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Abstract
This paper applies an extended Box-Cox model to test the functional form of the purchasing power parity hypothesis for Canada, the EU, Japan, and the U.K. Both the relative CPI and the relative PPI are considered in empirical work. The log-log form commonly chosen by most researchers can be rejected for Japan when the relative PPI is considered and the EU and the U.K. when the relative CPI or PPI is considered. The value of R2 is higher based on the relative PPI than that based on the relative CPI. None of the estimated elasticities is statistically equal to unity, suggesting that proportionality may not apply.

Keywords: PPP, functional forms, proportionality, elasticity

JEL Classification: F31

1. Introduction
The purchasing power parity (PPP) hypothesis is one of the most important foundations in international economics. Monetary models of the exchange rate are based on the validity of PPP and other assumptions. Lothian and Taylor (1996) rejected the random walk hypothesis. Sarno, Taylor, and Chowdhury (2004) and Lothian and Taylor (2006) found evidence of nonlinear reversion towards PPP. Sarno and Valente (2006) revealed that long-run PPP holds, there is evidence of nonlinear reversion towards PPP, and in adjusting toward PPP, relative roles of price and exchange rates vary with exchange rate regimes and over time.

There are several recent studies examining the PPP hypothesis for Asian and related countries. Based on monthly wholesale or produce price indexes for 35 manufacturing goods in the U.K. during 1975-1980, Fraser, Taylor and Webster (1991) showed very weak evidence of the PPP hypothesis that the exchange rate responds proportionally to the relative price in the long run. Rogoff (1996) maintained that the adjustment speed of PPP is very slow ranging from 3 to 5 years due to sticky wages and prices. Lothian and Taylor (1996) rejected the hypothesis of a random walk and found the coefficient of adjustment speed to be between 0.76 and 0.89. Bahmani-Oskooee and Mirzai (2000) applied the KPSS (1992) test and found evidence of PPP for most of 20 developing countries. However, based on the data of black market exchange rates for eight Asian countries, Bahmani-Oskooee and Goswami (2005) revealed that PPP can be rejected even if the exchange rate and the relative price have a cointegrating relationship. According to Razzaghipour, Fleming, and Heaney (2001), there was clear evidence of appreciation relative to PPP before 1997. Corrections occurred relative to PPP from 1997 in short time periods. Mean reversion towards PPP is not statistically significant whereas change in variance from 1997 is significant. Enders and Chumrusphonlert (2004) applied the threshold cointegration technique and showed that there is evidence for long-run PPP for

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most of the Asian nations and that there is an asymmetric process in making adjustments towards long-run PPP. Using a panel cointegration technique, Basher and Mohsin (2004) found that PPP for 10 selected Asian countries can be rejected. Breitung and Candelon (2005) indicated that before 1997, PPP holds for Asian countries but not for Latin American countries and that long-run PPP holds for Asian countries due to a flexible exchange rate system and breaks down for South American countries because of long-time pegging to the dollar. Yotopoulos and Sawada (2006) indicated that PPP holds for 132 countries in a 20-year time period and for 105 countries in a 10-year time period. Alba and Papell (2007) reported that PPP is valid for Latin American and European panel data, but not for Asian and African panel data. They also indicated that stronger evidence of PPP was found for countries with moderate volatility of exchange rates, lower inflation rates, more openness, similar rates of economic growth as the U.S., and less distance from the U.S. Taylor and Taylor (2004) and Taylor (2006) reviewed major previous works, presented issues and challenges in verifying PPP, and maintained that as the gaps between data and theory and deviations of exchange rates from PPP have narrowed, long-run PPP has gained more support.

Sarno, Taylor, and Chowdhury (2004) found strong support for nonlinear reversion when deviating from PPP. Cheung, Chinn, and Pascual (2005) predicted exchange rates based on three models including PPP and concluded that these models may forecast well in a period but show less desirable outcomes in a different period. Coakley and Snaith (2005) indicated that there is evidence of proportionality and symmetry for the U.S. dollar exchange rates for 15 European nations. Lothian and Taylor (2006) found evidence of the Harrod-Balassa-Samuelson effect for the sterling/dollar exchange rate and a nonlinear reversion to long-run PPP. Sarno and Valente (2006) revealed that PPP holds in the long run, there was a nonlinear reversion to PPP, and relative roles of price and exchange rates in adjusting toward PPP vary over time and exchange rate regimes.

This paper attempts to test whether the widely used double-log functional form is appropriate and whether the elasticity of the nominal exchange rate with respect to the relative price would be equal to unity for Canada, the EU, Japan, and the U.K. As Taylor and Taylor (2004), Taylor (2006), Engel and Rogers (1996) and others indicated, nonlinearity of PPP may arise due to central bank interventions, heterogeneity of opinions, and transactions cost and is an area that needs to be further explored. Because most previous studies did not consider the Box-Cox model in examining the PPP hypothesis for developing countries, empirical results in this study are expected to contribute to more understanding of PPP.

2. The Model

According to PPP, the nominal exchange rate in terms of domestic currency per unit of foreign currency is determined by the ratio of the domestic price to the foreign price and can be expressed as

\[ \ln \varepsilon = \delta + \phi \ln (P / P^*) + e \]  

(1)

Where

- \( \varepsilon \) = the nominal exchange rate (units of domestic currency per U.S. dollar),
- \( P \) = the domestic price level,
- \( P^* \) = the foreign price level, and \( e \) = the error term.
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If proportionality of PPP holds, the elasticity of the exchange rate with respect to the relative price is equal to unity in a double-log functional form. This paper attempts to test whether the double-log form is the correct functional form and whether the elasticity of the nominal exchange rate with respect to the relative price is unitary. Applying the extended Box-Cox model (Box and Cox, 1964; Greene, 2003; Hsing, 1997, 2007) in empirical estimation we can transform the nominal exchange rate and the relative price as:

\[
\frac{(e^\lambda - 1)}{\lambda} = \alpha_0 + \alpha_1 \left(\frac{(P/P^*)^\lambda - 1}{\lambda}\right) + \mu
\]

where \( \lambda \) is the Box-Cox transformation parameter and \( \mu \) is the error term. It can be shown that when \( \lambda \) approaches zero, equation (2) reduces to a double-log form. The elasticity of the nominal exchange rate with respect to the relative price at the means is given by:

\[
E_{e} = \alpha_1 \left(\frac{(P/P^*)}{\hat{\epsilon}}\right)^2
\]

where \( \alpha_1 \) is the estimated slope coefficient based on the extended Box-Cox model. The log-likelihood function is calculated as:

\[
L(\lambda; \epsilon, P/P^*) = -\frac{N}{2} (\ln 2\pi + 1) - \frac{N}{2} \sigma^2 (\lambda) + (\lambda - 1) \sum_{t=1}^{N} \ln \epsilon_t
\]

where \( \sigma^2 \) is the estimated variance. The procedure is to find the value of \( \lambda \) that will maximize the function in equation (4). Then the estimated \( \lambda \) is employed to derive an estimate of the parameters in equation (2). The estimate of \( \sigma^2 \) is given by:

\[
\hat{\sigma}^2 = \frac{1}{N - K} [\epsilon - (P/P^*) \hat{\alpha}] [\epsilon - (P/P^*) \hat{\alpha}]
\]

where \( \epsilon \) and \( P/P^* \) are vectors of the transformed dependent and independent variables and \( \hat{\alpha} \) is a vector of estimated parameters. The likelihood-ratio test is employed to determine whether the double-log form can be rejected in favor of the nonlinear form based on the extended Box-Cox model. The test statistic can be written as:

\[
J(\hat{\lambda}) = 2[L(\hat{\lambda}) - L(\lambda = 0)] \sim \chi^2_{(1)},
\]

where \( L(\hat{\lambda}) \) is the maximized log-likelihood function without any restriction on \( \lambda \) and \( L(\lambda = 0) \) is the log-likelihood function with the restriction that \( \lambda = 0 \). The critical value is 6.635 at the 1% level and 3.841 at the 5% level. Note that relative PPP is not considered because the percentage change in the exchange rate and the consumer or producer price index in relative PPP may have negative values, which cannot be transformed in the extended Box-Cox model.

3. Empirical Results

All the data were collected from the International Financial Statistics that is published by the International Monetary Fund. Monthly data are used in order to increase the sample size and reduce the standard error. The nominal exchange rate is expressed as units of...
domestic currency per unit of the U.S. dollar. Hence, an increase is a depreciation of the domestic currency. Both the consumer price index (CPI) and the producer price index (PPI) are used to represent the domestic and foreign price levels because the PPI may contain more tradable goods (Taylor and Taylor, 2004). To be consistent with the beginning of the floating exchange rate system, the data start in 1973.M3 and end in 2007.M6 except that the data for the EU begin in 1999.M1 and end in 2007.M4 or 2007.M6 depending on whether the PPI or the CPI is used in empirical work.

Unit root tests show that the nominal exchange rate and the relative price for each of the countries are stationary in first-difference even though some of them have unit roots in levels. The Johansen test indicates that there is one cointegrating relationship between the nominal exchange rate and the relative price for each of the eight countries under study.

Estimated parameters and related statistics for these three countries and the EU are presented in Table 1. Figures in the parenthesis are t ratios. The Newey-West (1987) method is applied in empirical work in order to generate consistent estimates for the covariance and standard errors when the forms of autocorrelation and heteroskedasticity are unknown.

The value of $R^2$ is lower when the relative CPI is used and higher when the relative PPI is used. The value of $R^2$ for Canada and the EU increases a great deal when the relative PPI is used. As shown, in the double-log form, when the relative CPI is used, estimated coefficients vary widely from a low of 0.620 for the U.K. to a high of 6.250 for the EU. When the relative PPI is employed, estimated coefficients range from 0.485 for the U.K. to 2.875 for the EU. The null hypothesis that the elasticity is equal to unity can be rejected for at the 5% level except for the coefficient of the relative CPI for Canada. Hence, if the conventional double-log form is selected without a priori test, PPP would
not hold for the EU, Japan, and the U.K. and would hold for Canada when the relative CPI is used.

In the extended Box-Cox model, the value of $\lambda$ ranges from -2.81 for the EU when the relative CPI is used to 0.50 for Japan when the relative PPI is employed. According to the likelihood ratio test, the null hypothesis that $\lambda = 0$ or the correct function form is a double-log form cannot be rejected for Canada when the relative CPI or PPI is considered and for Japan when the relative CPI is used and can be rejected for the other cases at the 5% level. The null hypothesis that the elasticity is equal to unity in the extended Box-Cox model can be rejected for all the cases at the 5% level. The use of the extended Box-Cox model has increased the value of $R^2$ in most cases and provided more accurate estimates of the elasticity of the nominal exchange rate respect to the relative price.

4. Summary and conclusions
This paper has examined functional forms of PPP for Canada, the EU, Japan, and the U.K. If the correct functional form is employed, the hypothesis of a unitary elasticity can be rejected for all the cases. Except for Canada (relative CPI or PPI) and Japan (relative CPI), the hypothesis that the correct functional form is a double-log form can be rejected for the other cases. Therefore, the double-log form may not be the proper functional form in testing the PPP hypothesis in empirical work. Because the value of $R^2$ is higher in using relative PPI than that in using the relative CPI, the consideration of the relative PPI in addition to the use of the relative CPI would provide a broader prospective for testing PPP.

There are several implications. Although none of the countries shows evidence of PPP, the relative PPI is a major determinant of the nominal exchange rate for Canada, the EU, and Japan. In studying exchange rates for these countries, we need to explore several models and theories in order to compare their performance and provide us with more insights into the subject.

References


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