EFFICIENCY FLOODING BLACK-BOX FRONTIERS AND POLICY IMPLICATIONS

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Abstract
This research aims to contribute to the discussion on the importance of theoretically consistent modelling for stochastic efficiency analysis. The robustness of policy suggestions based on inferences from efficiency measures crucially depends on theoretically well-founded estimates. The theoretical consistency of recently published technical efficiency estimates for different sectors and countries is critically reviewed. The results confirm the need for a posteriori checking the regularity of the estimated frontier by the researcher and, if necessary, the a priori imposition of the theoretical requirements.

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1. Introduction

Parametric techniques as the stochastic production frontier model dominate the empirical literature on efficiency measurement.[1] The availability of estimation software – freely distributed via the internet and relatively easy to use – recently inflated the number of corresponding applications.[2] The application of the econometric methods provided by these ‚black box'-tools are mostly not accompanied by a thorough theoretical interpretation. The estimation results are further used without a critical assessment with respect to the literature on theoretical consistency, flexibility and the choice of the appropriate functional form. The robustness of policy suggestions based on inferences from efficiency measures nevertheless crucially depends on proper estimates. Most applications, however, do not adequately test for whether the estimated function has the required

[1] For a detailed review of different measurement techniques see e.g. COELLI ET AL., 1998 or KUMBHAKAR/LOVELL, 2000.
[2] Here e.g. the software FRONTIER.

regularities, and hence run the risk of making improper policy inferences. By exemplary reviewing some more recent contributions this paper shows the importance of testing for the regularities of an estimated efficiency frontier based on flexible functional forms. The basic results of the discussion on theoretical consistency and functional flexibility are therefore briefly summarized and applied to the case of the translog production function. Stochastic efficiency measurement is discussed to the background of these findings and essential implications are shown. Some stochastic frontier applications with respect to developing countries are exemplary reviewed with respect to the theoretical requirements. It is in particular argued that the economic properties of the estimation results have to be critically assessed, that the interpretation and calculation of efficiency have to be revised and finally that a basic change in the interpretation of the estimated function is required.

2.The magic triangle – theoretical considerations

One of the essential objectives of empirical research is the investigation of the relationship between an endogenous (or dependent) variable y and a set i of exogenous (or independent) variables x_{ij} where subscript j denotes the *j-th* observation:

$$y_j = f(x_{ij}, ß_i) + e_j \qquad (1)$$

In general the researcher has to make two basic assumptions with regard to the examination of this relationship: The first assumption specifies the functional form expressing the endogenous variable as a function of the exogenous variables. The second assumption specifies a probability distribution for the residual e capturing the difference between the actual and the predicted values of the endogenous variable. These two major assumptions about the underlying functional form and the probability distribution of the error term are usually considered as maintained hypotheses (see FUSS ET AL., 1978)[3]. Statistical procedures such as maximum likelihood estimation

[3] "[…] one should not attempt to test a hypothesis in the presence of maintained hypotheses that have less commonly accepted validity. […] An implication of this principle is the need for general, flexible functional

are used to estimate the relationship, i.e. the vector of the parameters $ß_i$.

Lau's Criteria

In general, economic theory provides no a priori guidance with respect to the functional relationships. However, LAU (1978, 1986) has formulated some principle criteria for the ex ante selection of an algebraic form with respect to a particular economic relationship:[4] - *theoretical consistency*: the algebraic functional form chosen must be capable of possessing all of the theoretical properties required by the particular economic relationship for an appropriate choice of parameters. With respect to a production possibility set this would mean that the relationship in (1) is single valued, monotone increasing[5] as well as quasi-concave implying that the input set is required to be convex[6].[7] However, this indicates no particular functional form. - *domain of applicability*: most commonly the domain of applicability refers to the set of values of the independent variables x_i over which the algebraic functional form satisfies all the requirements for theoretical consistency. LAU (1986) refers to this

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forms, embodying few maintained hypotheses, to be used in tests of the fundamental hypotheses of production theory." (FUSS ET AL., 1978, p. 223).

[4] The ax ante choice problem has to be distinguished from that of ex post choice which belongs to the realm of specification analysis and hypothesis testing.

[5] This simply implies that additional units of any input can never decrease the level of output. Hence this equals the statement that all marginal productivities dy/dx_i are positive and is finally derived from the basic assumption of rational individual behaviour.

[6] This is essentially equivalent to assuming that the law of the diminishing marginal rate of technical substitution $(dy/dx_i)/(dy/dx_k)$ for i = 1, .., n and k = 1, .., m holds. It implies that if x_i and x_k are both elements of V(y), then their convex combination x_l = ?x_i + (1-?)x_k is also an element of V(y) and capable of producing y.

[7] In the following we only consider a production function relationship. However, the same arguments apply for a cost, profit, return or distance function each showing different exogenous variables. A general discussion would require relatively complex arguments without providing any further insights.

concept as the *extrapolative domain* since it is defined on the space of the independent variables with respect to a given value of the vector of parameters *ß*$_i$.[8] If, for given *ß*$_i$, the algebraic functional form $f(x_i, ß_i)$ is theoretically consistent over the whole of the applicable domain, it is said to be globally theoretically consistent or globally valid over the whole of the applicable domain. FUSS ET AL (1978) stress the *interpolative robustness* as the functional form should be well-behaved in the range of observations, consistent with maintained hypotheses and admit computational procedures to check those properties, as well as the *extrapolative robustness* as the functional form should be compatible with maintained hypotheses outside the range of observations to be able to forecast relations. – *flexibility*: a flexible algebraic functional form is able to approximate arbitrary but theoretically consistent economic behaviour through an appropriate choice of the parameters.[9] The production function in (1) can be said to be *second-order flexible* if at any given set of non-negative (positive) inputs the parameters *ß* can be chosen so that the derived input demand functions and the derived elasticities are capable of assuming arbitrary values at the given set of inputs subject only to theoretical consistency.[10] "Flexibility of a functional form is desirable because it allows the data the opportunity to provide information about the critical parameters." (LAU, 1986, p. 1544). – *computational facility*: this criteria implies the properties of

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[8] The set of *k*'s for which a given functional form $f(x, ß(k)) = f(x, k)$ will have a domain of theoretical consistency (in *x*) that contains the prespecified set of *x*'s is called the interpolative domain of the functional form characterizing "[…] the type of underlying behaviour of the data for which a given functional form may be expected to perform satisfactorily." (LAU, 1986, p. 1539).

[9] Alternatively flexibility can be defined as the ability to map different production structures at least approxiamately without determining the parameters by the functional form. The concept of flexibility was first introduced by DIEWERT (1973) and (1974), LAU (1986) and CHAMBERS (1988) discuss local and global approximation characteristics with respect to different functional forms.

[10] This implies that the gradient as well as the Hessian matrix of the production function with respect to the inputs are capable of assuming arbitrary non-negative and negative semidefinite values respectively.

‘linearity-in-parameters’, ‘explicit representability’, ‘uniformity’ and ‘parsimony’. For estimation purposes the functional form should therefore be linear-in-parameters, possible restrictions should be linear.[11] With respect to the ease of manipulation and calculation the functional form as well as any input demand functions derivable from it should be represented in explicit closed form and linear in parameters. Different functions in the same system should have the same ‘uniform’ algebraic form but differ in parameters. In order to achieve a desired degree of flexibility the functional form should be parsimonous with respect to the number of parameters. This to avoid methodological problems as multi-collinearity and a loss of degrees of freedom. - *factual conformity*: the functional form should be finally consistent with established empirical facts with respect to the economic problem to be modelled.[12]

The Concept of Flexibility

A functional form can be denoted as `flexible` if its shape is only restricted by theoretical consistency. This implies the absence of unwanted a priori restrictions and is paraphrased by the metaphor of „providing an exhaustive characterization of all (economically) relevant aspects of a technology“ (see FUSS ET AL., 1978).

Each relevant aspect of the concept of second order flexibility is assigned to exactly one parameter: the level parameter, the gradient parameters associated with the respective first order variable, and the Hessian-parameters associated with the second order terms. As a functional form cannot be second-order flexible with fewer parameters, the number of free parameters provides a necessary condition for flexibility. With respect to a single-product technology with an n-dimensional input vector, a function exhaustively characterizing all of its relevant aspects should contain information

[11] If necessary a known transformation should be applied. FUSS ET AL. (1978) nevertheless stress that the tradeoff between the computational requirements of a functional form and the thoroughness of empirical analysis has to be weighted carefully.

[12] Here e.g. the well confirmed fact that the elasticities of substitution between all pairs of inputs are not all identical in the three or more-input case.

about the quantity produced (one level effect), all marginal productivities (n gradient effects) as well as all substitution elasticities (n^2 substitution effects). As the latter are symmetric beside the main diagonal with n elements, only half of the off-diagonal elements are needed, i.e. $½n(n - 1)$. The number of effects an adequate single-output technology function should be capable of depicting independently of each other and without a priori restrictions amounts to a total of $½(n + 2)(n + 1)$. Hence a valid flexible functional form must contain at least $½(n + 2)(n + 1)$ independent parameters.[13] Finally it has been shown that the function value as well as the first and second derivatives of a primal function can be approximated as well by the dual behavioural representation of the same technology (see BLACKORBY/DIEWERT, 1979). With respect to the relation between the supposed true function and the corresponding flexible estimation function the following concurring hypotheses can then be formulated (see MOREY, 1986):

(I) The estimation function is a local approximation of the true function.

This simply means that the approximation properties of flexible functional forms are only locally valid and therefore value, gradient and Hessian of true and estimated function are equal at a single point of approximation. As only a local interpretation of the estimated parameters is possible, the forecasting capabilities with respect to variable values relatively distant from the point of approximation are severly restricted.[14] In this case e.g. at least the necessary condition of local concavity with respect to global concavity can be tested for every point of approximation.

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[13] See HANOCH (1970) and following him FEGER (2000).

[14] In the immediate neighbourhood of the approximation point each flexible functional form provides theoretically consistent parameters only if the true structure is theoretically consistent (see MOREY, 1986 and CHAMBERS, 1988).

(II) The estimated function and the true structure are of the same functional form but show the desired properties only locally.

Most common flexible functions can either not be restricted to a well-behaved function without losing their flexibility (e.g. the translog function) or cannot be restricted to regularity at all (e.g. the Cobb-Douglas function). Points of interest in the true structure can be examined by testing the respective points in the estimation function. However, a positive answer to the question whether the estimation function and the true structure are still consistent with the properties of a well-behaved production function if the data does not equal the examined data set is highly uncertain. This uncertainty can only be illuminated by systematically testing all possible data sets.

(III) The estimated function and the true structure are of the same functional form and show the desired properties globally.

A flexible functional form which can be restricted to global regularity (e.g. the Symmetric Generalized McFadden Function[15]) without losing its flexibility allows for the inference from the estimation function to the true structure and hence allows for meaningful tests of significance as the model is theoretically well founded (see MOREY, 1986).[16] This approach of a flexible functional form promotes a concept of flexibility where the functional form has to fit the data to the greatest possible extent, subject only to the regularity conditions following from economic theory and independently depicting all economically relevant aspects.

The Magic Triangle

Hence, it is evident that the quality of the estimation results crucially depends on the choice of the functional form. The latter has to be chosen so that:

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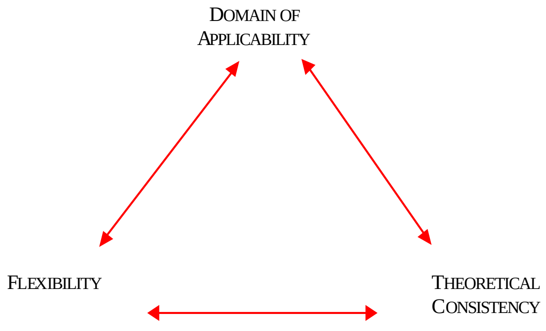
[15] See DIEWERT/WALES (1987). Applications can be found in RASK (1995) and FROHBERG/WINTER (2003). Khumbhakar (1989), PIERANI/RIZZI (1999), TSIONAS ET AL. (2001) as well as SAUER/FROHBERG (2005) applied it to estimate efficiency.

[16] On the other side, a serious problem arises for the postulates of economic theory if a properly specified flexible function which is globally well-behaved is not supported by the data (see FEGER, 2000).

- it provides all economically relevant information about the economic relationship(s) investigated,
- shows a priori consistency with the relevant economic theory on producer behaviour to the greatest possible extent,
- it includes no, or as few as possible, unwanted a priori restrictions, i.e. is flexible,
- it is relatively easy to estimate,
- it is parsimonious in parameters,
- it is robust towards changes in variables with respect to intra- as well as extrapolation,
- it finally includes parameters which are easy to interprete.

However, as was already noted by LAU (1978), one should not expect to find an algebraic functional form satisfying all of these criteria (in general cited as LAU'S `incompatibility theorem`). As one should not compromise on (at least) local theoretical consistency, computational facility or flexibility of the functional form, he suggests the domain of applicability as the only area left for compromises with respect to functional choice.[17]

Figure 1 The Magic Triangle of Functional Choice



(own figure)

[17] Hence, even if a functional form is not globally theoretically consistent, it can be accomodated to be theoretically consistent within a sufficiently large subset of the space of independent variables. Even so it has to be stressed that the surest way to obtain a theoretically consistent representation of the technology is to make use of a dual concept such as the profit, cost or revenue function.

As figure 1 summarizes, for most functional forms there is a fundamental trade-off between flexibility and theoretical consistency as well as the domain of applicability. Production economists propose two solutions to this problem, depending on what kind of violation shows to be more severe (see LAU, 1986 or CHAMBERS, 1988):

1) the choice of functional forms which could be made globally theoretically consistent by corresponding parameter restrictions, here the range of flexibility has to be investigated;
2) to opt for functional flexibility and check or impose theoretical consistency for the proximity of an approximation point[18] only;

However, a globally theoretical consistent as well as flexible functional form can be considered as an adequate representation of the production possibility set. Locally theoretical consistent as well as flexible functional forms can be considered as an i-th order differential approximation of the true production possibilities. Hence, the translog function is considered as a second order differential approximation of the true production possibilities.

3. The Case Of The Translog Production Function

A prominent textbook example as well as one of the most often used functional forms with respect to stochastic efficiency measurement the translog production function has to be noted:

$$f(x) = a_0 + \sum_{i=1}^{n} a_i \ln x_i + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij} \ln x_i \ln x_j$$

(2)

where symmetry of all Hessians by Young's theorem implies that $a_{ij} = a_{ji}$. It has $(n^2 + 3n + 2)/2$ distinct parameters and hence just as many as required to be flexible. By setting ? $_{ij}$ = $S_{i=1}^{n} S_{j=1}^{n} a_{ij}$ equal to a null matrix reveals that the translog function is a generalization of the

[18] Usually at the sample mean.

Cobb Douglas functional form.[19] The theoretical properties of the second order translog are well known (see e.g. LAU, 1986): it is easily restrictable for global homogeneity as well as homotheticity, correct curvature can be implemented only locally if local flexibility should be preserved, the maintaining of global monotonicity is impossible without losing second order flexibility.[20] Hence, the translog functional form is fraught with the problem that theoretical consistency can not be imposed globally. This is subsequently shown by discussing the theoretical requirements of monotonicity and curvature.

Monotonicity

As is well known with respect to a (single output) production function monotonicity requires positive marginal products with respect to all inputs:[21]

$$\frac{dy}{dx_i} > 0 \qquad (3)$$

and thus non-negative elasticities. However, until most recent studies the issue of assuring monotonicity was neglected. BARNETT ET AL. (1996) e.g. showed that the monotonicity requirement is by no means automatically satisfied for most functional forms, moreover

[19] The translog is probably the best investigated second order flexible functional form and certainly the one with the most applications.

[20] FEGER (2000) claims that the translog entertains two advantages over all other specifications: first, it is extremely convenient to estimate, and second, it is likely to be a good specification for economic processes. TERRELL (1996) applied a translog, generalized Leontief, and symmetric generalized McFadden cost function to the classical BERNDT and WOOD data. The results suggest that translog extensions to higher order could frequently outperform the Asymptotically Ideal Model (AIM) which is considered as today’s state of the art.

[21] BARNETT (2002) notes: “In specifications of tastes and technology, econometricians often impose curvature globally, but monotonicity only locally or not at all. In fact monotonicity rarely is even mentioned in that literature. But without satisfaction of both curvature and monotonicity, the second-order conditions for optimizing behaviour fail, and duality theory fails.” (p. 199).

violations are frequent and empirically meaningful. In the case of the translog production function the marginal product of input i is obtained by multiplying the logarithmic marginal product with the average product of input i. Thus the monotonicity condition given in (3) holds for the translog specification if the following equation is positive:

$$\frac{dy}{dx_i} = \frac{y}{x_i} * \frac{d\ln y}{d\ln x_i} = \frac{y}{x_i} * \left(a_i + \sum_{j=1}^{n} a_{ij} \ln x_j \right) > 0 \qquad (4)$$

Since both y and x_i are positive numbers, monotonicity depends on the sign of the term in parenthesis, i.e. the elasticity of y with respect to x_i. If it is assumed that markets are competitive and factors of production are paid their marginal products, the term in parenthesis equals the input i's share of total output, s_i.

By adhering to the law of diminishing marginal productivities, marginal products, apart from being positive should be decreasing in inputs implying the fulfillment of the following expression:

$$\frac{d^2y}{dx_i^2} = \left[a_{ii} + \left(a_i - 1 + \sum_{j=1}^{n} a_{ij} \ln x_j \right) * \left(a_i + \sum_{j=1}^{n} a_{ij} \ln x_j \right) \right] * \left(y / x_i^2 \right) < 0 \quad (5)$$

Again, this depends on the nature of the terms in parenthesis. These should be checked a posteriori by using the estimated parameters for each data point. However, both restrictions (i.e. ?y/?x_i > 0 and ?^{2}y/?x_i^2 < 0) should hold at least at the point of approximation.

Curvature

Whereas the first order and therefore non-flexible derivative of the translog, the Cobb Douglas production function, can easily be restricted to global quasi-concavity by imposing $a_i = 0$, this is not the case with the translog itself. The necessary and sufficient condition for a specific curvature consists in the semi-definiteness of its bordered Hessian matrix as the Jacobian of the derivatives ?y/?x_i with respect to x_i: if ∇^2Y(x) is negatively semi-definite, Y is quasi-concave, where ∇^2 denotes the matrix of second order partial derivatives with respect to (•). The Hessian matrix is negative semi-definite at every unconstrained local maximum[22], it yields with respect to the translog:

$$H = \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn} \end{pmatrix} - \begin{pmatrix} s_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & s_n \end{pmatrix} + \begin{pmatrix} s_1 s_1 & \cdots & s_1 s_n \\ \vdots & \ddots & \vdots \\ s_n s_1 & \cdots & s_n s_n \end{pmatrix} \quad (6)$$

where here s_i denote the elasticities of production:

$$s_i = \frac{d \ln y}{d \ln x_i} = a_i + \sum_{j=1}^{n} a_{ij} \ln x_j \qquad (7)$$

The conditions of quasi-concavity are related to the fact that this property implies a convex input requirement set (see in detail e.g. CHAMBERS, 1988). Hence, a point on the isoquant is tested, i.e. the properties of the corresponding production function are evaluated subject to the condition that the amount of production remains constant. Given a point $\mathbf{x^0}$, necessary and sufficient for curvature correctness is that at this point $\mathbf{v'Hv} = 0$ and $\mathbf{v's} = 0$ where $\mathbf{v}$ denotes the direction of change.[23] Hence, contrary to the Cobb Douglas function quasi-concavity can not be checked for by simply considering the parameter estimates.

A matrix is negative semi-definite if the determinants of all of its principal submatrices are alternate in sign, starting with a negative one (i.e. $(-1)^k D_k = 0$ where D is the determinant of the leading principal minors and k = 1, 2, …, n).[24] However, this criterion is only rationally applicable with respect to matrices up to the format 3 x 3 (see e.g. STRANG, 1976), the most operational way of testing square

[22] Hence, the underlying function is quasi-concave and an interior extreme point will be a global maximum. The Hessian matrix is positive semi-definite at every unconstrained local minimum.

[23] Which implies that the Hessian is negative semi-definite in the subspace orthogonal to s ? 0.

[24] Determinants of the value 0 are allowed to replace one or more of the positive or negative values. Any negative definite matrix also satisfies the definition of a negative semi-definite matrix.

numerical matrices for semi-definiteness is the eigen - or spectral decomposition:[25] Let **A** be a square matrix. If there is a vector **X** ? R^n ? 0 such that

$$AX = l\,X \qquad (8)$$

for some scalar ?, then ? is called the eigenvalue of **A** with the corresponding eigenvector **X**. Following this procedure the magnitude of the m + n eigenvalues of the bordered Hessian have to be determined.[26]

With respect to the translog production function curvature depends on the input bundle, as the corresponding bordered Hessian **BH** for the 3 input case shows:

$$BH = \begin{pmatrix} 0 & f_1 & f_2 & f_3 \\ f_1 & f_{11} & f_{12} & f_{13} \\ f_2 & f_{21} & f_{22} & f_{23} \\ f_3 & f_{31} & f_{32} & f_{33} \end{pmatrix} \qquad (9)$$

where f_i is given in (4), f_{ii} is given in (5) and f_{ij} is

$$\frac{d^2y}{dx_i dx_j} = \left[a_{ij} + \left(a_i + \sum_{j=1}^{n} a_{ij} \ln x_j \right) * \left(a_j + \sum_{i=1}^{n} a_{ij} \ln x_i \right) \right] * \left(y / x_i x_j \right) < 0 \quad (10)$$

For some bundles quasi-concavity may be satisfied but for others not and hence what can be expected is that the condition of negative-semidefiniteness of the bordered Hessian is met only locally or with respect to a range of bundles.

Theoretical Consistency And Flexibility

The preceeding discussion hence shows that there is a a trade-off between flexibility and theoretical consistency with respect to the translog as well as most flexible functional forms. Economists propose different solutions to this problem:

[25] The eigen decomposition relates to the decomposition of a square matrix A into eigenvalues and eigenvectors and is based on the eigen decomposition theorem which says that such a decomposition is always possible as long as the matrix consists of the eigenvectors of A is square.

[26] Checking the definiteness of a 2+x x 2+x bordered Hessian (x = 1, .., n) is not feasible as the determinant D_1 equals always zero.

1) Imposing globally theoretical consistency destroys the flexibility of the translog as well as other second-order flexible functional forms[27], as e.g. the generalized Leontief. However, theoretical consistency can be locally imposed on these forms by maintaining their functional flexibility. Further, RYAN and WALES (2000) even argue that a sophisticated choice of the reference point could lead to satisfaction of consistency at most or even all data points in the sample.[28] JORGENSON/FRAUMENI (1981) firstly propose the imposition of quasi-concavity through restricting **A** to be a negative semidefinite matrix.

Imposing curvature at a reference point (usually the sample mean) is attained by setting $a_{ij} = -(\mathbf{DD}')_{ij} + a_i d_{ij} + a_i a_j$ where i, j = 1, …, n, $d_{ij} = 1$ if i = j and 0 otherwise and $(\mathbf{DD}')_{ij}$ as the ij-th element of **DD**' with **D** a lower triangular matrix. The approximation point could be the data mean. However, the procedure is a little bit different. First, all data are divided by their mean. This transfers the approximation point to an (n + 1)-dimensional vector of ones. At the approximation point the terms in (7) and (12) do not depend on the input bundle anymore. It can be expected that input bundles in the neighbourhood also provide the desired output. The transformation even moves the observation towards the approximation point and thus increases the likelihood of getting theoretically consistent results (see RYAN/WALES, 2000). Imposing curvature globally is attained by setting $a_{ij} = -(\mathbf{DD}')_{ij}$. Alternatively one can use LAU'S (1978) technique by applying the Cholesky factorization $\mathbf{A} = -\mathbf{LBL}'$ where **L** is a unit lower triangular matrix and **B** as a diagonal matrix. However, the elements of **D** and **L** are nonlinear functions of the

[27] Second-order flexibility in this context refers to DIEWERT'S (1974) definition where a function is flexible if the level of production (cost or profit) and all of its first and second derivatives coincide with those of an arbitrary function satisfying linear homogeneity at any point in an admissable range.
[28] In fact RYAN/WALES (1998, 1999, 2000) could confirm this for several functional forms in a consumer demand context as well as for the translog and generalized Leontief specification in a producer context. See also FEGER (2000) and the recent example by TERRELL (1996).

decomposed matrix, and consequently the resulting estimation function becomes nonlinear in parameters. Hence, linear estimation algorithms are ruled out even if the original function is linear in parameters.

However, by imposing global consistency on the translog functional form DIEWERT/WALES (1987) note that the parameter matrix is restricted leading to seriously biased elasticity estimates.[29] Hence, the translog function would lead its flexibility. Any flexible functional form can be restricted to convexity or (quasi-)concavity with the above method – i.e. to local convexity or (quasi-)concavity. The Hessian of most flexible functional forms, e.g. the translog or the generalized Leontieff, are not structured in a way that the definiteness property is invariant towards changes in the exogenous variables (see JORGENSON/FRAUMENI, 1981). However, there are exceptions: e.g. the Hessian of the Quadratic does not contain exogenous variables at all, and thus a restriction by applying the Cholesky factorization suffices to impose regular curvature at all data points.[30]

2) Functional forms can be chosen which could be made globally theoretical consistent through corresponding parameter restrictions and by simultaneously maintaining flexibility. This is shown for the symmetric generalized McFadden cost function by DIEWERT/WALES (1987) following a technique initially proposed by WILEY ET AL. (1973). Like the generalized Leontief, the symmetric generalized McFadden is linearily homogenous in prices by construction, monotonicity can either be implemented locally only or, if restricted

[29] DIEWERT/WALES (1987) illustrate that the JORGENSON-FRAUMENI procedure for imposing concavity will lead to estimated input substitution matrices which are “too negative semidefinite”, i.e. the degree of substitutability will tend to be biased in an upward direction. However, if the elasticities would be independent of the input vector by transformation (assuming $a_{ij} = 0$ for all i and j) the translog function looses its flexibility as it collapses to the Cobb Douglas form.

[30] It is woth noting, that the Quadratic is disqualified for its incapability of being restricted with respect to linear homogeneity.

for globally, the global second-order flexibility is lost (see FEGER, 2000). However, if this functional form is restricted for correct curvature the curvature property applies globally.[31] Furthermore regular regions following GALLANT and GOLUPS (1984) numerical approach to account for consistency by using e.g. Bayesian techniques can be constructed with respect to flexible functional forms.[32]

4. Stochastic Efficiency Measurement

In recent years the primary interest shifted to the technical and allocative efficiency of individual netput bundles. A typical representation of the production possibilities can be given by the production frontier:

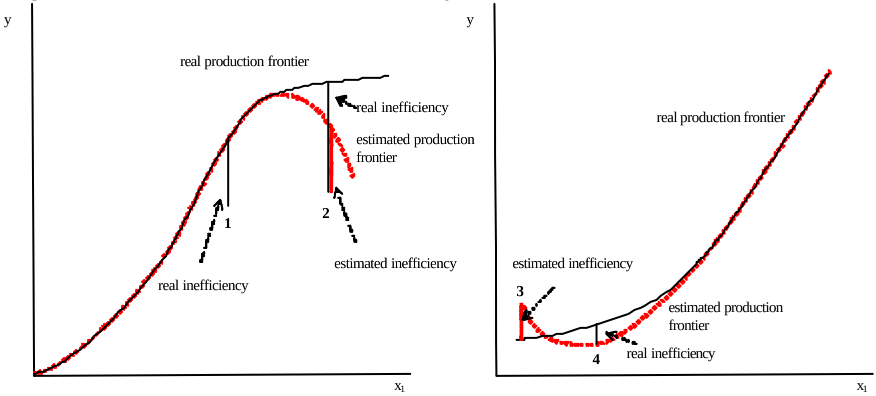
$$y = f(x) - e, with\ 0 < e < \infty \qquad (11)$$

This trend is accompanied by a shift in the interpretation insofar as the estimated results are not interpreted for the approximation point but for all input values. However, this in turn requires that the properties of the production function have to be investigated for every observable netput vector. The consequences of a violation of theoretical consistency for the relative efficiency evaluation will be discussed using figure 2 to 5 by showing the effect on the random error term:

[31] Unfortunately, the second order flexibility property is in this case restricted to only one point.

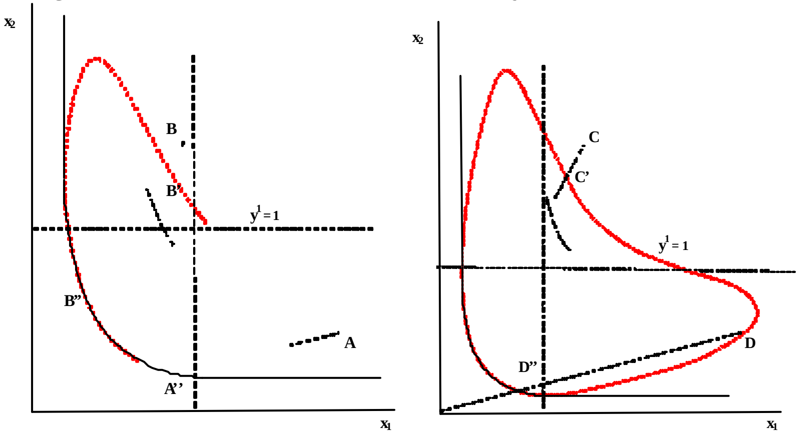
[32] To avoid the disturbing choice between inflexible and inconsistent specifications this approach imposes theoretical consistency only over the set of variable values where inferences will be drawn. Here the model parameters are restricted in a way that the resulting elasticities meet the requirements of economic theory for the whole range of variable constellations that are a priori likely to occur, i.e. a regular region is created.

Figure 2 & 3 Violation of Monotonicity



As becomes clear the estimated relative inefficiency equals the relative inefficiency for the production unit 1 with respect to the real production function. As the estimated function violates the monotonicity critera for parts of the function the estimated relative inefficiency of production unit 2 understates the real inefficiency for this observation. The same holds for production unit 3 which actually lies on the real production frontier, whereas the estimated relative inefficiency for production unit 4 again understates the real inefficiency. Figure 4 and figure 5 show the implications as a result of irregular curvature of the estimated efficiency frontier:

Figure 4 & 5 Violation of Quasi-Concavity



The red dotted line describes an isoquant of the estimated production function. The relative inefficiency of the input combination at production unit B measured against the estimated frontier (at B') understates the real inefficiency which is obtained by measuring the input combination against the real production frontier at point B''. Observation A lies on the estimated isoquant and is therefore measured as full efficient (point A). Nevertheless this production unit produces relatively inefficient with respect to the real production frontier (see point A''). The same holds for production unit D (real inefficiency has to be measured at point D''). Finally relative inefficiency of observation C detected at the estimated frontier (C') corresponds to real inefficiency for this production unit as the estimated frontier is theoretically consistent.

The graphical discussion clearly shows the implications for efficiency measurement: theoretically inconsistent frontiers over- or understate real relative inefficiency and hence lead to severe misperceptions and finally inadequate as well as counterproductive policy measures with respect to the individual production unit in question.[33]

[33] However, a few applications exist considering the need for theoretical consistent frontier estimation: e.g. KHUMBHAKAR (1989), PIERANI/RIZZI

5. Theoretically Inconsistent Efficiency Estimates - Examples

Although the majority of applications with respect to stochastic efficiency estimation uses the Cobb-Douglas functional form (see in a development context e.g. ESTACHE (1999), DERANIYAGALA (2001), ESTACHE/ROSSI (2002), AJIBEFUN/DARAMOLA (2003), KAMBHAMPATI (2003), OKIKE ET AL. (2004)) we subsequently focus on applications using the translog production function to derive efficiency judgements. This, as we outlined earlier, because of the relative superiority of flexible functional forms: to our opinion the Cobb-Douglas functional form should not be used for stochastic efficiency estimations any longer.

Theoretical consistency of the estimated function should be ideally tested and proven for all points of observation which requires for the translog specification beside the parameters of estimation also the output and input data on every observation. Most contributions fail to satisfactorily document the applied data set at least with respect to the sample means (see e.g. HOSSAIN/KARUNARATNE, 2004). However, the following exemplary analysis uses a number of translog production function applications published in recent years focusing on development related issues. Here monotonicity - via the gradient of the function with respect to each input by investigating the first derivatives - as well as quasi-concavity - via the bordered Hessian matrix with respect to the input bundle by investigating the eigenvalues - are checked for the individual local approximation

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(1999), CHRISTOPOULOS ET AL. (2001), CRAIG ET AL. (2003) as well as SAUER/FROHBERG (2004) estimated a symmetric generalized McFadden cost frontier by imposing concavity and checking for monotonicity (whereas KUMBHAKAR, CHRISTOPOULOS ET AL. as well as SAUER/FROHBERG uses a non-radial approach, CRAIG ET AL. uses a shadow cost frontier to efficiency measurement). Here global curvature correctness is assured by maintaining functional flexibility. O'DONNELL (2002) applies Bayesian methodology to impose regularity constraints on a system of equations derived from a translog shadow cost frontier. However, the vast majority of existing efficiency studies uses the error components approach by applying an inflexible CobbDouglas production function or a flexible translog production function without checking or imposing monotonicity as well as quasi-concavity requirements.

point at the sample mean or, if available, for the individual observations.

"A PRIMER ON EFFICIENCY MEASUREMENT"

The World Bank Institute's publication "A Primer on Efficiency Measurement for Utilities and Transport Regulators" by COELLI, ESTACHE, PERELMAN and TRUJILLO (2003) is intended to assist infrastructure regulators to learn about the tools needed to measure efficiency.[34] It aims to provide "[…] an overview of the various dimensions of efficiency that regulators should be concerned with" (p. v) and in particular focuses on policymakers interested in measuring relative efficiency and in implementing regulatory mechanisms based on the measurement of efficiency, as e.g. yardstick competition. To give an empirical example on estimating a stochastic production frontier COELLI ET AL. attempt to estimate a translog production function for 20 railway companies using panel data for a period of five years.[35] However, for all 29 observations the estimated frontier showed to be monoton only with respect to the variable input labor. It is not adhering to the requirement of diminishing marginal productivity as well as not quasi-concave for all input-bundles as required by economic theory (see table 1 for the results of the regularity tests for the 29 observations published[36] and table 2 for the numerical details of the tests performed).

[34] It is mainly based on lecture notes from courses the World Bank Institute offers for policy actors from developing countries.

[35] Although the authors point to the relative superiority of flexible functional forms they do not explicitly discuss the potential consequences of irregular efficiency estimates for regulatory measures.

[36] See COELLI ET AL. (2003), pp. 54.

Table 1 Example I - Regularity

Coelli/ Estache/ Perelman/ Trujillo (2003)[37]	Data Set (No. Obs., Years) Model Output Inputs	Monotonicity (for every Input)	Diminishing Marginal Productivity (for every input)	Quasi-Concavity (of the input-bundle)	Local Regularity (monoton & quasi-concave)
	100, 5 years Railway Output Capital Labor Other	0 x 0	0 0 0	0	0

x - fulfilled; 0 - not fulfilled

Table 2 Example I - Numerical Details of Regularity Tests[38]

OBSER-VATION	(1) FIRST DERIVATIVES			(2) SECOND DERIVATIVES			(3) HESSIAN			
	Capital	Labor	Other	Capital	Labor	Other	E1	E2	E3	E4
1	-1.85	0.59	-2.59	0.23	0.02	0.39	-3.02	0.25	-0.06	3.47
2	-2.17	0.78	-2.62	0.27	0.01	0.35	-3.27	0.23	-0.06	3.73
3	-2.75	1.01	-1.95	0.48	0.00	0.21	-3.25	3.81	-0.07	0.20
4	-2.20	0.88	-1.89	0.34	0.00	0.23	-2.81	3.29	-0.07	0.19
5	-2.26	0.70	-2.53	0.27	0.01	0.31	-3.31	0.20	-0.06	3.76
6	-2.55	0.93	-1.97	0.41	0.00	0.22	-3.12	3.62	-0.07	0.20
7	-2.09	0.66	-2.96	0.26	0.02	0.45	-3.44	0.28	-0.07	3.95
8	-2.05	0.75	-2.48	0.30	0.02	0.38	-3.07	0.29	-0.08	3.55
9	-2.11	0.87	-2.16	0.32	0.01	0.30	-2.92	0.24	-0.08	3.38
10	-2.15	0.88	-2.48	0.27	0.01	0.33	-3.18	0.22	-0.06	3.63
11	-1.62	0.32	-2.27	0.19	0.02	0.32	-2.63	0.20	-0.04	3.00
12	-2.09	0.61	-2.76	0.24	0.01	0.38	-3.30	0.23	-0.05	3.76
13	-1.85	0.69	-2.08	0.28	0.01	0.30	-2.67	0.25	-0.07	3.08
14	-1.72	0.45	-2.75	0.21	0.03	0.45	-3.04	0.27	-0.06	3.53
15	-2.23	0.75	-2.97	0.26	0.02	0.42	-3.55	0.25	-0.07	4.055

[37] Here evaluated for 29 observations published. The estimated frontier showed the same regularity results for every observation.
[38] 29 observations out of 100 are published in COELLI ET AL. (2003).

16	-2.13	0.63	-1.92	0.36	0.01	0.23	-2.73	3.15	-0.07	0.25
17	-2.15	0.95	-2.68	0.27	0.01	0.38	-3.33	0.25	-0.081	3.88
18	-2.30	0.81	-2.35	0.33	0.00	0.31	-3.15	0.25	-0.07	3.62
19	-1.73	0.47	-2.70	0.23	0.04	0.46	-3.00	0.31	-0.07	3.49
20	-2.27	0.79	-2.72	0.27	0.01	0.36	-3.40	0.23	-0.06	3.87
21	-2.68	0.95	-2.29	0.41	0.00	0.27	-3.40	3.91	-0.07	0.23
22	-2.52	1.14	-2.41	0.34	0.00	0.30	-3.43	3.93	-0.07	0.21
23	-2.51	0.97	-2.41	0.36	0.00	0.30	-3.37	3.87	-0.07	0.24
24	-2.38	0.88	-2.75	0.33	0.01	0.39	-3.50	0.28	-0.08	4.02
25	-3.33	1.73	-1.80	0.73	0.013	0.19	-3.769	4.629	-0.13	0.20
26	-4.15	2.16	-2.20	0.81	0.02	0.22	-4.71	5.69	-0.13	0.20
27	-3.34	1.60	-1.71	0.661	0.02	0.162	-3.71	4.49	-0.09	0.15
28	-2.29	1.31	-2.74	0.31	0.00	0.43	-3.54	0.29	-0.10	4.10
29	-2.62	1.37	-2.92	0.34	0.00	0.41	-3.88	0.27	-0.09	4.45

(1) MONOTONICITY
FIRST DERIVATIVES
$(?Y/?X_I > 0)$

(2) DIMINISHING MARGINAL PRODUCTIVITY
SECOND DERIVATIVES
$(?^2Y/?X_I^2 < 0)$
(3) QUASI –CONCAVITY
EIGENVALUES OF BORDERED HESSIAN MATRIX
$(E_I = 0)$

bold – not consistent with economic theory

OTHER EXEMPLARY FRONTIERS

BATTESE/BROCA (1997) estimated technical efficiencies of 109 wheat farmers in Pakistan over the period 1986 to 1991 using land, labor, fertilizer and seed as inputs (see table 3). Only model 2 fulfilled the monotonicity requirements for all four inputs. Both models evaluated at the sample means failed to adhere to quasi-concavity.

Table 3 Example II[39] - Regularity

Battese/ Broca (1997) Pakistan	Data Set Model Output Inputs	Monotonicity	Diminishing Marginal Productivity	Quasi-Concavity	Local Regularity
	330,				
	1986-				
	1991				
	Model	x	x	0	0
	1*	0	0		
	Wheat	x	x		
	Output	x	0		
	Land				
	Labour				
	Fertiliser	x	0	0	0
	Seed	x	x		
	Model	x	x		
	2*	x	x		
	Wheat				
	Output				
	Land				
	Labour				
	Fertilizer				
	Seed				

x - fulfilled; 0 - not fulfilled

[39] Due to lacking data on each observation for study II) to VI) evaluated at the sample means.

Table 4 Example II - Numerical Details of Regularity Tests

Battese/ Broca (1997) Pakistan	Monotonicity First Derivatives	Diminishing Marginal Productivity Second Derivatives	Quasi –Concavity Eigenvalues of Bordered Hessian Matrix
IIA) MODEL 1	Land: 1115.82115 Labour: -1.17838 Fertiliser: 5.23465 Seed: 26.37129	Land: -47.18914 Labour: 0.00133 Fertiliser: -0.01544 Seed: 0.00042	E1: 1298.53011 E2: -1321.70761 E3: 0.01271 E4: -0.02751 E5: -23.99859
IIB) MODEL 2	Land: 1015.04819 Labour: 2.35394 Fertiliser: 4.39806 Seed: 14.95299	Land: 2424.33423 Labour: -0.02503 Fertiliser: -0.012672 Seed: -0.01413	E1: -382.95155 E2: 2814.24112 E3: -0.00444 E4: -0.02995 E5: -6.97277

bold – not consistent with economic theory

ESTACHE ET AL. (2001) attempted to measure the efficiency gains from reforming ports' infrastructure by using panel data on Mexico for the period 1996 to 1999 and the modelling of production with and without technical change. However, both model specifications showed monotonicity only for the inputs labour and intermediates and failed with respect to correct curvature.

Table 5 Example III - Regularity

Estache/ González/ Trujillo (2001) Mexico	Data Set Model Output Inputs	Monotonicity	Diminishing Marginal Productivity	Quasi-Concavity	Local Regularity
	56, 1996-1999 Model 1 Harbour Output Labour Capital Intermediate Inputs Model 2 Harbour Output Labour Capital Intermediate Inputs	x 0 x x 0 x	0 0 0 0 0 x	0 0	0 0

x - fulfilled; 0 - not fulfilled

Table 6 Example III - Numerical Details of Regularity Tests

Estache/ González/ Trujillo (2001) Mexico	Monotonicity First Derivatives	Diminishing Marginal Productivity Second Derivatives	Quasi –Concavity Eigenvalues of Bordered Hessian Matrix
IIIA) MODEL 1	Input 1: 9.92808 Input 2: -2377936.216 Input 3: 3.62655	Input 1: 1.21123E-05 Input 2: 869070.6498 Input 3: 1.77912E-06	E1: -9.92808 E2: 9.92809 E3: 6.7825E+14 E4: -6.7825E+14

IIIB) MODEL 2	Input 1: 5.19119 Input 2: -206283.4505 Input 3: 0.12041	Input 1: 2.71923E-06 Input 2: 11711.38547 Input 3: -3.01675E-09	E1: -5.19119 E2: 5.19119 E3: 3.12125E+12 E4: -3.12125E+12

bold – not consistent with economic theory

AJIBEFUN ET AL. (2002) aimed to investigate factors influencing the technical efficiency of 67 crop farms in the Nigerian state of Oyo for the year 1995. The authors used land, labor, capital as well as hired labour to estimate a translog production frontier. However, the estimated function showed to be monoton in all inputs but not quasi-concave for the input bundle.

Table 7 Example IV - Regularity

Ajibefun/ Battese/ Daramola (2002) Nigeria	Data Set Model Output Inputs	Monotonicity	Diminishing Marginal Productivity	Quasi-Concavity	Local Regularity
	67, 1995 Total Crop Output Land Labour Capital Hired Labour	 x x x x	 0 x x x	 0	 0

x - fulfilled; 0 - not fulfilled

Table 8 Example IV - Numerical Details of Regularity Tests

Ajibefun/ Battese/ Daramola (2002) Nigeria	Monotonicity First Derivatives	Diminishing Marginal Productivity Second Derivatives	Quasi –Concavity Eigenvalues of Bordered Hessian Matrix
	Input 1: 545.52 Input 2: 63.40 Input 3: 210.65 Input 4: 1.22	Input 1: 325.60 Input 2: -0.08 Input 3: -2.32 Input 4: -0.00	E1: -473.83 E2: 756.15 E3: -0.62 E4: 41.49 E5: -0.00

bold – not consistent with economic theory

SHERLUND ET AL. (2002) used panel data from 464 rice plots in Cote d'Ivoire to estimate technical efficiency by including the inputs land, fertilizer, adult -, child -, and hired labour. The estimated efficiency frontier fulfills the monotonicity as well as diminishing marginal returns criteria for all inputs but nevertheless showed to be not quasi-concave.

Table 9 Example V - Regularity

Sherlund / Barrett/ Adesina (2002) Cote d'Ivoire	Data Set Model Output Inputs	Monotonicity	Diminishing Marginal Productivity	Quasi-Concavity	Local Regularity
	464, 1993-1995 Rice Production Land Adult	 x x x x	 x x x x	 0	 0

	Labour Hired Labour Child Labour Fertilizer	x	x		

x - fulfilled; 0 - not fulfilled

TABLE 10 Example V - Numerical Details of Regularity Tests

Sherlund/ Barrett/ Adesina (2002) Cote d'Ivoire	Monotonicity First Derivatives	Diminishing Marginal Productivity Second Derivatives	Quasi –Concavity Eigenvalues of Bordered Hessian Matrix
	Input 1: 545.52 Input 2: 63.40 Input 3: 210.65 Input 4: 1.22	Input 1: 325.60 Input 2: -0.08 Input 3: -2.32 Input 4: -0.00	E1: -473.83 E2: 756.15 E3: -0.62 E4: 41.49 E5: -0.00

bold – not consistent with economic theory

Finally KWON and LEE (2004) estimated stochastic production frontiers for the years 1993 to 1997 with respect to Korean rice farmers. All efficiency frontiers showed to be non-monoton for the input fertilizer and do not fulfill the curvature requirement of quasi-concavity. To sum up: 100% of all arbitrarily selected translog production frontiers fail to fulfill (at least) local regularity at the sample means.

Table 11 Example VI - Regularity

Kwon/ Lee (2004) Korea	Data Set Model Output Inputs	Monotonicity	Diminishing Marginal Productivity	Quasi-Concavity	Local Regularity

	1026, 1993-1997 Models 1993 - 1997 Rice Output Land Labour Capital Fertilizer Pesticides Others	x x x 0 x x	x x x 0 x x	0	0

x - fulfilled; 0 - not fulfilled

TABLE 12 Example VI - Numerical Details of Regularity Tests

Kwon/ Lee (2004) Korea	Monotonicity First Derivatives	Diminishing Marginal Productivity Second Derivatives	Quasi – Concavity Eigenvalues of Bordered Hessian Matrix
VIA) MODEL 1993	Input 1: 2483.90 Input 2: 1.57 Input 3: 6.03 Input 4: -0.83 Input 5: 5.90 Input 6: 9.52	Input 1: -1973.77 Input 2: -0.01 Input 3: -0.00561 Input 4: 0.01 Input 5: -0.01 Input 6: -0.08	E1: 1685.90 E2: -3659.58 E3: -18709.41 E4: 18709.53 E5: 0.00 E6: -0.02 E7: -0.33
VIB) MODEL 1994	Input 1: 2150.90 Input 2: 6.50 Input 3: 5.92 Input 4: -0.76 Input 5: 6.47 Input 6: 10.05	Input 1: -1247.37 Input 2: -1391.39 Input 3: -0.01 Input 4: 0.00 Input 5: -0.01 Input 6: -0.08	E1: 24561.32 E2: 1615.87 E3: 0.00 E4: -0.03 E5: -0.35 E6: -2863.19 E7: -25952.49

VIc) Model 1995	Input 1: 1799.93649 Input 2: 7.28249 Input 3: 5.39876 Input 4: -0.86076 Input 5: 5.83771 Input 6: 10.40969	Input 1: -1025.09236 Input 2: -0.02257 Input 3: -0.00483 Input 4: 0.00481 Input 5: -0.00929 Input 6: -0.08251	E1: 24112.16 E2: 1359.09 E3: 0.01 E4: -0.02 E5: -0.39 E6: -2384.06 E7: -24111.985
VId) Model 1996	Input 1: 1800.85281 Input 2: 9.75850 Input 3: 5.70050 Input 4: -1.04981 Input 5: 6.06115 Input 6: 11.08452	Input 1: -1009.05752 Input 2: -0.03173 Input 3: -0.00507 Input 4: 0.00558 Input 5: -0.00879 Input 6: -0.08038	E1: 31260.111 E2: 1365.8201 E3: 0.00538 E4: -0.02140 E5: -0.41888 E6: -2374.7521 E7: -31259.922
VIe) Model 1997	Input 1: 1596.88 Input 2: 11.45 Input 3: 5.55 Input 4: -1.27 Input 5: 5.67 Input 6: 11.66	Input 1: -874.601 Input 2: -0.03836 Input 3: -0.00498 Input 4: 0.01 Input 5: -0.01 Input 6: -0.08	E1: 33613.80 E2: 1218.59 E3: 0.01 E4: -0.02 E5: -0.46 E6: -2093.02 E7: -33613.63

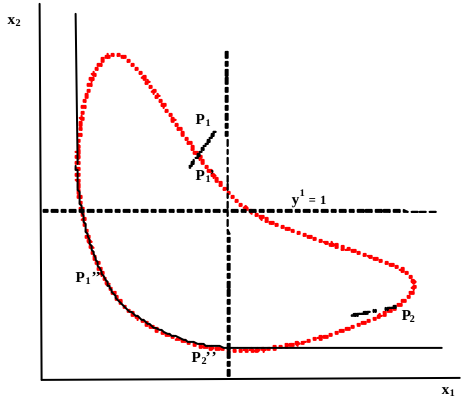
bold – not consistent with economic theory

Hence, as the investigated frontiers are flexible but not regular (at least at the sample mean) derived efficiency scores are not theoretically consistent and therefore are not an appropriate basis for the formulation of policy measures focusing on the relative performance of the investigated decision making units.

6. Policy Implications

A short exemplary discussion of the conclusions drawn by ESTACHE ET AL. (2001) with respect to their (theoretical incorrect) relative efficiency scores for the Mexican port sector should highlight the severity of potential policy implications. The authors draw three main conclusions: (1) the preceeding sector reforms would have resulted in significant performance improvements of ports on average and detected efficiency gains could be passed on to port users, (2) performance rankings by port specific efficiency measures would promote yardstick competition as they are superior to those based on partial productivity indicators, and (3) the quality of the data would be crucial for the model specification. As shown above, the efficiency estimates generated by ESTACHE ET AL. (2001) are not theoretical consistent at the sample mean by not adhering to monotonicity and quasi-concavity requirements. Hence conclusion (1) can not be drawn as the estimated production frontier is not quasi-concave at the sample means. Whether there are efficiency gains at all and if yes, how great such gains are, can not be answered by these (theoretical inconsistent) results. If the estimated relative 'efficiency position' of a reformed port is at P_1 in figure 6 its estimated efficiency score (graphically the distance between P_1 and P_1') evidently understates its real relative inefficiency (graphically the distance between P_1 and P_1''). If the estimated relative 'efficiency position' of a reformed port is at P_2 and hence on the estimated frontier its estimated efficiency score does not account for its real relative inefficiency (graphically the distance between P_2 and P_2''). In both cases positive efficiency effects by liberalisation measures are much lower in reality and hence "significant performance improvements of ports on average" are also much lower. If such improvements can be linked to preceeding policy actions remains unclear and can not be answered by such results. The same holds with respect to the possibility of passing cost savings by ports to the final port users via lower prices.

Figure 6 Quasi-Concave and Not Quasi-Concave Frontier Regions



With respect to conclusion (2) it is to say that global efficiency measures as e.g. multivariate stochastic efficiency frontiers are superior to partial productivity indicators as long as they are adhering to the requirements by economic theory. Regulatory measures based on theoretical consistent partial performance indicators are superior to efficiency estimates invalid because of theoretical inconsistencies. Finally it is true that the quality of the available data on a specific performance measurement problem is crucial for the significance of the policy inferences made. However, the specification of the efficiency model should be at first oriented at ensuring that the production possibility set T – all inputs x, exogenous factors z and output combinations y - of each production unit shows the properties corresponding to the aforementioned requirements of monotonicity and quasi-concavity of the estimated efficiency frontier (see e.g. CHAMBERS, 1988).

7. Conclusions

Existing black box estimation tools foster incorrect and unsound efficiency estimations lacking theoretical consistency and hence lead to inadequate and potentially counterproductive development policy actions. The preceeding discussion hence aims at highlighting the compelling need for a critical assessment of efficiency estimates with respect to the current evidence on theoretical consistency, flexibility as well as the choice of the appropriate functional form. The application of a flexible functional form as the translog specification by the majority of technical efficiency studies is adequate with respect to economic theory.[40] However, most applications do not adequately test for whether the estimated function has the required regularities of monotonicity and quasi-concavity, and hence run the risk of making improper policy recommendations. The researcher has to check a posteriori for the regularity of the estimated frontier which means checking these requirements for each and every data point with respect to the translog specification. If these requirements do not hold they have to be imposed a priori to estimation as briefly outlined in the text. Imposing global regularity nevertheless leads to a significant loss of functional flexibility, local imposition requires a differentiated interpretation: if theoretical consistency holds for a range of observations, this 'consistency area' of the estimated frontier should be determined and clearly stated to the reader. Estimated relative efficiency scores hence only hold for observations which are part of this range. Alternatively flexible functional forms – as e.g. the symmetric generalized McFadden – could be used which can be accomodated to global theoretical consistency over the whole range of observations. Furthermore one should always check for a possibility of using dual concepts such as the profit or cost function with respect to the efficiency measurement problem in question.[41] Hence, policy measures based on such efficiency estimates are not subject to possible inadequacy and a waste of scarce resources. Here exemplary applications already exist in the literature. The test for theoretical consistency of an arbitrary selected sample of translog production frontiers published in development relevant literature in

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[40] Unless there is strong a priori information on the true functional form, flexibility should be maintained as much as possible (see e.g. LAU, 1986).

[41] As LAU (1986) notes: „With regard to specific applications, one can say that as far as the empirical analysis of production is concerned, the surest way to obtain a theoretically consistent representation of the technology is to make use of one of the dual concepts such as the profit function, the cost function or the revenue function." (p. 1558).

recent years revealed the significance of this problem for daily efficiency measurement as well as policy formulation.

7. Literature

Ajibefun, I. A., Battese, G. E., Daramola, A. G. (2002); Determinants of Technical Efficiency in Smallholder Food Crop Farming: Application of Stochastic Frontier Production Function, in: *Quarterly Journal of International Agriculture*, Vol. 41, No. 3, pp. 225 – 240.

Ajibefun, I. A., Daramola, A. G. (2003); Determinants of Technical and Allocative Efficiency of Micro-Enterprises: Firm-Level Evidence from Nigeria, in: *African Development Review*, Vol. 15, 2/3, pp. 353 – 395.

Anderson, R. G., B. Jones, T. Nesmith (1996); An Introduction to Monetary Aggregation Theory and Statistical Index Numbers, Federal Reserve Bank of St. Louis, Research Division Working Papers, No. 9, St. Louis, 1996.

Banker, R. D., S. Janakiraman, R. Natarajan (2002); Evaluating the Adequacy of Parametric Functional Forms in Estimating Monotone and Concave Production Functions, in: *Journal of Productivity Analysis*, Vol. 17, pp. 111 – 132.

Barnett, W. A. (2002); Tastes and Technology: Curvature is not Sufficient for Regularity, in: *Journal of Econometrics*, Vol. 108, pp. 199 – 202.

Barnett, W., M., Kirova, M., Pasupathy (1996); Estimating Policy Invariant Deep Parameters in the Financial Sector, When Risk and Growth Matter, in: *Journal of Money, Credit, and Banking*, Vol. 27, pp. 1402 – 1430.

Battese, G. E., S. S. Broca (1997); Functional Forms of Stochastic Frontier Production Functions and Models for Technical Inefficiency Effects: A Comparative Study for Wheat Farmers in Pakistan, in: *Journal of Productivity Analysis*, Vol. 8, pp. 395 – 414.

Blackorby, C., W. E. Diewert (1979); Expenditure Functions, Local Duality, and Second Order Approximations, in: Econometrica: *Journal of the Econometric Society*, Vol. 47, 1979, No. 3, pp. 579 – 601.

Casler, S. D. (1997); Applied Production Theory: Explicit, Flexible and General Functional Forms; in: *Applied Economics*, Vol. 29, 1997, pp. 1483 - 1492.

Chambers, R. (1988); Applied Production Analysis: A Dual Approach, Cambridge University Press, Cambridge, 1988.
Christopoulos, D., J. Loizides, E. G. Tsionas (2001); Efficiency in European Railways: Not as Inefficient as One Might Think, in: *Journal of Applied Economics*, Vol. 4, 2001, No. 1, pp. 63 - 88.
Coelli, T., Estache, A., Perelman, S., Trujillo, L. (2003); A Primer on Efficiency Measurement for Utilities and Transport Regulators, WBI Development Studies, The World Bank, Washington D. C.
Craig, S. (2002); The Effect of Institutional Form on Airport Governance Efficiency. Department of Economics, University of Houston, Houston, USA.
Deraniyagala, S. (2001); The Impact of Technology Accumulation on Technical Efficiency: An Analysis of the Sri Lankan Clothing and Agricultural Machinery Industries, in: *Oxford Development Studies*, Vol. 29, 1, pp. 101 – 114.
Diewert, W. E., T. J. Wales (1987); Flexible Functional Forms and Global Curvature Conditions, in: *Econometrica*, 55, 1, pp. 43 - 68.
Diewert, E. W. (1973); Functional Forms for Profit and Transformation Functions, in: *Journal of Economic Theory*, Vol. 6, 1973, No. 3 pp. 284 – 316.
Diewert, E. W. (1974); Functional Forms for Revenue and Factor Requirements, in: *International Economic Review*, Vol. 15, 1974, No. 1, pp. 119 – 130.
Estache, A., M. A. Rossi (1999); Comparing the Performance of Public and Private Water Companies in Asia and Pacific Region - What a Stochastic Costs Frontier Shows, Policy Research Working Paper 2152, World Bank, 1999.
Estache, A., González, M., Trujillo, L. (2001); Technical Efficiency Gains from Port Reform: The Potential for Yardstick Competition in Mexico, Working Paper (to be published), July 2001, pp. 20.
Estache, A., M. A. Rossi (2002); How different is the Efficiency of Public and Private Water Companies in Asia?, in: *The World Bank Economic Review*, Vol. 16, 2002, No. 1, pp. 139 – 148.
Feger, F. (2000); A Behavioural Model of the German Compound Feed Industry: Functional Form, Flexibility, and Regularity; Göttingen, 2000.

Frohberg, K., E. Winter (2003); Impacts of Croatia's Bi- and Multilateral Trade Agreements: Experiments with Different Trade Model Specifications. Zentrum für Entwicklungsforschung, University of Bonn, Bonn, Germany.

Gallant, A. R., G. H. Gollup (1984); Imposing Curvature Restrictions on Flexible Functional Forms, in: *Journal of Econometrics*, Vol. 26, 1984, No. 3, pp. 295 – 321.

Giannakas, K., K. C. Tran, V. Tzouvelekas (2003); On the Choice of Functional Form in Stochastic Frontier Modelling, in: *Empirical Economics*, Vol. 28, pp. 75 – 100.

Hanoch, G. (1970); Generation of New Production Functions Through Duality, Harvard Institute of Economic Research, Discussion Paper, No. 118, Cambridge Mass.

Hossain, M. A., Karunaratne, N. D. (2004); Trade Liberalisation and Technical Efficiency: Evidence from Bangladesh Manufacturing Industries, in: The Journal of Development Studies, Vol. 40, 3, pp. 87 – 114.

Jorgenson, D. W., B. M. Fraumeni (1981); Relative Prices and Technical Change; in: Berndt, E. R.; Modeling and Measuring Natural Resource Substitution, pp. 17 – 47.

Kambhampati, U. S. (2003); Trade Reform and the Efficiency of Firms in India, in: *Oxford Development Studies*, Vol. 31, 2, 2003, pp. 219 – 233.

Kumbhakar, S. (1989); Estimation of Technical Efficiency Using Flexible Functional Form and Panel Data, in: *Journal of Business & Economic Statistics*, Vol. 7, 1989, No. 2, pp. 253 – 258.

Kumbhakar, S., C. A. Lovell (2000); Stochastic Frontier Analysis, Cambridge University Press, Cambridge.

Kwon, O. S., H., Lee (2004); Productivity Improvement in Korean Rice Farming: Parametric and Non-Parametric Analysis, in: *The Australian Journal of Agricultural and Resource Economics*, Vol. 48, No. 2, pp. 323 – 346.

Lau (1978); Testing and Imposing Monotonicity, Convexity and Quasi-Convexity Constraints, in: FUSS, M., D. MCFADDEN; Production Economics: A Dual Approach to Theory and Applications, Vol. 1: The Theory of Production, Vol. 2: Applications of the Theory of Production; North-Holland, New York, 1978.

Lau, L. J. (1986); Functional Forms in Econometric Model Building, in: GRILICHES, Z., M. D.
INTRILIGATOR; Handbook of Econometrics, Vol. III, pp. 1516 – 1566.
Marcus, M., H., Minc (1988); Introduction to Linear Algebra, New York: Dover, 1988.
Morey, E. R. (1986); An Introduction to Checking, Testing, and Imposing Curvature Properties: The
True Fundtion and the Estimated Function; in: *Canadian Journal of Economics*, Vol. 19, No. 2, 1986, pp. 207 – 235.
O'Donnell, C. J. (2002); Parametric Estimation of Technical and Allocative Efficiency in U.S. Agriculture, in: Ball, E., G. W. Norton; Agricultural Productivity: Measurement and Sources of Growth, Boston: Kluwer, pp. 137 – 166.
Okike, I., Jabbar, M. A., Manyong, V. M., Smith, J. W., Ehui, S. K. (2004); Factors Affecting Farm-Specific Production Efficiency in the Savanna Zones of West Africa, in: Journal of African Economies, Vol. 13, 1, pp. 134 – 165.
Pierani, L., Rizzi, P. (2001); Technology and Efficiency in a Panel of Italian Dairy Farms : A SGM
Restricted Cost Function Approach, Manuscript, Department of Economics, University of
Siena, Siena, Italy.
Rask, K. (1995); The Structure of Technology in Brazilian Sugarcane Production, 1975-87 – An Application of a Modified Symmetric Generalized McFadden Cost Function., in: *Journal of Applied Econometrics*, Vol. 10, 1995, pp. 221 – 232.
Ryan, D. L., T. J. Wales (1998); A simple Method for Imposing Local Curvature in some Flexible Consumer Demand Systems, in: *Journal of Business and Economic Statistics*, Vol. 16, 1998, pp. 331 – 338.
Ryan, D. L., T. J. Wales (1999); Flexible and Semiflexible Consumer Demands with Quadratic Engel Curves, in: *The Review of Economics and Statistics*, Vol. 81, No. 2, pp. 277 – 287.
Ryan, D. L., T. J. Wales (2000); Imposing Local Concavity in the Translog and Generalized Leontief Cost Functions, in: *Economics Letters*, Vol. 67, 2000, pp. 253 – 260.

Salvanes, K. G., S. Tjotta (1998); A Note on the Importance of Testing for Regularities for Estimated Flexible Functional Forms, in: *Journal of Productivity Analysis*, Vol. 9, pp. 133 – 143.

Sauer, J., K. Frohberg (2005); Input Specific Allocative Efficiency of Rural Water Suppliers – A Flexible and Globally Concave Cost Frontier, in: *Journal of Productivity Analysis*, 2005 (forthcoming).

Skinner, J. (1994); What do Stochastic Frontier Cost Functions tell us about Inefficiency?, in: *Journal of Health Economics*, Vol. 13, No. 1, pp. 323 – 328.

Sherlund, S. M., Barrett, C. B., Adesina, A. (2002); Smallholder Technical Efficiency Controlling for Environmental Production Conditions, in: Journal of Development Economics, Vol. 69, 2002, pp. 85 – 101.

Strang, G. (1976); Linear Algebra and its Applications, Academic Press, New York, 1976.

Terrell, D. (1996); Incorporating Monotonicity and Concavity Conditions in Flexible Functional Forms, in: *Journal of Applied Econometrics*, 1996, pp. 179 – 194.

Thiam, A., B. E. Bravo-Ureta, T. E. Rivas (2001); Technical Efficiency in Developing Country Agriculture: A Meta-Analysis, in: *Agricultural Economics*, Vol. 25, No. 2 – 3, pp. 235 – 243.

Tzouvelekas, E. (2000); Approximation Properties and Estimation of the Translog Production Function with Panel Data, in: *Agricultural Economics Review*, Vol. 1, No. 1, 2000, pp. 27 – 41.

Wales, T. J. (1977); On the Flexibility of Flexible Functional Forms, in: *Journal of Econometrics*, 1977, pp. 183 – 193.

Wiley, D. E., W. H. Schmidt, W. J. Bramble (1973); Studies of a Class of Covariance Structure Models, in: *Journal of the American Statistical Association*, Vol. 68, 1973, pp. 317 – 323.

Zellner, A., H. Ryu (1998); Alternative Functional Forms for Production, Cost and Returns to Scale Functions, in*: Journal of Applied Econometrics*, Vol. 13, 1998, No. 2, pp. 101 – 127.