DETERMINATION OF VOLATILITY AND MEAN RETURNS: AN EVIDENCE FROM AN EMERGING STOCK MARKET
KIANI, Khurshid M. *

Abstract: In the present research we work with excess returns for an emerging stock market i.e. Jamaican Stock Price Index for the determination of volatility persistence and persistence in the mean returns series. We model excess returns in this stock market using state space or unobserved component models, which is a signal extraction approach. Our model encompass stable distributions to account for fat tails and GARCH-like effects to account for time varying volatility that may be present in the series. The study results that are obtained using the most general as well as the restricted versions of the state space models reveal statistically significant evidence of volatility persistence in the excess returns series. Further, there exist persistent predictable signals in returns series at 5 percent level of significance, and the value of an efficiently estimated excess returns series is 1.7 percent per month (20.4 percent per annum). Further, the series encompass a stable characteristic exponent $\alpha$ of 1.634 showing a non-normal behavior in this market.

JEL codes: C22, C53, G14

Keywords: stock return predictability, unobserved components, fat tails, stable distributions

1. Introduction

A wide-ranging literature appears on stock return predictability since high profits can be obtained with accurate stock return predictions particularly when suitable trading strategies are employed (Xu, 2004). A survey article by Fama (1991) shows earlier empirical work in this area in addition to many recent studies that employ data from developed countries. However, the present

* Address for correspondence: Department of Finance, BCB, KIMEP, Room #204, Dostyk Building, 2 Abai Avenue, Almaty 050010, Republic of Kazakhstan, E-mail: mkkiani@yahoo.com, kkiani@kimep.kz
research focuses on predictability is excess returns particularly in an emerging stock market in Caribbean region i.e. the Jamaican Stock Price Index, where empirical work pertaining to predictability in stock returns is sparse.

Researcher who studied predictability in stock returns focused mainly on the two aspects of stock returns predictability i.e. non-normality or fat tails, and volatility persistence. For example Nelson (1991) Danielsson (1994), Pagan and Schwert (1990), Diebold and Lopez (1995), and Goose and Kroner (1995) showed existence of volatility persistence in stock returns, whereas Akgiray and Booth (1986), Jensen (1991), de Vries (1991), Buckel (1995), Mantegna and Stanley (1995), and McCulloch (1997) concluded that stock returns are non-normal with fat tails showing that the errors come from a non-normal family. Therefore, the models employed for predicting stock returns should incorporate measures to account for non-normality as well as conditional heteroskedasticity.

State space or unobserved component model in addition to many other models have been employed for stock returns predictability. For instance, Conard and Kaul (1988), Harvey (1985) and Watson (1986) employed state space or unobservable component model to predict stock returns, however, they assumed errors to follow normal distributions which is contrary to the findings presented in the above paragraphs. Mantegna and Stanley (1995), Buckel (1995), McCulloch (1996a), McCulloch (1997), and Bidarkota and McCulloch (2004) modeled stock returns within the framework of Parisian stable distributions using non-Gaussian state space models that encompass non-normality and conditional heteroskedasticity.

Adequate forecasting models with such features that would account for fat-tails and time varying volatility have not been employed so far in the context of the emerging countries’ stock markets like Jamaica, therefore, we believe that the present study will fill this gap adequately. Therefore, we investigate possible existence of persistent predictable signal in monthly Jamaica Stock Price Index (JSPI) excess returns over the respective risk free rates using non-Gaussian state space models with stable distributions and
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GARCH-like effect that take into account fat tails and time varying volatility in the returns series. As in Bidarkota and McCulloch (1998), we relax normality assumption in favor of stable distributions because the powerful Kalman filter does not work efficiently with stable distributions.

The remaining study is organized as follows. Section 2 shows the most general state space model employed and its estimation issues. Section 3, shows data sources, empirical results, and hypotheses tests, and finally section 4 includes conclusions that can be drawn from this study.

2. State space model for stock returns

A state space model represents a multivariate time series model through auxiliary variables, some of which might not be directly observable, which are also called ‘state vectors’. State vectors summarize all the information from the present and the past values relevant to the prediction of the future values of series. The state space models (SS) are also called Markovian representations, canonical representations, or multivariate time series processes. The state space approach to model a multivariate stationary time series process is summarized by Akaike (1976). Any Gaussian multivariate stationary time series can be written in SS form provided the dimension of the predictor space is finite.

SS models are alternative formulation of time series with a number of advantages for forecasting. All ARMA models can be written as SS models. Non-stationary models, e.g., ARMA with time varying coefficients, are SS models as well. Multivariate time series can be handled more easily with SS models and these are consistent with Bayesian methods. In general, a SS model consists of an observation and a state equation. In the following Gaussian SS model we assume Equation 1 to be an observation or measurement Equation whereas Equation 2 is a state Equation:

\[ y_t = H_t z_t + Gx_t + v_t \]  \hspace{1cm} (1)

\[ z_t = Bz_{t-1} + Fx_{t-1} + w_{t-1} \]  \hspace{1cm} (2)
where \( v_t \sim N(0, \Sigma_v) \) and \( w_t \sim N(0, \Sigma_w) \) for all \( t = 1,2,\ldots,n \). Here, both the input matrix and transition matrices are time invariant and unknown. The measurement matrix \( H_t \) is assumed to be known and non-stochastic at time \( t \). The white noise processes, \( v_t \) and \( w_{t-1} \) for \( t = 1,2,\ldots,n \) are independent of each other and are Gaussian with time invariant covariances. Because of the Gaussian nature of shocks, the powerful Kalman filter works efficiently, so we use it as our estimation algorithm for estimating Gaussian SS models.

Conard and Kaul (1988) modeled weekly stock returns on size-based portfolios using SS model considering that shocks in observation as well as state equations are independently and identically distributed (iid) normal. They assumed stock returns to develop from first order autoregressive process. Likewise, SS models were employed by Harvey (1989) and Watson (1986) with the assumptions that underlying errors are iid normal. However, Bidarkota and McCulloch (2004) used SS models with the assumptions that the errors are non-normal which is in conformance with many studies that showed that stock returns encompass non-normality. Therefore, in the present research, we employ non-Gaussian SS models that account for fat tails and GARCH-like effects in the return series, which is shown in following thee Equation:

\[
\begin{aligned}
r_t &= x_t + \varepsilon_t, \quad \varepsilon_t \sim c_t z_{1t}, \quad z_{1t} \sim iid \text{ } s_{\alpha}(0,1) \\
(x - \mu) &= \phi(x_{t-1} - \mu) + \eta_t, \quad \eta_t \sim c_{\eta} c_t z_{2t} \\
z_{2t} &\sim iid \text{ } S_{\alpha}(0,1) \\c_t^\alpha &= \omega + \beta c_{t-1}^\alpha + \delta |r_{t-1} - E(r_{t-1} | r_1, \ldots, r_{t-2})|^\alpha \\
&+ \varphi d_{t-1} |r_{t-1} - E(r_{t-1} | r_1, \ldots, r_{t-2})|^\alpha
\end{aligned}
\]  

(1a)

(1b)

(1c)

where,

\[
d_{t-1} = \begin{cases} 
1 & \text{if } r_{t-1} - E(r_{t-1} | r_1, \ldots, r_{t-2}) < 0 \\
0 & \text{otherwise}
\end{cases}
\]
Here $r_t$ is the observed one-period excess return, $x_t$ is an unobserved persistence components in the series, and $Z_1$ and $Z_2$ are independent white noise processes.

Model 2 is obtained restricting $\alpha = 2$ in model 1, which can be written as:

$$r_t = x_t + \varepsilon_t, \quad \varepsilon_t \sim \sqrt{2c_t} Z_t, \quad Z_t \sim iid \, N(0, 1) \tag{2a}$$

$$(x_t - \mu) = f(x_{t-1} - \mu) + \eta_t, \quad \eta_t \sim \sqrt{2c_t} Z_2, \quad Z_2 \sim iid \, N(0, 1) \tag{2b}$$

$$c_t^2 = \omega + \beta c_{t-1}^2 + \delta |r_{t-1} - E(r_{t-1} | r_1, r_2, ..... r_{t-2})|^2 + \gamma d_{t-1} - E(r_{t-1} | r_1, r_2, ..... r_{t-2})|^2 \tag{2c}$$

Setting $\beta = \delta = \gamma = 0$ in model 1 gives model 3, which is shown in Equation 3:

$$r_t = x_t + \varepsilon_t, \quad \varepsilon_t \sim S_\alpha(0, c) \tag{3a}$$

$$(x_t - \mu) = f(x_{t-1} - \mu) + \eta_t, \quad \eta_t \sim S_{\alpha}(0, c_\eta, c) \tag{3b}$$

When restricting $\phi = 0$ in model 1, the shocks $\varepsilon_t$ and $\eta_t$ are not separately identified so $c_\eta$ is also not identified. The resulting model is model 4, which is shown in Equations 4:

$$r_t = \mu + \varepsilon_t, \quad \varepsilon_t \sim c_t z_t, \quad z_t \sim iid S_\alpha(0, 1) \tag{4a}$$

$$c_t^\alpha = \omega + \beta c_{t-1}^\alpha + \delta |r_{t-1} - \mu|^\alpha + \gamma d_{t-1} |r_{t-1} - \mu|^\alpha \tag{4b}$$

where, $$d_{t-1} = \begin{cases} 1 & \text{if } r_{t-1} - \mu \leq 0 \\ 0 & \text{otherwise} \end{cases}$$

Model 5 shown in Equation 5 is obtained setting $\alpha = 2$ in model 4.

$$r_t = \mu + \varepsilon_t, \quad \varepsilon_t \sim \sqrt{2c_t} z_t, \quad z_t \sim iid \, N(0, 1) \tag{5a}$$

$$c_t^2 = \omega + \beta c_{t-1}^2 + \delta |r_{t-1} - \mu|^2 + \gamma d_{t-1} |r_{t-1} - \mu|^2 \tag{5b}$$

Restricting $\beta = \delta = \gamma = 0$ in model 4 results in model 6 which is presented in Equation 6:

$$r_t = \mu + \varepsilon_t, \quad \varepsilon_t \sim S_\alpha(0, c) \tag{6}$$

A random variable $x$ will have stable distribution $S_\alpha(0, c)$ when its log characteristic function can be represented as

$$\ln E \exp(i \lambda x) = i \lambda \delta - |c_t|^\alpha.$$ The parameter $c > 0$ measures scale whereas the parameter $\delta(-\infty, \infty)$ measures location, and $\alpha \in (0, 2]$ is
the characteristic exponent that governs the tail behavior. A small value of $\alpha$ indicates thicker tail, however, normal distribution pertaining to symmetric stable family results when $\alpha = 2$ whose variance is equal to $2c^2$.

In the process contained in Equation (1c) we restrict $\omega > 0, \beta \geq 0, \delta \geq 0$, and $\gamma \geq 0$. The theoretical term involving dummy variable $d_{t-1}$ captures leverage effects that are transmitted from negative shock to increase in future volatility more than a positive shock of equal magnitude (Nelson 1991, and Hamilton Susmel 1994). However, when the errors are normal, the model of volatility persistence reduces to GARCH-normal process.

Any predictable variation in excess return is because of persistent component $x_t$, which are assumed to follow a simple AR (1) process. When predictable component in Equation 1 becomes significant, than $E(r_t | r_{t-1}, \ldots, r_{t-1})$ provides a useful forecast of returns. However, when $c_\eta$ and $\phi$ or one of these is negligible, the returns are purely random, so these may display spurious predictions.

2.1. Estimation issues

Non-Gaussianity of the SS model in Equation 1a–1c creates complication in estimation even without the presence of conditional heteroskedasticity. This happens because the Kalman filter is no longer optimal due to the non-Gaussian nature of shocks.

The general recursive-filtering algorithm due to Sorenson and Alspach (1971) provides optimal filtering and predictive densities under any distribution for the errors and the formula for computing the log likelihood function. These formulae are presented in Bidarkota and McCulloch (2004). The recursive equation that is employed to compute filtering and predicting densities are given in the form of integrals whose close form analytical expressions are generally obstinate, especially in our case. Therefore, in this study, we numerically approximate these integrals.

Although Zolotriv’s (1986) recommended that the stable distributions and density be evaluated by taking inverse Fourier
transformation of the characteristic function or by proper integral representation, we restrict our characteristic exponent $\alpha$ to a range determined by McCulloch (1996b) to facilitate computational convenience because we employ his fast numerical approximations to stable distribution and density that has an expected relative density of the precision of $10^{-6}$ for $\alpha \in [0.84, 2]$.

3. Empirical results

3.1. Data sources.

We employ monthly stock prices for Jamaican Stock Price Index (JSPI) over the risk free rates i.e. Treasury bill rates of the relevant frequency from 1993:3 to 2005:6. The stock prices were obtained from Jamaican Stock Exchange whereas the relevant risk free rates were obtained from the Bank of Jamaica. Figure 1 shows plots of excess return series for Jamaican Stock Price index (JSPI).

![Figure 1: Jamaica excess returns](image)

3.2. Estimation Results

Table 1 show estimation results for JSPI for different models estimated. This Table shows parameter estimates for characteristic exponent $\alpha$, volatility persistence parameter $\beta$, ARCH parameter $\delta$, leverage parameter $\gamma$, signal to noise ratio $c_\eta$, and AR coefficient of persistent component of returns $\phi$ for the most general state space or unobserved component model.
Table 1: Model Estimates with Leverage Effects: JSP I Excess Returns

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Model 1</th>
<th>Model 2</th>
<th>Model 3</th>
<th>Model 4</th>
<th>Model 5</th>
<th>Model 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha )</td>
<td>1.426 (0.032)</td>
<td>2 (rest.)</td>
<td>1.482 (0.078)</td>
<td>1.634 (0.132)</td>
<td>2 (rest.)</td>
<td>1.699 (0.110)</td>
</tr>
<tr>
<td>( \mu    )</td>
<td>0.019 (0.009)</td>
<td>0.029 (0.024)</td>
<td>0.021 (0.009)</td>
<td>0.017 (0.003)</td>
<td>0.017 (0.009)</td>
<td>0.059 (0.005)</td>
</tr>
<tr>
<td>( \omega )</td>
<td>0.000 (0.000)</td>
<td>0.005 (0.034)</td>
<td>0.004 (0.002)</td>
<td>0.016 (0.061)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \beta )</td>
<td>0.109 (0.433)</td>
<td>0.136 (6.120)</td>
<td>0.239 (0.312)</td>
<td>0.006 (0.001)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \delta )</td>
<td>0.000 (0.000)</td>
<td>0.000 (0.000)</td>
<td>0.134 (0.097)</td>
<td>0.000 (0.000)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \gamma )</td>
<td>0.395 (0.398)</td>
<td>0.000 (0.000)</td>
<td>0.000 (0.000)</td>
<td>0.000 (0.000)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( c_\eta )</td>
<td>11.480 (1.917)</td>
<td>0.000 (0.000)</td>
<td>11.362 (5.546)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( c )</td>
<td></td>
<td>0.005 (0.002)</td>
<td></td>
<td>0.019 (0.008)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \phi )</td>
<td>0.289 (0.077)</td>
<td>0.976 (0.052)</td>
<td>0.287 (0.079)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log L</td>
<td>126.545</td>
<td>109.184</td>
<td>126.375</td>
<td>121.229</td>
<td>108.179</td>
<td>118.760</td>
</tr>
<tr>
<td>LR ((\alpha = 2))</td>
<td>34.722</td>
<td></td>
<td>26.100</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LR ((\beta = \delta = \gamma = 0))</td>
<td>0.340</td>
<td></td>
<td>4.938</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LR ((\phi = c_\eta = 0))</td>
<td>10.632 (0.001)</td>
<td>2.010 (0.156)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: The unobserved component or state space model with non-normality and conditional heteroskedasticity that is shown in Equations 1a–1c is employed to estimate the results shown in this Table. Normality is tested using the likelihood ratio test statistic LR \((\alpha = 2)\) that gives the value of the likelihood ratio test statistic for the null hypothesis of normality. The small-sample critical value at the 0.01 significance level for a sample size of 300 is reported to be 4.764 from simulations in McCulloch (1997). The LR \((\beta = \delta = \gamma = 0)\) is used to test no volatility persistence in this series. This test is evaluated at \(\chi^2_3\) p-values. Finally, the test for no predictable component in US stock excess returns is evaluated using the LR \((\phi = c_\eta = 0)\). Under this null, the distribution of the LR test statistic is non-standard so the test statistics are evaluated using \(p\)-values generated by estimating Gaussian versions of Models 1 and 2 with data simulated from the estimated Gaussian Model 2 are reported in parentheses. Restricted: (rest.)
The most general non-Gaussian state space model that is shown in Equations 1a−1c is model 1. Relaxing non-normality i.e. $\alpha = 2$ in the most general model, we get model 2.

Similarly, model 3 is obtained relaxing conditional volatility in model 1. Finally model 4 is obtained when restricting predictable component ($\phi = 0$) in model 1. For considering additional tests on non-normality and conditional heteroskedasticity we restrict non-normality ($\alpha = 2$) in model 4 that gives model 5 and likewise relaxing conditional heteroskedasticity in model 4 gives model 6.

Figures 2 shows filter mean $E(x_t | r_1, r_2, r_3, \ldots, r_t)$ for JSMI which reveal that predictable component appear to be constant showing that variation in its parameter estimates might not be component in forecasting access returns.

3.2. Hypotheses test
We test four types of hypotheses for this research i.e. tests for normality, test for persistence in time varying volatility, test for persistence in mean, and tests for leverage effects in addition to the additional tests for volatility persistence and non-normality. These tests are explained in the following paragraphs.
The test for normality is based on the null of $\alpha = 2$ in model 1. The LR test statistics for this test has non-standard distribution because the null hypothesis lies on the boundary of the admissible values for $\alpha$; therefore, standard regularity conditions are not satisfied. The inferences for this test are derived from test statistics based on the critical values due to McCulloch (1997). The null hypothesis for normality for JSPI can easily be rejected using critical values from McCulloch (1997). The results indicate that even after accounting for GARCH-like behavior, the excess returns are significantly non-normal.

The test for the null of homoskedasticity can be constructed by restricting $\beta = \delta = \gamma = 0$ in model 1. The statistical inferences for this test are based on $\chi^2$ distributions. The LR for the null of "no GARCH" that is to test homoskedasticity ($\beta = \delta = \gamma = 0$) in the series is reported in Tables 1. Bases on the critical values that are obtained from $\chi^2$, homoskedasticity in this market is strongly rejected.

The null hypothesis for no persistence in predictable components in mean returns can be obtained setting $\phi = 0$ in model 1, which assumes that return series are purely random. In this case the standard likelihood ratio test statistics for this test are not applicable because the two shocks $\varepsilon_t$ and $\eta_t$ are not separately identified so the scale ratio $c_\eta$ is also not identified either. Similarly, the bound for the asymptotic distribution of a standardized likelihood ratio test statistics due to Hansen (1992) which is applicable in such cases may result in under-rejection of the null or a subsequent power loss as was noticed by Hansen himself. In addition, the test statistics is computationally very intense especially for the present study, so we abstain using it. Therefore, the inferences are drawn based on $p$-values that are generated by estimating Gaussian versions of Models 1 and 2 with data simulated from the estimated Gaussian Model 2.

The null hypothesis of no persistence in mean returns is rejected at 5 percent level of significance using ($LR = (\phi = c_\eta = 0)$ that is evaluated using critical values from $\chi^2$ as well as $\chi^2$ distributions.
Therefore, even after accounting for normality and volatility persistence, there exist statistically significant persistent predictable signals in this market.

The additional tests for non-normality and volatility persistence are constructed considering model 4 as an alternative model. In this case model 5 is the null model for non-normality whereas model 6 is the null model for homoskedasticity. The intuition behind these additional tests is to test the impact of excluding predictable component (from state space model) on the inferences from our models employed.

LR test statistics for normality, volatility persistence, and persistence in predictable components are reported in last three rows in Table 1. Based on the tests results we failed to reject hypotheses of normality and no volatility persistence as well as the null of no predictable component in Jamaican Stock Price index.

Figure 3 plot scales from model 4 for JSPI, which show an evidence of highly non-constant scales in this market.

The fourth hypothesis test for this research is the test for leverage effects. Absence of leverage effect imply that negative shock do not necessarily lead to negative increase in future volatility than positive shocks of the same magnitude. This hypothesis can be tested setting $\gamma = 0$ in Equation 1c showing that no leverage effect exists versus the alternative hypotheses that $\gamma > 0$ demonstrating that the leverage effect does exist in JSPI. The results (not reported for brevity) failed to reject the null hypothesis in favor of no leverage effects in JSPI.
3.3. Discussions on results
The study results on hypothesis tests reveal that monthly Jamaican stock market index excess returns series from March 1993 through June 2005 do possess significant non-normality that is predictable even after accounting for conditional heteroskedasticity. Similarly, volatility persistence is also statistically significant. Leverage effects in volatility is insignificant, however, there is an evidence of statistically significant predictable component in JSPI at 5 percent level of significance using $p$-values that are generated by estimating Gaussian versions of Models 1 and 2 with data simulated from the estimated Gaussian Model 2.

As shown in Figure 3 the index show highly non-constant scales and the Figure also reveal random spikes in the neighborhood of 2001. The plausible cause of these spikes appear to be due to the external events during these years e.g. 2001 bubble blast in the US economy and crises after September 11, 2001 respectively that caused slump in tourism industry in Caribbean countries in general and in Jamaica in particular. However, these plots do not reveal instability after that era even though the Jamaican economy suffered from major hurricane in the year 2004.

4. Conclusions
In this study non-Gaussian state space or unobserved component models are employed to find possible existence of predictable components in Jamaican Stock Price Index (JSPI). The state space models fully account for non-normality and volatility persistence that might be present in return series. The estimated value of characteristic exponent $\alpha$ shows non-normal behavior that demonstrates significant leptokurtosis in this market. The estimated value of characteristic exponent is well away from the value pertaining to normal behavior in this market, and excess stock returns exhibit persistence in stock return volatility that can be characterized by a GARCH-like process. Moreover, there is insignificant leverage effect in the stock return volatility in this market indicating that the negative shocks do not necessarily lead to greater increases in future volatility than the positive shocks of the
equal magnitude. The study results on predictability of monthly stock returns are statistically significant in Jamaican stock price index. The efficiently estimated excess returns for this market are 1.7 percent per month (20.4 percent per annum).

References


