DYNAMIC MODELS IN ECONOMETRICS: CLASSIFICATION, SELECTION AND THE ROLE OF STOCK VARIABLES IN ECONOMIC DEVELOPMENT

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Abstract

We analyze the specification and selection of econometric models with dynamic components for explaining economic growth of one or more variables: models in levels, models in first differences and several kinds of mixed models, including the simple mixed dynamic model and the EC Model. We have into account goodness of fit, significance of parameters, cointegration, contemporaneous and lagged relations and forecasting performance, with particular focus on the role of stock variables, through bilateral dynamic relationships, in explaining propagation movements of great importance for economic development. This research has been performed from a disequilibrium approach to the analysis of causal relations in economic growth and development, having into account both demand and supply sides.

JEL classification: A1, B4, C5, O1, O51, O52

Keywords: Dynamic causal models, Stock Variables, Development

1. Introduction

Accordingly to Bancoc, Baxter and Rees(1979) and other authors Economic Dynamics is the part of Economics which analyses the movement of economic variables and systems thorough time, and the three main areas of economic dynamics are: 1) growth and development theory, 2) stability analysis, and 3) the theory of trade cycles (or more generally the theory of economic cycles and fluctuations).

Analysis of stability is well explained in several books as Pindyck and Rubinfeld(1976 and 1998) and Greene(1993) among others.

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Stability analysis in the short run and trade cycles have been very often analyzed in methodological and applied publications of Econometrics, but less attention have generally received the studies addressed to analyze dynamic relationships of growth and development, in spite of the great importance of these studies.

Some experts on Economic Thought, like Hutchison(1992) and (2000) and Mayer(1994), among other prestigious authors, have shown regret about the increasing priority of formalist sophistication in many academic publications and the low degree of interest of many of those publications towards relevant questions that affect real economy, such as economic development. From the point of view of econometrics researchers we may find also prestigious voices, such as Klein(1980) and (1997) recommending priority to Economics relevance of econometric applications instead of excess of formalism with little interest for economic policies.

The purpose of this study is to point to several relevant questions that may be of interest for researchers interested in realistic modeling dynamic relationships of growth and development, well at sector level or at a general macroeconomic level. Section 2 is devoted to concepts and types of dynamic models with special focus on causal and contemporaneous dynamic models. Section 3 is related with models selection, including comparison of goodness of fit, analysis of causality and cointegration. Section 4 analyzes the important role of stock variables in development dynamics, as well as the existence of feedback, and the cases of coefficients higher than unity in lagged variables. Sections 3 and 4 include examples of some applications to OECD countries. Finally section 5 presents the main conclusions.

2. Concepts and types of dynamic models.

2.1. Causal and contemporaneous dynamic models.

Causal dynamic models are those models where a change in one explanatory variable in one moment of time has an impact, on one or more explained variables, which is transmitted through many moments of time (may include current and future moments or only future ones).
The table 1 presents a classification of causal dynamic models with at least one contemporaneous relationship between $y_t$ and $x_t$.

### Table 1. Causal dynamic models with contemporaneous relation

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Types of models</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>a) with explicit lags in $y_t$: causal autoregressive</td>
</tr>
<tr>
<td>1. Lagged regressors</td>
<td>b) with explicit lags in one explanatory variable</td>
</tr>
<tr>
<td></td>
<td>c) without explicit lags in the regressors but with implicit ones: stock variable or other indirect effects.</td>
</tr>
<tr>
<td>2. Levels of variables</td>
<td>a) levels: $Y$ and $X$ in levels</td>
</tr>
<tr>
<td></td>
<td>b) simple dynamic: $Y$ and $X$ in first differences</td>
</tr>
<tr>
<td></td>
<td>c) mixed dynamic models: $Y$ in levels $X$ in differences</td>
</tr>
<tr>
<td></td>
<td>d) ECM: long term in levels, short term in differences</td>
</tr>
<tr>
<td>3. Direction of dynamic effects</td>
<td>a) one dynamic equation and unidirectional causality</td>
</tr>
<tr>
<td></td>
<td>b) one dynamic equation and bidirectional causality</td>
</tr>
<tr>
<td></td>
<td>c) two dynamic equations and unidirectional causality</td>
</tr>
<tr>
<td></td>
<td>c) two dynamic equations and bidirectional causality</td>
</tr>
<tr>
<td>4. Persistence of propagation effect</td>
<td>a) declining</td>
</tr>
<tr>
<td></td>
<td>b) constant,</td>
</tr>
<tr>
<td></td>
<td>c) increasing</td>
</tr>
</tbody>
</table>

Note: the models of this table are causal models and thus they include at least one exogenous variable, well in the context of a single equation model or in a multiple equation system.

The propagation effect through time is the main feature of this type of models. The effect does not necessarily have to vanish and we consider situation where it may have a constant or an increasing effect through time. The increasing effect may be particularly relevant when there is bilateral causality as we will see in section 2.3.

All the models presented in these tables may belong to different kinds of econometric models that we may consider accordingly to other criteria such as the hypotheses of the random shock, number of equations, linearity and other features. The dynamic models may be estimated with time series samples, pools of time series and cross-sections and even with only cross-section samples if they include the appropriate explanatory variables with explicit or implicit lags.
Examples of the models in table 1:

1) Lagged regressors: We distinguish three cases: explicit lags of the explained variable, explicit lags of one or more explanatory variables, and the effects of lagged values of the explanatory variables in models without explicit lags, but with implicit ones. We may consider the following relations between the endogenous variable $y_t$ and one, or more, exogenous variable $x_i$:

**Lagged endogenous variable**: $y_t = F(x_t, y_{t-1})$ \hspace{1cm} (1a)

**Lagged explanatory variable**: $y_t = F(x_{t}, x_{t-1}, x_{t-2}, \ldots x_{t-n})$ \hspace{1cm} (1b)

In the case of a geometric distributed lags (1b) may be converted in equation (1a) accordingly to the Koyck transformation, in which the final random shock follows a MA(1) model.

**Not explicit lagged regressors**: $y_t = F(x_{1t}, x_{2t}, \ldots x_{kt})$ \hspace{1cm} (1c)

Where one explanatory variable may be explained as a function of its own lagged valued and other variables:

$x_{kt} = \delta x_{k,t-1} + F(z_{1t}, z_{2t}, \ldots z_{nt})$ \hspace{1cm} (1c′)

In model (1c) $x_{kt}$ may be a stock variable or other variable explained by an autoregressive causal model. When $x_{kt}$ is a stock variable: $0 < \delta < 1$ is the parameter of survival at moment $t$ of its lagged value given by $1 - \gamma$, where $\gamma$ is the depreciation rate. When $x_{kt}$ is not a stock variable the coefficient $\delta$ may not be restricted to the interval $(0,1)$ as it may happen in a mixed dynamic model as we will see below.

It is important to consider (1c) as a dynamic causal model, for example in the case of the Cobb-Douglas production function, because an increase in the stock variable $x_{kt}$ in one moment of time, activated by a change in one of the explanatory variables $z_{it}$, will have a persistent effect on $y_t$ for many years. Often this type models is not included in many books related with the analysis of dynamic relations.
in Econometrics but in our view it should be included because it may have an important role in explaining dynamic relations of growth and development.

2) Levels of the variables: The cointegration analysis has often focused in the comparison between models in levels and in first differences in order to choose the most adequate. Some researchers found also interesting to consider mixed models where some variables are explained in levels and others in first differences, because these model may be in many cases most adequate. Examples for the four cases of the second row of table 1 are as follows:

Model in levels: \( y_t = \beta_0 + \beta_1 x_{1t} + \beta_2 x_{2t} + \beta_3 y_{t-1} + \epsilon_t \) (2a)

Interpretation of coefficients is not immediate in model (2a). The estimators of the parameters are called short-run coefficients, and they usually show a value rather low in comparison with the long-run coefficients of the model in levels, which are calculated as:

\[ \beta_i^* = \frac{\beta_i}{1 - \beta_3} \quad (i=1,2) \quad \text{if} \quad \beta_3 < 1. \]

The long-run effect is more representative of the importance of each explanatory variable than the short-run effect.

Model in first differences: \( D(y_t) = \beta_1 D(x_{1t}) + \beta_2 D(x_{2t}) + \epsilon_t \) (2b)

This model may include also a term \( \beta_3 D(y_{t-1}) \), where \( \beta_3 \) may be different from zero. Even if this parameter is null, and the term is not include, model (2b) is dynamic because an increases in one of the explanatory variables at moment \( t \) is transmitted to \( y_t, y_{t+1}, y_{t+2}, ...., \) because \( y_t \) may be written as:

\[ y_t = \beta_1 D(x_{1t}) + \beta_2 D(x_{2t}) + y_{t-1} + \epsilon_t \]

and any increase in one of the explanatory variables is transmitted as a constant to the current and all the future levels of the endogenous variable.
Mixed dynamic model: \( y_t = \beta_1 D(x_{1t}) + \beta_2 D(x_{2t}) + \beta_3 y_{t-1} + \epsilon_t \) \hspace{1cm} (2c)

In comparison with (2a) the mixed dynamic model (2c) has the advantage of a more clear interpretation of the coefficients, and usually better results regarding goods of fit and forecasting capacity.

In comparison with (2b) model (2c) has the advantage that the effect of a change in one of the explanatory variables, for example \( x_{1t} \), on current and future values of \( y_t \) has not to be constant and takes the following value:

\[
\frac{\delta y_{t+s}}{\delta x_{qt}} = \beta_1 \left( \beta_3 \right)^s
\]

Thus the effect of a change in \( x_t \) on future values of \( y_{t+s} \) is as follows: decreasing if \( 0 < \beta_3 < 1 \), constant if \( \beta_3 = 1 \) and increasing if \( \beta_3 > 1 \). We may found the three types of models in reality as we will see below. The case of \( \beta_3 > 1 \) usually is due to the effects of some missing explanatory variables on the coefficients of the regressors included in the model.

We may notice that model (2c) coincides with model (2b) when \( \beta_3 = 1 \). In general (2c) is a better model than (2a) and (2b) if \( \beta_3 \) is different from unity and it is very similar to (2b) when \( \beta_3 \) is close to unity. In a few cases models (2a) or (2b) may give better results than (2c) if the data present some particular features as we shall see below.

CE model: This model presents two versions with and without contemporaneous relation between \( D(y_t) \) and \( D(x_t) \) in the short run relationship. In both cases the long run relationship is the same. We write, for simplification, the case of only one explanatory variable:

long run relationship: \( y_t = \beta_0 + \beta_1 x_{1t} + \epsilon_t \) and short run relationship:

\[
D(y_t) = \beta_1 D(x_{t-1}) + \beta_2 D(y_{t-1}) + \beta_3 D(x_t) + \gamma e_{t-1} + u_t \hspace{1cm} (2d)
\]

Where \( e_{c1} \) is the lagged residual of the long run relationship and \( u_t \) is the random shock of the short run relationship. This model in the original version did not include the current \( \beta_3 D(x_t) \), because it was
developed under the approach of non contemporaneous relations in the short run, but the experience shows that including the current value of $D(x_t)$ usually is a better option because the goodness of fit and forecasting capability increases, and many researchers follow this second approach.

3) Direction of dynamic effects. We distinguish four cases: a) one dynamic equation and unidirectional causality: only unilateral direction of dynamic effects, b) one dynamic equation and bidirectional causality: only unilateral direction of dynamic effects, c) two dynamic equations and unidirectional causality: bilateral direction of dynamic effects without feedback, d) two dynamic equations and bidirectional causality: bilateral direction of dynamic effects with feedback. This last case is of uppermost importance in development dynamics.

One dynamic equation and unidirectional causality

\[ y_t = F(x_t, y_{t-1}) \quad \text{and} \quad x_t = F(z_t) \]  

(3a)

being $x_t$ independent of $y_t$ and $z_t$ independent of $x_t$ and $y_t$

In (3a) a change in $z_t$ has effect on current $x_t$ and dynamic effects on $y_t$ (current and future values), but there are not dynamic effects on $x_t$

One dynamic equation and bidirectional causality

\[ y_t = F(x_t, y_{t-1}) \quad \text{and} \quad x_t = F(y_t, z_t) \]  

(3b)

being $z_t$ independent of $y_t$ and $x_t$. A change in $z_t$ will have a contemporaneous effect of $x_t$, and a dynamic effect on the current and future values of $y_t$. The change in $y_t$ will have only a contemporaneous effect on $x_t$ but not in its future values.

Two dynamic equations and unidirectional causality

\[ y_t = F(x_t, y_{t-1}) \quad \text{and} \quad x_t = F(z_t, x_{t-1}) \]  

(3c)
In (3b) a change in \( z_t \) has dynamic effects both on \( x_t \) and \( y_t \) but there is not feedback between the changes in \( y_t \) and those in \( x_t \).

Two dynamic equations and bilateral causality:

\[
y_t = F(x_t, y_{t-1}) \quad \text{and} \quad x_t = F(z_t, y_{t-1}, x_{t-1}) \quad (3d)
\]

being \( z_t \) independent of \( y_t \) and \( x_t \),

\[
\text{or } x_t = F(z_t, x_{t-1}) \quad \text{and} \quad z_t = f(y_{t-1}, \text{other variables}) \quad (3d')
\]

A change in \( z_t \) in (3d), or in other variables in (3d'), has dynamic effects both on \( x_t \) and \( y_t \). Besides any change in \( y_t \) is an activator of future changes in \( x_t \), which new dynamic effects on future values of \( y_t \). This feedback may be very important in economic development dynamics.

4) Propagation effect. Many studies of dynamic models addressed to analyze the stability of the model after a shock consider that the dynamic effects may be decreasing through time and that after some years, or months, since the initial shock the effect will cease. In the case of dynamic models addressed to analyze the growth and development of the variables through time, we should consider also the case of constant and permanent effects and the case of increasing effects. We have seen the three cases for the mixed dynamic models and similarly we could analyze the three situations in other specifications.

In model (2c) we have seen the following cases, regardint the effect of a change in \( x_t \) on future values of \( y_{t+s} \):

- decreasing If \( 0 < \beta_3 < 1 \); constant if \( \beta_3 = 1 \); increasing if \( \beta_3 > 1 \)

In some cases the value of \( \beta_3 \) is expected to be less than one (due to depreciation or to other effects) or equal to unity (if all the relevant exogenous variables do not change the expected value of the endogenous variable is expected to remain equal to its lagged value) but the effect of some missing explanatory variables linearly related
with the lagged value of the endogenous variable, may explain that the value of $\beta_3$ could be higher than unity.

A value of $\beta_3$ higher than unity usually do not imply any chaotic evolution of the model, because the value of this parameter may be moderated in the future and to lower down until a value close to one, if the effects of the missing variables diminish, or may be for many years slightly higher than one without provoking any chaotic effects on the evolution of the endogenous variable.

### 2.2. Non causal and non contemporaneous dynamic models

**ARIMA models:** The Autoregressive Integrated and Moving Averages models, are purely autoregressive models and non causal models, where there is not any exogenous variable and the evolution of $y_t$ is explained only as a function of their own lagged values and the current and lagged values of the random shock, are used by some authors to analyze the dynamic effect of an exogenous shock. To do that they include an exogenous increase in the variable at moment $t$, what it is similar to add an exogenous variables multiplied by a dummy variables which only takes value equal to one in a moment of time, and analyze the effect through future years. For the case of an integrated variable of order zero the equation is:

$$
y_t = F(y_{t-1}, \ldots, y_{t-p}, \varepsilon_t) \quad \text{and} \quad \varepsilon_t = F(u_t, u_{t-1}, \ldots, u_{t-q}) \quad 5(a)
$$

where $\varepsilon_t$ and $u_t$ are random shocks and there are not exogenous variables. Equation (5a) may include an intercept. When the variable is integrated $(d+s.D)$ instead of $y_t$ we include in relation the corresponding ARMA variable, $y^*_t$, after differentiation:

$$
y^*_t = (1-B)^d (1-B^s)^D,
$$

where $d$ is the degree or non-seasonal differentiation, $D$ is the degree of seasonal differentiation, $B$ is the back operator ($B^s y_t = y_{t-s}$). We need to choose the values of $d=0,1,2,\ldots$, $D=0,1,2,\ldots$ accordingly to the identification rules of ARIMA models.
VAR models. The Vector Auto-regression models are considered by some authors as non causal models where a vector including two or more variables in moment $t$ is a function of the past values of the vector and a random shock. This approach is usually based on the joint analysis of the evolution of several variables but without setting any order of causality among them. Nevertheless Granger has used this approach to analyze the direction of causality between variables: a) When there is unilateral direction of causality, for example $y_t$ depends causally on the lagged values of $x_t$ but $x_t$ does not depend on the past of $y_t$, we may consider that lagged values of $x_t$ are exogenous variables and thus the model is causal dynamic model and non contemporaneous. b) When there is bilateral direction of causality, then there are not exogenous variables and the model is a non causal dynamic model where we can study only the effects of an exogenous shock similarly to the case of the ARIMA models.

The main limitation of VAR models is the lack of contemporaneous relationships which limits their interest in many real applications. These models may be estimated for the variables in levels or in first differences, so the relationship between $y_t$ and $x_t$ may be expressed as:

$$y_t = F(y_{t-1}, \ldots y_{t-n}, x_{t-1}, \ldots x_{t-n})$$

(5b)

or:

$$D(y_t) = F((D(y_{t-1}), \ldots D(y_{t-n}), D(x_{t-1}), \ldots D(x_{t-n}))$$

(5b′)

Causal and non contemporaneous dynamic models: This type of models includes a short number of lags in the exogenous variable, and at least one lagged value of the endogenous variable. Models with many lags in the exogenous variables without the lagged value of the endogenous one may be transformed in a model with one lag in the exogenous variable and one lag in the endogenous, similarly to the Koyck transformation mentioned in 2.1 for the contemporaneous case. An example of this type of models is the following one:

$$y_t = F(y_{t-1}, x_{t-1})$$

(5c)

An increase in $x_{t-1}$ has a dynamic effect on $y_t$ and its future values.
2.3. Non dynamic models

Having into account all the kinds of dynamic models abovementioned we may define non dynamic models as those where the changes in \( x_t \) are only transmitted to \( y_t \) in the same or in the following period but without important future effects. An example is the following one:

\[
y_t = \beta_0 + \beta_1 x_{1t} + \beta_2 x_{2t} + \beta_3 x_{3,t-1} + \varepsilon_t \tag{6}
\]

where the explanatory variables are exogenous, and thus do not dependent on \( y_t \), and none of the exogenous variables may be expressed as autoregressive causal models type (1c’) and do not influence in any other way the future values of \( y_t \) for many periods.

Suppose that equation (6) includes a few lags of one exogenous variable, for example if we add as regressors two additional lagged values of \( x_3 : x_{3, t-2} \) and \( x_{3, t-3} \). This type of model with a very short number of lagged values of the exogenous variables has only a transitory dynamic effect but it is not strictly a dynamic model, only in a very wide sense it could be considered partially dynamic.

3. Selection of dynamic models.

The mixed dynamic model is very often a better specification than the other dynamic and non dynamic models, not only regarding goodness of fit and forecasting capability but also regarding cointegration, significance of parameters and analysis of causality. The EC model with contemporaneous causality very often behaves similarly to the mixed dynamic model regarding goodness of fit and forecasting capability, but the interpretation of coefficients is less clear and the method is less simple, and thus the mixed dynamic model is usually preferable. In order to compare the results of several models some useful suggestions are the following ones:

1) **Goodness of fit and forecasting capability:** When we wish to compare a model with the dependent variable in levels with another model with the dependent variable in first differences it is important to have into account that some statistics are comparable and other
ones are not directly comparable. The Standard Error of Regression is comparable because the residual is the same for a variable $y_t$ and for its first difference $D(y_t)$, while the determination coefficient is not directly comparable because although the Sum of Squares of Residuals is comparable the Sum of Total Squares is quite different in both cases. We must calculate the determination coefficient of $y_t$, and not of $D(y_t)$, in the model where this variable is not in levels if we wish to compare the $R^2$, or the adjusted $R^2$, of both models. Other statistics like Schwartz criteria are comparable. Regarding the forecasting capability RMSE (Root of the Mean Square Error) are comparable but the percentage of RMSE in relation with the mean of $y_t$ is not comparable to the percentage in relation with the mean of $D(y_t)$, and thus we should calculate the percentage in relation with the same mean (usually mean of $y_t$ during the sample period).

Table 3 presents the results of the estimation of several models for the relation between real Private Consumption (C90) and real Gross Domestic Product (GDP90) in the United States.

### Table 2. Estimated relation between C90 and Gdp90, US, 1961-2003

<table>
<thead>
<tr>
<th>Model</th>
<th>Regressors</th>
<th>Coeff. gdp90</th>
<th>Coeff. c90(-1)</th>
<th>Prmse in sample</th>
<th>Prmse in forecast</th>
<th>RS</th>
<th>RF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mixed Dynamic with ar(1)</td>
<td>D(gdp90)</td>
<td>0.4793</td>
<td>1.0111</td>
<td>0.70</td>
<td>2.74</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>c90u(-1)</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>ar(1)</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mixed Dynamic</td>
<td>D(gdo90)</td>
<td>0.4994</td>
<td>1.0101</td>
<td>0.72</td>
<td>3.71</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>c90(-1)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Levels With lag</td>
<td>c gdp90</td>
<td>0.4350</td>
<td>0.4202</td>
<td>0.92</td>
<td>3.84</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>c90(-1)</td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Levels without lag</td>
<td>c gdp90</td>
<td>0.7330</td>
<td>-</td>
<td>1.26</td>
<td>4.17</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>Levels with ar(1)</td>
<td>c gdp90</td>
<td>0.7307</td>
<td>-</td>
<td>0.87</td>
<td>4.41</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>AR(1)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>First differences</td>
<td>D(gdp90)</td>
<td>0.6634</td>
<td>1</td>
<td>0.89</td>
<td>4.83</td>
<td>3</td>
<td>5</td>
</tr>
</tbody>
</table>

Note: Prmse is the percentage of the Root of Mean Square Error on the mean of the explained variable. RS and RF are, respectively, the ranking positions of each model in sample and forecasting period.
The sample period was 1961-2000 and the forecasting period 2001-2003. Data and some comments are included in the Annex.

The estimations in table 1 are single equation models. Although there is some degree of bidirectional causality between C90 and GDP90, as seen in Guisan(2002), and a more detailed multiple equation model should be more complete, for this example we consider that the comparisons are of interest because the strongest direction of causality is that in which Consumption is the variable explained by Gdp. In this comparison the mixed dynamic models ranked in the best positions. The values of the coefficient of C90(-1) higher than unity are explained in the Annex.

2) Cointegration: As stated in Guisan(2001) and (2002) the analysis of cointegration should be addressed with caution, having into account the following questions: a) The non rejection of the hypothesis of “non cointegration” in the ADF test does not always imply evidence against non cointegration, simply it may happen that there is an slight degree of uncertainty. For example the interval of confidence for the coefficient of autocorrelación $\rho$ may be between 0.75 and 1.01, so we can not reject the hypothesis of non cointegration but there is more evidence in favour of cointegration ($\rho<1$) that in favour of no cointegration ($\rho=1$). b) Non cointegration is not equivalent to spurious regressions, because we may found non cointegration, between two variables with strong causality, due to incorrect specification of the relation: for example if the variables are in levels and the best specification is a mixed dynamic model. In that case we change the specification of the equation and the problem is easily solved.

3) Joint regression; It is a simple and useful method to distinguish between true and spurious relationships in many cases, which may have clear advantages in comparison with cointegration tests in some cases. The cointegration tests results may present both the problem of rejection of true relations and the acceptance of false ones. They are useful in some cases but they should be used with caution. The joint regression method very often allows us to solve a problem of selection of explanatory variables with more accuracy. For example
if we wish to decide if $x_{1t}$ or $x_{2t}$ is the main explanatory variable in the model $y_t$, we may estimate a model where $y_t = f(x_{1t}, x_{2t})$ and test the significance of any individual parameter in the joint regression. If the sample size is correct and there is not a problem of very high correlation among the explanatory variables, the results of the regression will show clearly which of both variables has a significant parameter. Table 3 shows the results of a study of 625 regressions to analyse causality between Private Consumption and Gross Domestic Product in 25 OECD countries during the period 1961-97, presented in Guisan(2001) and (2002).

In table 4 we may notice that for the mixed dynamic model both the ADF test of cointegration and the joint regression allowed to recognize the true relationship in 100% of the cases, while the EG test have only 88% of right results. Regarding the rejection of the false models the two cointegration tests failed with a percentage of false acceptances between 100% in the case of ADF and 38% in the case of EG, while the joint regression got a 0% of false acceptances. For the model in levels the results where also in favour the joint regression with 100% of acceptance of the true model and 0% of acceptance of the false model.

Table 4. Summary of acceptances, OECD countries, 1961-97

<table>
<thead>
<tr>
<th>Acceptance of true and false models</th>
<th>Levels</th>
<th>Mix.Dyn</th>
</tr>
</thead>
<tbody>
<tr>
<td>true model EG cointegration test</td>
<td>0 %</td>
<td>88 %</td>
</tr>
<tr>
<td>true model ADF cointegration test</td>
<td>84%</td>
<td>100%</td>
</tr>
<tr>
<td>true model joint regression</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>false model EG cointegration test</td>
<td>19%</td>
<td>38%</td>
</tr>
<tr>
<td>False model ADF cointegration test</td>
<td>66%</td>
<td>100%</td>
</tr>
<tr>
<td>False model joint regression</td>
<td>0%</td>
<td>0%</td>
</tr>
</tbody>
</table>

Source: Guisan(2001) and (2006): Relationship between C90 (Private Consumption) and Gdp90 (Gross Domestic Product) at 1990 prices and exchange rates. Data source: OECD National Accounts Statistics.

The results correspond to the Engle-Granger (EG) and to the Augmented Dickey and Fuller (ADF) tests of cointegration, with the McKinnon critical values for both tests, and we present the results of
own cointegration (the cointegration between $C90_{it}$ and $GDP90_{it}$ of a same country) and the cross cointegration (the cointegration between $C90_{it}$ of a country and $GDP90_{jt}$ of any of the other countries). It is clear that the joint regression was more helpful to recognize the strong true relationship existing between both variables, which it is also shown in the following graphs.

The graphs 1 to 3 show the strong relationship that exists between the level of real Private Consumption per inhabitant (CH) and real Gross Domestic Product per inhabitant (PH) in 25 OECD countries, data are in thousand dollars at constant prices and exchange rates.

In this graphs we may notice an increasing dispersion the higher is the level of production per inhabitant. As explained in Guisan(2001) and (2002) two causes of this increase of dispersion are: 1) the usual increase of opportunities for consumers with high incomes to choose between consumption and saving. 2) increase of public consumption per inhabitant in richest countries: citizens of countries with high levels of public provision of services of health and education need to expend less of their private income in this regard. This does not mean that public consumption has a negative effect on private one, because there are also many important positive effects of public services on the demand of private goods and services, as stated in Klein(1999).

Graph 1. Relation between CH and PH in Group 1 (PH < 7)
Graph 2. Relation between CH and PH in Group 2 (7< PH <14)

Graph 3. Relation between CH and PH in Group 3 (PH>14)
d) Causality tests and other alternatives to cointegration

The Hausman test in the context of an interdependent system is, in our experience, a good test for distinguishing between unilateral and bilateral causality. In the case of the relation between real Consumption and Gdp this test and other analyses lead to the conclusion that the main address of causality is from Gdp to Consumption, from the supply side, although there is some degree of bilateral relation as Consumption influences Gdp from the demand side, as seen in Guisan(2002) and other studies.

Regarding the Granger test for analyzing the causal relation between two variables, \( y_t \) and \( x_t \), I would like to say that it is of interest because it is easy to perform and gives some orientation about causality relations, although we may notice that the test has some limitations mainly due to two causes: 1) it does not include the actual value of the main explanatory variable and only includes lagged values. 2) the strong relation between the lagged values of \( y_t \) and \( x_t \) leads to a great degree of multicollinearity and thus it explains how, in many cases, the test fails to recognize causal relations. In some cases a slightly modified version of this test, as seen in Guisan(2002) may be useful.

4. Stock variables, feedback and increasing dynamic effects.

4.1. Stock variables and feedback

As we have pointed out there are very relevant dynamic relationships in models without explicit lags, as it may be the case of the Cobb-Douglas production function, in the case of full capacity:

\[
Q_t = A K_t^\beta L_t^\alpha e^{\lambda t}
\]  

\[
K_t = KA_t
\]

\[
KA_t = \delta KA_{t-1} + I_{t-1}
\]

\[
I_t = F (Q_t, \text{other variables})
\]
being $K_A$, available stock at the beginning of year $t$, $Q_t$ is real Gdp, $K_t$ is utilized stock of physical capital, $L$ is labor, $t$ is time, and $I$ is investment. The parameter $\delta$ is equal to $1$ less $\gamma$, being $\gamma$ the rate of depreciation.

Equation (7) does not show explicit lags but the dynamic impact $K$ on $Q$ is very important and for many periods. Really there is feedback because there are important bilateral relationships: If we get an increase in $K_A$ in one period this, under the conditions of full capacity, will have a positive effect on $Q_t$, which will have a positive effect on $I_t$ and also in $K_A$ and so for many periods. All the accumulation of $K_A$ will have, under the full capacity conditions, positive effects on the future values of $Q_t$ for many periods. Of course full capacity is not always the regime that we may find in real world, and for a more deep analysis we should have into account other restrictions to development as seen in Guisan(2005) and (2006), or in other studies related with the roles of demand and supply in development.

### 4.2. Increasing effects of lagged variables

Some dynamic studies of stability imposes the condition of coefficient less than unity for the lagged value of the explained variable, but this restriction des not hold in the study of dynamics of development, because we may found several relationships in which this coefficient is higher than unity, due to the effects of missing explanatory variables or to other circumstances. This means that the dynamic effects will be very important and increasing through time, but this does not mean that they will be explosive, because other factors and restrictions to economic growth and development will prevent this type of situation.

In the Annex we include the results of the estimation of a mixed dynamic model between Non Manufacturing and Manufacturing real Gdp in four OECD countries: United States, Japan, Germany and Spain for the period 1966-93. The coefficient of the lagged dependent variable is significantly higher than unit in the individual regressions and also in the pool of the four countries.
5. Conclusions

The main conclusions of this study are related with three questions:
1) Existence of important dynamic effects in models without explicit lags, as it happens in the case of one stock explanatory variable or in other cases, which may be highly persistent through time.
2) Good general results of mixed dynamic models with contemporaneous causality, in comparison other specifications, The EC model in the version that includes contemporaneous causality also gets a good position in the comparisons.
3) The great importance of analyzing the direction of causality and the persistence of the dynamic effects, both with contemporaneous and lagged causal relationships, in order to improve our knowledge about development processes and the effects of economic policies.
4) There are interesting alternative to cointegration analysis, in order to distinguish between true and untrue causal relations, like causality tests, specification tests, forecasting performance, and joint regressions. Cointegration tests are also useful but the results must be interpreted having into account that non cointegration does not always imply spurious relationships and that very often the results of cointegration tests show uncertainty with evidence in favor of cointegration and that it is not correct to interpret those results as evidence against cointegration.

Bibliography


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