NEUTRAL NETWORKS TO DETECT NONLINEARITIES IN TIME SERIES: ANALYSIS OF BUSINESS CYCLE IN FRANCE AND THE UNITED KINGDOM.

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Abstract

In this research we investigate possible existence of nonlinearities in business cycle fluctuations in France and United Kingdom (U.K.) real gross domestic product (GDP). We model the relationship between the real GDP in these countries using neural network linearity tests via in-sample as well as jackknife out-of-sample testing. Our results based on neural network linearity tests for possible existence of nonlinearities due to Terasvirta el al. (1993) using in-sample forecasts from neural nets in France and U.K. show statistically significant evidence of nonlinearities in both the series. Similarly, the results on linearity tests based on jackknife out-of-sample forecast also show statistically significant evidence of nonlinearities in both France and U.K. series. Moreover, the results based on neural network test for neglected nonlinearities that was proposed by Lee el al. (1993) also show statistically significant evidence of nonlinearities in both the countries. Therefore, policymakers are not able to evaluate the impact of monetary policy or any other shocks on output in these countries that are based on predictions from linear models.

Key phrases: asymmetries; neural networks; nonlinearities; principal components; real GDP;

JEL codes: B22, C32, C45, E32;

1. Introduction

With the repeated episodes of decline in economic activity in the United State of America (U.S.A) and other industrialized countries in 1970s and 1980s due to oil price cartel, in the year 1987 due to stock market crash in the U.S.A., in 1990s due to Gulf war, and in the year 2001 due to bubble blast in U.S.A economy followed by undesirable incidence of September 11 affected the U.S.A economy in particular and the economies of other industrialized countries including the group of seven (G7) industrialized nations (Canada, France, Germany, Italy, Japan, U.K., and U.S.A.) which in turn deviated these countries' economies from their long term growth path (trend).

In such types of unprecedented downturns in the economy the stabilization policies are usually called for to set back the course of the economy towards its long term growth trend otherwise situation like 1930s Great Depression might arise. Monetary policy being one of the stabilization policies can be helpful in such situations but the policymakers would be interested to know the impact of monetary policy or other shocks on output which may require appropriate forecasting models that are based on underlying data generating process. However, one may not be able to forecast the impact of such shocks based on linear models when the underlying data generating process is nonlinear. Therefore, it is imperative to investigate possible existence of nonlinearities in data series

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so that appropriate forecasting models are employed to anticipate the impact of monetary policy or other shocks on output.

In this context the relatively older notion reveals that business cycles are like alternate current cycles which means that positive half of a business cycle will have its span and amplitude equal to that of a negative half (rendering it to be its mirror image) which means that linear forecasting models can be used to forecast monetary policy and other shocks on output. However, Beaudry and Koop (1993) including other showed that the span of the positive half of a business cycle is more than that of negative half and the amplitude of the positive half of a business cycle is less than its negative half. This means that nonlinearities do prevail in business cycle fluctuations and these can not be anticipated using linear models including linear vector autoregression because the underlying data generating process is nonlinear. This necessitates knowing the types of mathematical models that could be used to forecast the impact of monetary policy or other shocks on output. In this context a number of empirical studies including Neftci (1984), Brunner (1997), Potter (1995), and Ramsey and Rothman (1996), Bidarkota (2000), Anderson and Vaheed (1998), and Anderson and Ramsey (2002), conclude that due to existence of significant nonlinearities in economic time series, linear forecasting models can not be used to forecast the behavior of economic activity.

It is of interest to know whether the fluctuations in economic activity are alike across the countries starting with the group of seven highly industrialized (G7) countries was empirically investigated by other researchers including Andreano and Savio (2002) who were not able to detect nonlinearities in France, Germany, and U.K GDP despite using Markov switching models in their research. Similarly Kiani and Bidarkota (2004) also studied nonlinearities in real GDP rates in G7 countries using nonlinear and switching time series models with stable distributions and long memory but they were also not able to detect existence of nonlinearities in France and U.K. real GDP. Thus, the basic question "whether the economic fluctuations in all the countries starting G7 countries are alike" remaining answered poses a challenge for macro-theorists to develop new theories of economic fluctuations provided empirical research does not show an evidence of nonlinearities in France and U.K. time series that are two prominent members of G7 countries as well as the European Union.

We feel that the present study can contribute to fill this gap adequately. Therefore, to further this work we employ data series pertaining to France and U.K synonymous to Kiani and Bidarkota (2004), however, contrary to Andreano and Savio (2002), Kiani and Bidarkota (2004), and others, we prefer to use artificial neural networks for modeling asymmetries for France and U.K. real GDP growth rates that we approximated using genetic algorithm because neural networks are flexible form of nonlinear models which can fit data well even when distribution of the data generating process as well underlying laws pertaining the data generating process are unknown.

Artificial neural networks are highly flexible functional form of nonlinear models that fit any data series without taking into account the distribution of the underlying data generating processes (White 1989b). Therefore, artificial neural networks (ANN) have been applied successfully in many disciplines including finance and economics. For example, Kuan and White (1994) and Swanson and White (1989, 1997a, 1997b), Hutchinson, Lo, and Poggio (1994), Garcia and Gencay (2000), and Qi and Madala (1999), Gencay (1999), Vishwakarma (1995), Qi (2001) and Kiani, Bidarkota, and Kastens (2005) employed neural networks in economics and finance. However, ANN are under heavy criticism because of their tendency to over fit the data that can be eliminated with careful construction of neural network architecture (Kiani 2005). Therefore, we employ artificial neural networks (ANN) to approximate in-sample as well as jackknife out-of-sample forecasts to construct neural network tests to investigate possible existence of business cycle asymmetries in France and U.K. real GDP growth rates. We use France and U.K. real GDP series[2] synonymous to Kiani and Bidarkota (2005) so as to obviate differences in results that might occur due to a different data set.

The remaining paper is split into the following sections. Section 2 discusses the structure of neural network models and neural network nonlinearity tests, and section 3 incorporates empirical results on hypothesis tests. Finally, section 4 incorporates conclusions that can be drawn from this empirical investigation.

2. Neural Networks

An artificial neural network (ANN) is an advanced artificial intelligence technology that mimics human brain's learning and decision making process. According to Reilly and Cooper (1990), ANN are capable to be powerful computational devices that can learn from examples to solve the problems that are never seen before. ANN being dissimilar to traditional model based methods are data driven, therefore, ANN can easily be used for solving problems where data is available but data generating process is unknown. The makes ANN useful for forecasters and researchers in situations where the data is available but the distribution of the data as well as underlying laws governing the data generating process are unknown. ANN can approximate any continuous function with a desired level of precision (Hornik, Stinchcombe, and White, 1990). Moreover, ANN are treated as nonlinear, nonparametric statistical methods due to which these are independent of distributions of the underlying data generating processes (White, 1989b).

A typical single layer feed forward neural network as of Lee, et al. (1993) can be presented in the following Equation:

$$f(x,\xi) = \theta_0 + \sum_{i=1}^{k} \theta_i \{\psi(\gamma_i^{'} \widetilde{w}_t)\}, \qquad k \in N \qquad (1)$$

where, $w_t = (1, \widetilde{w}_t^{'})'$, *and* $\widetilde{w}_t = (y_{t-1}, y_{t-2}, y_{t-3}, ..., y_{t-k})'$. Equation 1 shows flexible functional forms and ψ is a transfer function that can be either sigmoid (logistic) or hyperbolic (tangent) cumulative distribution function. We use the sigmoid function as transfer function.

In this research we employ two types of neural network linearity tests for detecting possible existence of nonlinearities in France and U.K. real GDP. The first test is neural network test for possible existence of nonlinearities whereas the second test is neural network test for neglected nonlinearities. Both these tests are elaborated in the following sub-sections.

2.1. Neural Network Test for Possible Existence of Nonlinearities

[2] We obtained quarterly data on real GDP growth rates for France and UK from International Financial Statistic's CD-ROM for the month of September 2001. Data for Canada, Japan, UK and USA ranges from 1957:1 to 2000:4 whereas for France the data ranges from 1970:1 to 2000:4.

The neural network test for possible existence of nonlinearities (NNW1) is employed in this research for detecting neglected nonlinearities (if any) in France and UK real GDP growth rates. This type of test was originally proposed by Terasvista et al. (1993) and is based on the null hypothesis of linearity against an alternative hypothesis of nonlinearity. The test consists of the following two equations:

$$y_t = \pi' w_t + v_t \qquad (2.1)$$

where, $u_t \sim N(0,\sigma^2)$, $w_t = (1, \tilde{w}_t)'$, and $\hat{w}_t = (y_{t-1},....y_{t-p})'$, and $\pi = (\pi_0, \pi_1, ..., \pi_p)$

$$\hat{u}_t = \pi' w_t + \sum_{j=1}^{q} \theta_{oj} \{\psi(\gamma_j', w_t)\} + v_t \qquad (2.2)$$

where, $\psi(\gamma', w_t) = (1 + \exp\{-\gamma' w_t\})^{-1}$ and π_0 is intercept. Equation 2.1 shows a linear model whereas the Equation 2.2 shows a neural networks model that nests the linear model represented by Equation 2.1.

This test comprises of three steps procedure. In the first step we estimate in-sample forecasts from linear model of the form shown in Equation 2.1 for France and U.K. series to compute residual ($\hat{u}_t$), and residual sum of square ($SSE_1 = \sum_{t=1}^{T} \hat{u}_t$). In the second step we approximate in-sample forecasts from neural networks of the form of Equation 2.2 to compute residuals ($\hat{v}_t$) and residual sum of square ($SSE_2 = \sum_{t=1}^{T} \hat{v}_t$) for both France and U.K. series. Finally, in the third step, we compute a test statistics using Equation 2.3 which is constructed from residuals and residual sum of squares from previous two steps of this test.

$$TS = \{(SSE1 - SSE2)/m\}/\{SSE2/(n-p-m-1)\} \qquad (2.3)$$

where, m denotes the number of restrictions in the unrestricted model n is the number of observations, and p is the number of lags. The test statistics is distributed approximately[3] F under normality hypothesis with (n-p-m-1) and m degrees of freedom.

In addition to constructing neural network test for possible existence of nonlinearities using in-sample forecasts from linear model as well as neural networks, we construct this test using jackknife out-of-sample forecasts from both the models for France and U.K. series. Jackknife re-sampling technique is used when the distribution of the parameters under review is either unknown, when it can not be characterized by a mathematical function, or when the mathematical function is especially difficult to estimate. Standard jackknife out-of-sample forecasts were used by Quenouille (1949) to reduce the bias in estimators. Thereafter, Tuckey (1958) used jackknife re-sampling for estimating variances. However, the sub-sample jackknife technique was initially proposed by Wu (1990) and extended by Politis and Romeo (1994). In 1990s researchers including Politis et al. (1997), and Ziari et al. (1997) used sub-sample jackknife re-sampling in their empirical work. Compared to the standard jackknife, sub-sample jackknife drops more

[3] This test statistics is approximate because of the nuisance parameter that appears under the alternative hypothesis (Davis, R. 1997; Andrews, W. 2001).

than one observation to estimate out-of-sample forecast of the remaining $m = n - d$ observations, where, n is the total number of observations and $d = 2,3,.............,n-1$. The model and the estimated observations are than used to conditionally predict value for the deleted observation and this process continues until each observation in the data is predicted or until all possible sub-samples are considered, depending on the statistical assumptions and the application.

2.2. Neural Network Test for Neglected Nonlinearities

The neural network test for neglected nonlinearities (NNW1) was originally proposed by Lee et al. (1993) which we employ to identify possible existence of nonlinearities in France, and UK real GDP growth rates. The test in presented in the following two Equations:

$$y_t = \pi^{'} w_t + u_t \qquad (2.4)$$

where, $u_t \sim N(0,\sigma^2)$, $w_t = (1,\widetilde{w}_t)^{'}$, and $\hat{w}_t = (y_{t-1},....y_{t-p})^{'}$, and $\pi = (\pi_0, \pi_1,.....,\pi_p)$

$$y_t = \pi^{'} w_t + \sum_{j=1}^{q} \theta_{oj} \{\psi(\gamma^{'}, w_t)\} + v_t \qquad (2.5)$$

where, $\psi(\gamma^{'} w_t) = \{1 + \exp(-\gamma^{'}, w_t)\}^{-1}$, and π_0 is an intercept and

$$\Psi_t = [\psi(\gamma_1^{'}, w_t),.........,\psi(\gamma_q^{'}, w_t)]^{'}. \qquad (2.6)$$

The artificial neural network model which is shown in Equation 2.5 nests a linear model of the form shown in Equation 2.4. This neural network model belongs to the family of flexible functional form that is indexed by ψ matrix and q (White 1989a, 1990), however, when q is sufficiently large, this neural network model can approximate a wide class of functions Stinchecimbe and White (1989-90), Hornik Stinchcombe, and White (1989, 90).

The neural network test for neglected nonlinearities can be completed in four steps. In the first step we estimate a linear model of the form of Equation 2.4 for France and U.K. real GDP and compute residuals ($\hat{e}_t$) and forecasts for each series. In the second step using neural network of the form of Equation 2.5 we compute the matrix (Ψ_t) of the form shown in Equation 2.7 which comprises of a data matrix ($\widetilde{X}_t$) and transfer function (ψ_t). In the third step a test statistics of the form shown in Equation 2.7 could be constructed using residuals ($\hat{e}_t$) and data matrix ($\widetilde{X}_t$) as proposed by Lee et al. (1993).

$$M_n = (n^{-1/2} \sum_{t=1}^{n} \Psi_t \hat{e}_t)^{'} \hat{W}_n^{-1} (n^{-1/2} \sum_{t=1}^{n} \Psi_t \hat{e}_t) \qquad (2.7)$$

where in Equation 2.7 $\hat{W}_n$ is a consistent estimator of $W^* = \text{var}(n^{-1/2} \sum_{t=1}^{n} \Psi_t \hat{e}_t)$ and the test statistics is distributed $\chi^2(q)$ when $n \to \infty$ under linearity hypothesis. However, Lee et al. (1993) pointed out that computation of $\hat{W}_n$ being arduous makes this test statistics computationally very intensive. Moreover, elements of Ψ_t tend to be collinear with themselves and with $\widetilde{X}_t$. Alternatively Lee et al. (1993) proposed another test statistics using $q^* < q$ principal components[4] of Ψ_t where q is the number of hidden nodes in the neural network. To find the appropriate matrix of principal components (Ψ_t^*), we approximated several neural network models as per Equation 2.5 with $2 > q > 12$ hidden nodes for France and U.K. series and found that the matrix of principals components (Ψ_t^*) obtained from Ψ_t matrix with 5 hidden nodes encompasses the first two principal components that account for maximum variance of all variables in Ψ_t. It is imperative to note that the matrix of the principal components (Ψ_t^*) is not collinear with itself or the data matrix $\widetilde{X}_t$. Therefore, we regress $\hat{e}_t$ on $X_t^{'}$ and Ψ_t^* to compute R^2. The alternate test statistics for this test is nR^2 which is equivalent to the one reported in Equation 2.7 which is also distributed $\chi^2(q)$ using standard asymptotic arguments when n approaches infinity under linearity hypothesis.

3. Empirical Results

3.1. Estimation Results.

Table 2 shows results from neural network test for possible existence of nonlinearities (NNW1) in France and U.K. series. Column 2 in this Table shows test statistics for neural network linearity tests. In this Table, rows $1-2$ in column 2 shows test statistics for France and UK respectively. This test statistics has a F distribution. While critical values from F distribution for France and U.K. are shown in rows $3-4$ in column 2, p-values for each test statistics are also shown in rows $5-6$ in this column. This Table also shows test statistics for neural network test for neglected nonlinearities (NNW2). Test statistics for this test has a χ^2 distribution which is reported in rows $3-4$ in column 3. Relevant p-values for each of the test statistics (France and U.K.) are shown in rows $5-6$ in this column (column 3).

Figure 2 show comparative plots encompassing forecasts from linear models as well as artificial neural networks with real GDP growth rates for France and U.K series.

For neural network test for possible existence of nonlinearities (NNW1), we test the null hypothesis of linearity against the alternative of nonlinearities for France and UK real GDP growth rates. For this test if the null hypothesis of linearity is true against the alternative hypothesis of nonlinearity, linearity do prevail on the data series being tested, else the series encompass nonlinearities.

The neural network test for possible existence of nonlinearities (NNW1) rejects the null hypothesis of linearity against the alternative hypothesis of nonlinearities for France and U.K. at 5 percent level of significance. Similarly, the neural network test for neglected nonlinearities (NNW2) accepts the alternative hypothesis of nonlinearity against the null

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[4] See Dunteman (1989) for principal component analysis.

of linearity for France and UK at 5 percent level of significance. This shows the nonlinearities do prevail in France and U.K. real GDP.

For both the neural network linearity tests (NNW1 and NNW2), inferences do not change when we change significance level from 5 to 10 percent. This shows that compared to Andreano and Savio (2002) we are able to find an evidence of neglected nonlinearities in France, Germany, and U.K. However, when compared to Kiani and Bidarkota (2004), we are able to find an evidence of neglected nonlinearities in France and U.K. real GDP growth rates only.

4. Conclusions

In this research we employ neural network tests for possible existence of nonlinearities in France and U.K. real GDP growth rates using in-sample as well as jackknife out-of-sample forecasts from linear models as well as neural nets. Similarly, we also employ neural network tests for remaining nonlinearities in France and U.K. series to detect possible existence of remaining nonlinearities (if any) in both France and U.K. series.

Our results based on neural network tests for possible existence of nonlinearities (NNW1) using in-sample as well as jackknife out-of-sample forecasts from linear models as well as artificial neural networks show statistically significant evidence of nonlinearities in France and U.K. real GDP growth rates. Moreover, neural network tests for neglected nonlinearities also show statistically significant evidence of neglected nonlinearities in France and U.K. real GDP growth rates.

Based on our study results we can conclude that linear models including vector autoregression can not be used to forecast the impact of monetary policy or any other shocks in France and U.K. real GDP growth rates.

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Annex

Table 1: Neural Network Tests Results

Test Type	NNW Test 1		NNW Test 2
Forecast Type	In-Sample	Jackknife out-sample	
Test Statistics			
France	27.082	121.633	178.35
U.K.	40.694	268.614	268.47
Critical values			
France	2.800	2.800	135.807
U.K.	2.750	2.800	135.807
p-values			
France	0.000	0.000	0.000
U.K.	**0.000**	**0.000**	**0.000**

Notes on Table 1. 1. The neural network test for possible existence of nonlinearities (NNW1) is shown in the following Equations:

$$y_t = \pi' w_t + u_t \qquad (1.1)$$

where, $u_t \sim Nid(0,\sigma^2)$, $w_t = (1,\tilde{w}_t')'$, $\tilde{w}_t = (y_{t-1},...,y_{t-p})'$ *and* $\pi = (\pi_0, \pi_1, ...\pi_p)'$

$$\hat{u}_t = \pi' w_t + \sum_{j=1}^{k} \theta_{0j} \{\psi(\gamma_j' w_t)\} + v_t \qquad (1.2)$$

where, $\psi(\gamma' w_t) = (1 + \exp\{-\gamma' w_t\})^{-1}$ and π_0 is intercept.

2. Test statistics, critical values from the relevant distribution and p-values for NNW1 are shown in column 2 of this Table for France and UK respectively. The test statistics is constructed using Equation 1.3:

$$TS = \{(SSE1 - SSE2)/m\}/\{SSE2/(n-p-m-1)\} \qquad (1.3)$$

where in Equation 1.3, m denotes the number of restrictions in the unrestricted model, n is the number of observations, and p is the number of lags in the regression equations chosen by S.B.C. criterion.

The test statistics (TS) is distributed approximated $F\ [m,(n-p-m-1)]$ under normality hypothesis.

4. Neural network test for neglected nonlinearities (NNW2) is presented in the following two Equations:

$$y_t = \pi' w_t + u_t \tag{2.1}$$

Where, $u_t \sim N(0,\sigma^2)$, $w_t = (1,\tilde{w}_t')'$, *and* $\tilde{w}_t = (y_{t-1},\ldots,y_{t-p})'$ *and* $\pi = (\pi_0,\pi_1,\ldots\ldots\pi_p)'$

$$y_t = \pi' w_t + \sum_{j=1}^{k} \theta_{0j}\{\psi(\gamma' w_t)\} + v_t \tag{2.2}$$

Where, $w_t = (1,\tilde{w}_t')'$, $\tilde{w}_t = (y_{t-1}, y_{t-2}, y_{t-3}, \ldots\ldots, y_{t-k})'$, and ψ is a transfer function.

5. Test statistics, critical values from the relevant distribution and p-values for NNW2 are shown in column 3 of this Table. In this column the first row denotes test statistics for France and the second row for UK. The test statistics for NNW2 is nR^2, where R^2 is calculated from regressing regress $\hat{u}_t$ on X_t' and Ψ_t^* (a matrix of principal components). The test statistics is distributed $\chi^2(p^*)$ under the assumption of normality.Figure 1: Predictions from Linear Model and Artificial Neural Networks.Neural Networks to Detect Nonlinearities in Time Series (Overleaf) Author: Khurshid M. Kiani

Figure 1: Predictions from Linear Model and Artificial Neural Networks. Neural Networks to Detect Nonlinearities in Time Series(Overleaf) Author: Khurshid M. Kiani