEL classification code: G12, C16, C53.

1. Introduction
The capability of risk managers and policy makers of smoothing stock markets crises and their consequences in the real economy depends mainly on the ability of the econometric models to anticipate or forecast market prices and volatility. For this reason a large number of models have been proposed in recent decades with a view to improve risk measures and volatility forecasts for stock markets – e.g. Lo and MacKinlay (1988) for the NYSE or Al-Loughani and Campbell (1997) for the London Stock Exchange. This is also the main objective of this article, which intends to produce “accurate” forecasting confidence intervals for expected returns one period out-of-sample, by means of jointly estimating the full conditional density and its quantiles. However, there is not a consensus on the density specification, the Student’s t being the most widespread proposal (Praetz, 1972; Rogalski and Vinso, 1978; Bollerslev, 1987; among others). Nevertheless, the use of this distribution (as well as other alternatives which depend on a few number of parameters) may cause misspecified tail behaviour and, consequently, misleading confidence intervals and risk valuation, as stated by Mauleon and Perote (2000). To overcome this problem we propose the use of a more flexible distribution, the Edgeworth-Sargan (ES hereafter) density, which has rarely been used for such purposes despite its capability of incorporating thick tails, as well as asymmetries.¹

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¹ Details on both the empirical performance and theoretical properties of this distribution or other “positive” versions can be also found in Gallant and Nychka (1987), Harvey and Siddique (1999), Mauleon and Perote (2000) and Mencía et al. (2005).
In order to illustrate the performance of the ES compared to the Student’s t and its consequences on the forecasting confidence intervals (i.e. risk measures), we apply both specifications to forecast stock market indices during the days immediately before and after the 1987 stock market crash (we used both daily and hourly data). However, the results can be replicated for other market crashes occurred in recent decades, such as those triggered by the Asia crisis (October 1997), the failure of long term capital management and the Russia crisis (August 1998), the terrorist attack in New York (September 2001), the consequences of the political situation followed by the 11th September 2001 (July 2002), or the so-called subprime mortgage crisis (August 2007).

The rest of the article is structured as follows: Section 2 summarises the models and the forecasting methodology; Section 3 shows the empirical application; and Section 4 gathers the main conclusions.

2. Forecasting models

Let us suppose that the return on a particular asset at time t may be fairly explained by the following econometric model

\[ y_t = \beta_0 + \beta_1 y_{t-1} + \beta_2 x_{t-1} + u_t, \]  

\[ k_t^2 = \alpha_{0_t} + \alpha_{1_t} k_{t-1}^2 + \alpha_{2_t} u_{t-1}^2, \]

where \( x_{t-1} \) stands for an explanatory variable known at time \( t \) and \( u_t \) is a random variable with zero mean, GARCH(1,1) conditional variance and distributed either as Student’s t with \( n \) degrees of freedom (equation 2.3) or ES (equation 2.4).

\[ h_t(u_t) = \frac{1}{k_t\sqrt{n}} \Gamma \left( \frac{n+1}{2} \right) \left[ 1 + \frac{\varepsilon_t^2}{n} \right]^{-\frac{1}{2}(n+1)} \]  

\[ f_t(u_t) = \frac{1}{k_t} g(\varepsilon_t) \left\{ 1 + \sum_{s=2}^{q} d_{st} H_{st}(\varepsilon_t) \right\}, \]

where \( \varepsilon_t = u_t / k_t, \Gamma(\bullet) \) and \( g(\bullet) \) stand for the gamma function and standard Normal density, respectively, and \( H_{st}(\bullet) \) represents the \( s \)th order Hermite polynomial. These polynomials can be obtained recursively from the derivatives of the Gaussian density, as given in equation 2.5 (particularly we expand the density to the fourth term, and thus use \( H_{2t}(\varepsilon_t) = \varepsilon_t^2 - 1 \), \( H_{3t}(\varepsilon_t) = \varepsilon_t^3 - 3\varepsilon_t \) and \( H_{4t}(\varepsilon_t) = \varepsilon_t^4 - 6\varepsilon_t^3 + 3 \). The parameters of the models satisfy the necessary stationarity and positivity constraints.

\[ \frac{\partial}{\partial \varepsilon_t} g(\varepsilon_t) = (-1)^s g(\varepsilon_t) H_{st}(\varepsilon_t) \]  

For these models the one-step-ahead forecasting confidence intervals are those given in equation 2.6,

\[ \left[ y_{T+1} - \sigma_{e,T+1} \hat{\phi}^-(\alpha/2); y_{T+1} + \sigma_{e,T+1} \hat{\phi}^+(\alpha/2) \right], \]  

(2.6)
where $\hat{y}_{T+1}$ is the one-step-ahead linear predictor, $\sigma_{\epsilon,T+1}^2$ is the prediction error estimated variance and $\hat{\phi}_T^- (\alpha/2)$ and $\hat{\phi}_T^+ (\alpha/2)$ are the percentiles of the estimated standard distribution, which for the ES distribution are computed by implementing algorithms based on the following integral:

$$\int_{-\infty}^{\hat{\phi}_T^-} g(x) dx - g(\hat{\phi}_T^-) \sum_{s=2}^{a} \hat{\beta}_s H_{s-1}(\hat{\phi}_T^-) = \alpha/2.$$ (2.7)

3. Confidence intervals for the 1987 stock market crash

In this section we compare the forecast accuracy of the models presented in the last section in a highly volatile scenario, as it was October 1987. In fact, it was the largest stock prices decline ever recorded: S&P500 index fell a 20% in a single day, eclipsing the 12% decline in October 1929. For such period we compute the one-step-ahead forecasting confidence intervals using the Dow Jones index as the explanatory variable and the models shown in equations 2.1 to 2.4. We use daily data from the FTSE and Dow Jones indices (continuously compounded returns). Figure 1 depicts the historical daily prices for the FTSE index from the beginning of 1970 to the end of 1987 to illustrate the magnitude of the October 1987 stock market crash.

![FIGURE 1: FTSE DAILY PRICES](image)

The corresponding confidence intervals at 5 and 1 percent confidence level are plotted in Figures 2 and 3, respectively.²

² Note that the first observation corresponds to October 1st, whilst the market crash happened in October 19th (observation 13).
These figures highlight the fact that the ES confidence intervals are smaller than those of the Student’s t at both confidence levels. In particular, Student’s t intervals before the crash are about 28% and 57% larger at the 5 and 1 percent confidence levels, respectively, than those for the ES density. In this “low” volatility scenario, all the values are within the intervals bandwidths, and thus the ES intervals are clearly more accurate. On the other hand, the forecasting performance seems not to be that good when dealing with high volatility after the crash. Actually, note that neither the Student’s t nor the ES are capable of predicting the crash. However, once the market has impounded the effects of the crash, both distributions adapt to the new scenario through wider intervals. In this case, the Student’s t has larger intervals than the ES but it fails to capture the future values of the FTSE as many times as the ES at both confidence levels. Hence the ES reacts faster and more accurately to the crash as a consequence of its more flexible structure.

This feature is emphasised in Figures 4 and 5, where the forecasting confidence intervals at the 5 and 1 confidence levels, respectively, are depicted for the Dow Jones
index (modelled as an AR(1) process) and for every hour of the days 19, 20, 21 and 22 October 1987. These figures illustrate how the use of intraday data can help risk managers to anticipate and quantify risk even in a market crash scenario. For such data, the ES density forecasting describes the behaviour of the Dow Jones reasonably well (except for some “outliers”) and definitely better than the Student’s t.

4. Conclusions
This article shows the out-of-sample performance of the ES density quantifying the market risk in crisis and highly volatile scenarios. To calibrate its performance we compared it to that of the Student’s t, which is considered as the most popular distribution in finance. We show that the ES forecasting confidence intervals are narrower than those from the Student’s t for high confidence levels. Hence the ES is more accurate in accounting for market risk due to its general and flexible structure. The gains in accuracy obtained by the use of the ES distribution increase the higher the frequency of the data. Therefore, the combined use of the ES density and the tic-by-tic data can improve risk forecasting measures considerably. These new risk measures may help policy and
decision makers to optimise their risk management strategies and smooth the effects of market crises on the real economy.

References


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