THE RELATIONSHIP BETWEEN WAGES AND PRODUCTIVITY: TAR UNIT ROOT AND TAR COINTEGRATION APPROACH

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Abstract

We analyzed the relationship between wages and productivity in the 1990-2007 period in Turkey. At the first stage we followed the traditional unit root tests and apply the analysis followed by unit root test procedure proposed by Caner and Hansen (2001). Then we discussed the long run nonlinear relationship between wages and productivity by employing the TAR cointegration analysis developed by Hansen and Seo (2002).

JEL Classification: C32, J31

Keywords: wage, productivity, nonlinear, TAR unit root, TAR cointegration.

1. Introduction

In the US, the founder of the approach that wages be based on increases in productivity, the firms have considered productivity levels as the measure of wage rises, while the firms in England have adopted the practice of delivering stocks in return for productivity. The reflection of these instances on labor literature is raising wages after observing increases in productivity or the practice of prior raises in wages in order to increase productivity. The second approach that has been intensely debated after 1980 and known as effective wage approach, has been developed by the European branch of new Keynesians and been used in explaining the reason why wage levels differ from the Walrasian equilibrium values. The studies are not limited to these and the relation between wages and efficiency has been analyzed in different dimensions. Salop (1979), Shapiro and Stiglitz (1984), Hall (1986), Alexander (1993), Hadroyiannis (1997), Dibooglu and Enders (2001), Marcellino and

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Mizon (2001), Welfe and Majsterek (2002), Brügeman (2004), Wakeford (2004) and Christopoulos and Tsionas (2005) have used panel unit root and cointegration analysis in their studies that investigate the relationship between wages and productivity with an approach that also accounts for unemployment as a variable.

Bildirici (2004, 2005) has emphasized that there existed no relation between productivity and wages in Turkey. We also accept that peculiar circumstances after 1980 have caused deviations from productivity on a manufactural basis and the relation between labor productivity and wages\(^1\). However, these factors not only caused deviations but also have led the labor productivity to keep as one of the lowest among OECD countries. Consequently, the rate of increase in labor productivity for the period 1980-2000 is merely 2.7% while the same ratio for East and South Asian countries is 5.6% (OECD, IMD). Considering the period 2003-2005, the years by which Turkey has partially gotten rid of the heaviest crisis it has experienced, it still seems to be among the most unsuccessful performers in productivity increases among OECD countries (see figure 1).

![Figure 1. Labor productivity in OECD countries for the year 2005 (Purchasing power parity, value added per hour)](image)

\(^1\) Productivity on manufactural basis is calculated in conformity with the traditional method that requires the output obtained as the result of production be divided by the amount of input. If this value happens to be less than 1, this means that costs are rising. The most important reason for the increase in costs is the direct and indirect taxes and other liabilities incurred by the employer on behalf of the employee. In terms of costs, specifically labor costs will be considered since the most significant item in costs is this one.
Although there exists a productivity gap of 3.8 times (%283) between the most productive country, Ireland and Turkey, the differential in gross wages is merely 67%. Taxes on wages (direct taxes, indirect taxes and other liabilities) are an important reason for the gross wages being high in Turkey. Tax burden on the wages raise gross wages while they break the link between productivity and wage level. In economic literature, the effect of taxes on wages (especially on effective wage) has been discussed and Economists have come to a conclusion that there exists a strong effect. The literature on taxation in labor markets in context of efficiency wage find that the specification of taxes influences wages and employment [e.g. Malcomson and Sartor, 1987; Hoel, 1990; Delipalla and Sanfey, 2001, (see for efficiency wage, Akerlof and Yellen;1986, Shapiro and Stiglitz, (1984), Johnson and Layard, (1986),. Pisauro (1991) Rasmusen (1998) Faria (2001), Hoon and Phelps (1992), Lin and Lai (1994) and Faria (2000).

These studies are based on direct taxes as a consequence of the structure in the particular country subject to the study. On the other hand, direct taxes, indirect taxes and liabilities are significant in Turkey\(^2\) and payments made by the employer on behalf of the employee (employee’s cost to the firm) increase the gross wages and consequently overall input costs. In 2000, Turkey has been the third among 30 OECD countries in employment taxes (including income tax paid by the employer on behalf of the employee, dole payments, and all employee and employer payments made to the Social Security Foundation). By the year 2001, Turkey has become the first and kept this position in 2004 (calculated from the data by OECD, TISK and IMF). Figure 2 depicts the share of employment taxes in employers’ costs for the period 2001-2005. In this comparison, some indirect taxes peculiar to Turkey are overlooked in order to construct a linkage between Turkey and other countries.

\(^2\) Additionally, costly employment and firing practices, application of lump-sum wage increases in collective bargaining procedures, resistance to job evaluation systems, degeneration of the system and application of minimum wage laws are other important factors that lie beneath low productivity increases. The concept of “employee’s cost to the firm” which seems more important than other factors will be treated separately.
Figure 2. Share of Employment Taxes in Labor Cost, (The ratio of the sum of taxes levied on wages, social security premia of the employer and the employee to the overall labor cost (%) including familial tax subsidies) Source: OECD, Taxing Wages Statistics, various years; TISK, Working Statistics and Labor Cost, various years.

With the effect of mentioned factors, productivity seems to be following a path independent from wage. On the other hand, this does not exactly reflect the facts. The relation between productivity and the wage received by the worker must be analyzed. For this reason, we shall consider the wage exclusive of indirect taxes and other payments. Besides, in order to avoid the high cost of employment and to prevent productivity increases falling below wage increases, firms recourse to some practices such as informal employment\(^3\), foreign and illegal employment\(^4\), non-primary earnings.\(^5\) Such practices generate increases in production but this

\(^3\) Even though there are fluctuations between the years 1989-2006, 50% of overall employment is informal. Concentration of employment on agrarian sector has an important role on this outcome. 26.7% of those who are employed in non-agrarian sectors work at informal sectors (www.tuik.gov.tr)

\(^4\) In the year 1986, General Directorate of Turkish Employment Organization has declared that 50 thousand informal workers had been employed in public sector. In 1990, it had been figured out that 7 out of every 10 employers had informal employment. In 1993, it is estimated that more than 2 million workers are informally employed. And by 1999, it was estimated that, the volume of informal employment exceeded 4.5 million (Bildirici:2007).

\(^5\) “Non-primary earnings” include earnings from activities other than primary activities of firms (interest, stocks, repurchase agreements etc.). this issue that emerged after 1980 has reached its peak after 1995 and particularly in the years 2000-2001. In the crisis years of 1991, 1994, 1999 and 2001, these earnings reach
increase is not observed in national accounts since formal level of production is considered in productivity calculations. For this reason, we shall exclude such effects and base the analysis on formal firms and production figures.

In the second part of the study, we explain the model and the method we will use while we spare the third part for econometric outcomes.

2 Model

In the study, we investigated the TAR unit root and TAR cointegration analysis. In the literature, even though the nonlinear structure and the threshold effects of unemployment are discussed in certain studies, the relationship between productivity and wages do not seem to have been analyzed in accordance with the nonlinear cointegration methods. Rothman (1991); Chen and Lee (1995); Montgomery, Zarnowitz, Tsay, and Tiao (1998); Altissimo and Violante (1996); Chan and Tsay (1998); Hansen (1997); Tsay (1997) analyzed the unemployment accordingly. Further, Caner and Hansen (2001) investigated the unemployment series in accordance with the TAR unit root methodology.

2.1 TAR Unit Root. One point that cannot be overlooked is that, a significant reason of application of the TAR unit root model, as discussed by Caner and Hansen (2001) is the traditional unit root tests (KPSS, ADF, PP etc.) test the hypothesis of stationarity with the nonlinearity also tested within the relevant hypothesis. Thus, it is common that the stationarity is rejected in favor of integration of order d mostly resulting from nonlinearity. On the other hand, Pipenger and Goering (1993) showed that, DF test lost its testing power especially for the TAR models. Tsay (1997) modeled and applied unit root tests but failed to develop the relevant asymptotic distribution theory. Tsay’s model required the usage of constant their highest levels. In the year after the crisis they fall sharply and the next year they rapidly increase one more time. In the years of high “non-primary earnings”, the magnitude of the cost of the employee to the firm becomes unimportant while the growth rate of such earnings decrease (as in the 2001 crisis), firms recourse to practices mentioned above.
autoregressive lags. Gonzalez and Gonzalo (1998) investigated the TAR(1) model that has unit roots and tested the threshold in TAR(1).

In the study, firstly we will follow the method of Caner and Hansen (2001) and apply the unit root tests. Following the acceptance of the unit root in both regimes for the wage and productivity series for the period investigated, we develop the study followed by the TAR cointegration procedure developed by Hansen and Seo (2002) to investigate the cointegration relation among the variables.

Caner and Hansen (2001) put forth that, unemployment is a stationary nonlinear process. Our results confirm the results of Caner and Hansen for the Turkish Economy.

Two regime threshold autoregressive model can be written as follows,

\[
\Delta y_t = \theta_1' x_{t-1} I\{z_{t-1} < \lambda\} + \theta_2' x_{t-1} I\{z_{t-1} \geq \lambda\} + e_t,
\]

where \( x_{t-1} = (y_{t-1}, r_i \Delta y_{t-1}, \ldots, \Delta y_{t-k})' \) and \( I(.) \) is the indicator function.

In model (2.1) errors \( e_t \) are \( i.i.d. \), and \( Z_t = y_t - y_{t-m} \) for some \( m \geq 1 \). \( r_i \) is vector of deterministic components. \( Z_{t-1} \) is predetermined, strictly stationary and ergodic with a continuous distribution function. Threshold variable \( \lambda \) is unknown and takes the values in the interval \( \lambda \in \Lambda = [\lambda_1, \lambda_2] \). In this interval for \( \lambda_1 \) and \( \lambda_2 \), \( P(Z_t \leq \lambda_1) = \pi_1 > 0 \), \( P(Z_t \leq \lambda_2) = \pi_2 < 1 \) relations can be written.

In Caner and Hansen (2001) the components of parameters \( \theta_1 \) and \( \theta_2 \) are analyzed separately as,

\[
\theta_1 = \begin{pmatrix} \mu_1 \\ \beta_1 \\ \rho_1 \end{pmatrix}, \quad \theta_2 = \begin{pmatrix} \mu_2 \\ \beta_2 \\ \rho_2 \end{pmatrix}.
\]

In these \( \theta_1 \) and \( \theta_2 \) vectors \((\mu_1, \mu_2)\) are slope coefficients, \((\beta_1, \beta_2)\) are the slopes on the deterministic components, and \((\rho_1, \rho_2)\) are the slope coefficient on \( y_{t-1} \). For some \( \lambda > 2 \),
E\left| e_i \right|^{2\gamma} < \infty \text{ is assumed. For some matrix } \delta \text{ and continuous vector function } r(s), \ \delta r_{[t,s]} \Rightarrow r(s) \text{ relation is defined in their analyze. These assumptions apply that } \rho_1 = \rho_2 = 0, \text{ for constants } \mu_1 \text{ and } \mu_2, \ \beta'_1 r_t = \mu_1 \text{ and } \beta'_2 r_t = \mu_2, \text{ and when } i \text{ is a vector of ones than } |a'_i| < 1 \text{ and } |a'_{2i}| < 1.

For each } \lambda \in \Lambda \text{ (1) is estimated by OLS, }
\Delta y_t = \hat{\theta}_1(\lambda)'x_{t-1}I_{\{z_{t-1} < \lambda\}} + \hat{\theta}_2(\lambda)'x_{t-1}I_{\{z_{t-1} \geq \lambda\}} + \hat{\epsilon}_t(\lambda)
\text{OLS estimate of } \sigma^2 \text{ for fixed } \lambda, \sigma^2(\lambda) \text{ is, }
\sigma^2(\lambda) = T^{-1} \sum_{t=1}^{T} \hat{\epsilon}_t(\lambda)^2
\text{and OLS estimate of threshold } \lambda \text{ is found by minimizing } \sigma^2(\lambda),
\hat{\lambda} = \text{argmin}_{\lambda \in \Lambda} \sigma^2(\lambda).

Estimation of the other parameters are done by plugging in } \hat{\lambda} \text{. Then the estimated model is written as, }
\Delta y_t = \hat{\theta}_1'x_{t-1}I_{\{z_{t-1} < \hat{\lambda}\}} + \hat{\theta}_2'x_{t-1}I_{\{z_{t-1} \geq \hat{\lambda}\}} + \hat{\epsilon}_t
\text{In this study the main purpose is to find out the presence of threshold effect and unit root. For this purpose Caner and Hansen } (2001) \text{ set up a joint hypothesis for testing procedure. The null hypothesis is, }
H_0 : \theta_1 = \theta_2\text{.}
\text{For testing (4) standard Wald statistic } W_T \text{ and model (2.5) are used. This statistic is written as, }
W_T = T\left( \frac{\sigma^2_0}{\hat{\sigma}^2} - 1 \right).\text{ } \hat{\sigma}^2 \text{ is residual variance from estimated in model (2.5), }
\sigma^2 = T^{-1} \sum_{t=1}^{T} \hat{\epsilon}_t^2.$
and $\hat{\sigma}_0^2$ is the residual variance from OLS estimation of the null linear model. For fixed $\lambda$ in model (2.3), $W_T(\lambda)$ denote the Wald statistic than,

$$W_T(\lambda) = T \left( \frac{\hat{\sigma}_0^2}{\hat{\sigma}^2(\lambda)} - 1 \right)$$

can be written. Because of the fixed threshold parameter $\lambda$ isn’t identified under $H_0$, the relation which is called Sup-Wald statistic is,

$$W_T = W_T(\hat{\lambda}) = \sup_{\lambda \in \Lambda} W_T(\lambda).$$

This relation is valid when $W_T(\lambda)$ is decreasing function of $\sigma^2(\lambda)$ (Caner et. al., 2001).

Second part of Caner and Hansen (2001) procedure is testing for the stationarity. In this case the null hypothesis is,

$$H_0: \rho_1 = \rho_2 = 0$$

if (2.10) holds than the model (2.1) can be defined as a stationary TAR process for $\Delta y_t$. In this case $y_t$ can be described as a unit root process. In equation (2.1) the parameters $\rho_1$ and $\rho_2$ tests the stationarity of $y_t$ with the hypothesis defined in (2.10). On the other hand they investigate the stationarity and ergodicity of the process. In the model (2.3) when $\rho_1 < 0$, $\rho_2 < 0$ and $(1+\rho_1)(1+\rho_2) < 1$ than the process is stationary. So the alternative of the null hypothesis in (2.10) is

$$H_1: \rho_1 < 0 \text{ and } \rho_2 < 0.$$  

But there is another case of interest which mentioned in their analyses is partial unit root case. For this case they formed the following hypothesis,

$$H_2: \begin{cases} 
\rho_1 < 0 & \text{and } \rho_2 = 0, \\
\rho_1 = 0 & \text{or } \\
\rho_1 = 0 & \text{and } \rho_2 < 0.
\end{cases}$$
If \( H_2 \) holds than the process \( y_t \) will be nonstationary but not a classis unit root case (Caner, Hansen, 2001). They suggested using Wald statistic for testing these hypotheses. Unrestricted alternative is \( \rho_1 \neq 0 \) or \( \rho_2 \neq 0 \). In this case test statistic is,

\[
R_{2t} = t_1^2 + t_2^2
\]

In this equation \( t_1 \) and \( t_2 \) are the t - ratios for \( \hat{\rho}_1 \) and \( \hat{\rho}_2 \) from the estimation of model (2.5). Testing the alternatives \( H_1 \) and \( H_2 \) are one-sided. Two-sided Wald statistic has low power than one sided alternative. One sided Wald statistic alternative which is testing one sided alternative hypothesis \( \rho_1 < 0 \) or \( \rho_2 < 0 \) is,

\[
R_{1T} = t_1^2 I_{\{\hat{\rho}_1 < 0\}} + t_2^2 I_{\{\hat{\rho}_2 < 0\}},
\]

Caner and Hansen (2001) emphasized that \( R_{1T} \) and \( R_{2T} \) tests have power against alternative hypothesis \( H_1 \) and \( H_2 \). When the test statistic is significant and the hypothesis \( H_0 \) is rejected, this shows the process has not had a unit root but this result is not enough to discriminate if the process is stationary case \( H_1 \) or partial unit root case \( H_2 \). For this type of decision they build \( t_1 \) and \( t_2 \) test statistics for testing \( H_0 \) against \( H_1 \) and \( H_2 \). All those test statistics are continuous functions of t ratio and \( t_1 \) and \( t_2 \) statistics.

They defined the test statistics as \( R_T = R( t_1, t_2 ) \). In this equation \( R( x_1, x_2 ) \) is a continuous function of \( x_1 \) and \( x_2 \).

First type of test statistic they apply is \( R_{1T} \) which is a one sided Wald test statistic and tests the alternative hypothesis \( \rho_1 < 0 \) or \( \rho_2 < 0 \) against \( H_0 \). The second test is two sided Wald test statistic which tests \( \rho_1 \neq 0 \) or \( \rho_2 \neq 0 \) against \( H_0 \). These two tests help to decide if the process has a unit root or stationary.

Another test statistic is one sided Wald test statistic \( t_1 \) which tests the alternative hypothesis \( \rho_1 < 0 \) and \( \rho_2 = 0 \) against \( H_0 \). This test helps to decide if the process is not stationary or stationarity is only in first regime. The last test statistic is another one sided Wald test statistic \( t_2 \) which tests the alternative hypothesis \( \rho_1 = 0 \) and
\( \rho_2 < 0 \) against \( H_0 \). \( t_2 \) also helps to decide if the process is a partial unit root process.

### 2.2 TAR Cointegration.

Threshold cointegration was introduced by Balke and Fomby (1997). They made a joint analysis of nonlinearity and nonstationarity with cointegration. They showed that a process may follow a unit root in a middle regime whilst at the same time being globally geometrically ergodic in outer regimes. Another application about the joint issues of nonstationarity and nonlinearity are as follows: Balke and Wohar (1998), Baum et. al. (2001) Baum and Karasulu (1998), Enders and Falk (1998), Lo and Zivot (2001), Martens et. al (1997), O’Connell (1998), O’Connell and Wei (1997), Obstfeld and Taylor (1997), Taylor (2001), Michael et. al. (1997).


This paper examines the relation between wage and productivity on the basis of Hansen and Seo (2002). As in their approach we also use (MLE) Maximum likelihood estimation of the threshold model. Second step is to test the presence of a threshold effect. Under the null hypothesis the model transforms to the linear VECM.

Two regime threshold model where the \( \gamma \) is the threshold parameter takes the following form\(^6\).

\[
\Delta x_t = A_1' x_{t-1}(\beta) d_{1t}(\beta, \gamma) + A_2' x_{t-1}(\beta) d_{2t}(\beta, \gamma) + u_t \quad \text{where } I(.) \text{ denotes the indicator function, and this model can be written as}
\]

\[
d_{1t}(\beta, \gamma) = I(w_{t-1}(\beta) \leq \gamma) \quad \text{and} \quad d_{2t}(\beta, \gamma) = I(w_{t-1}(\beta) > \gamma).
\]

\( ^6 \)
\[ \Delta x_t = \begin{cases} A_1'x_{t-1}(\beta) + u_t, & w_{t-1}(\beta) \leq \gamma \\ A_2'x_{t-1}(\beta) + u_t, & w_{t-1}(\beta) > \gamma \end{cases} \]

There are two regimes defined by the error correction terms value. As described in Hansen and Seo (2002) the parameters \( A_1 \) and \( A_2 \) are coefficient matrices and requires the dynamics in these regimes. If \( P(w_{t-1} \leq \gamma) \) is has the relation \( 0 < P(w_{t-1} \leq \gamma) < 1 \) this shows threshold effect, otherwise the model characterizes linear cointegration. And also they form the following constraint,

\[ \pi_0 \leq P(w_{t-1} \leq \gamma) \leq 1 - \pi_0 \]

where the trimming parameter \( \pi_0 > 0 \).

3 Data and Econometric Results

The study aims to analyze the relationship between the wages and productivity for 1990:01 – 2007:03 period. To facilitate the relationship between the wages and productivity, indirect taxes and other types of payments are left out of the study; whereas, the real pay received by the worker is kept in the focus of analysis. In order to avoid the deviation in the analysis caused by the unregistered economy, we kept the illegal workers out of the study and we used the registered workers and their contribution to the production. Nonoperating revenues are also kept outside the study. As a result, to avoid the deviation mentioned, \( \text{ap} = \) the value added of marginal worker and \( \text{w} = \) wages; \( \text{w} \) does not include direct/indirect taxes or liabilities.

At the second stage, where the unit root procedure suggested by Caner and Hansen (2001) is implemented, we examined the relation between wage and productivity on the basis of Hansen and Seo (2002). As in that approach we also use (MLE) Maximum likelihood estimation of the threshold model. Further, we tested the presence of possible threshold effects, where the model transforms to the linear VECM under the null hypothesis if the alternative is statistically insignificant.

3.1 Unit Root Tests. We applied Augmented Dickey Fuller test to the variables and obtained the lowest AIC information criteria
at the 4th and 5th lags. As a result of the test, we failed to reject the hypothesis that the series are stationary and accepted the null hypothesis that the series are integrated of I(1) at the 0.01 significance level.

Table 1. Augmented Dickey-Fuller Test

<table>
<thead>
<tr>
<th>W</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.846160</td>
<td>0.242448</td>
</tr>
</tbody>
</table>

Test critical values: 1% level: -2.576; 5% level: -1.942; 10% level: -1.615

The nonlinear unit root test results are given in Table 1. We based our calculations on Caner and Hansen (2001) methodology. The m parameter for both variables are obtained as 2; whereas k is obtained as 4 for P (productivity) and 5 for W (wage). The lag length k is calculated by AIC information criteria. The m parameter, on the other hand, is calculated in accordance with the Caner and Hansen (2001).

3.2 Bootstrap Threshold Test. The bootstrap threshold test tests the threshold effect on the time series investigated. Consequently, we reported the p values resulting from the Wald test according to the k and m lag lengths obtained. As can be seen in the table, at m lag length both variables contain threshold effect. When the analysis is carried according to k lag length determined by AIC and for which we obtained the highest test statistic. As a result, although the variable w has threshold effect at 0.95 confidence level, the $H_0$ hypothesis that points out stationary cannot be accepted for the variable AP at k lag length. However, Caner and Hansen (2001) obtained similar results in their study and accepted to continue the analysis according to m lag length. Consequently, the alternative hypothesis that states threshold effect exists for the series is accepted against the null hypothesis of stationarity. Following the Bootstrap Threshold Test, we analyzed $R_1$ and $R_2$ tests in the study. After the theoretical explanation of $R_2$ test which investigates two sided Wald statistic; the $R_1$ is a one sided test. The test aims to test the null hypothesis of stationarity $H_0: ?_1 = ?_2 = 0$ against $H_1: ?_1 ? ?_2 ? 0$. As observed by the Wald statistics and p values, the null hypothesis of stationarity is rejected for both series by the $R_3$ test for both m and k lag lengths. According to the $R_1$ test we obtained similar results such
that the stationarity of the series $H_0: \beta_1 = \beta_2 = 0$ is rejected against the unit root in the series and the alternative hypothesis that series has unit root $H_1: \beta_1 < 0; \beta_2 < 0$ is accepted. For one tail $t_1$ test that represents stationarity at both regimes $H_0: \beta_1 = \beta_2 = 0$ hypothesis is tested against the unit root effect $H_1: \beta_1 < 0; \beta_2 = 0$ and concluded that both series have unit root effect. Further, we tested the $H_0: \beta_1 = \beta_2 = 0$ hypothesis against $H_1: \beta_1 = 0; \beta_2 < 0$ with one tail $t_2$ test. The results suggest that $t_1$ and $t_2$ tests confirm the result that both series have unit root at least at 95% significance level.

Table 2. Threshold and Unit Root Tests for TAR Model

<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>P(m)</td>
<td>20.71</td>
<td>0.035</td>
<td>0.047</td>
</tr>
<tr>
<td>P(k)</td>
<td>11.76</td>
<td>0.49</td>
<td>0.48</td>
</tr>
<tr>
<td>W(m)</td>
<td>25.73</td>
<td>0.028</td>
<td>0.022</td>
</tr>
<tr>
<td>W(k)</td>
<td>30.03</td>
<td>0.008</td>
<td>0.01</td>
</tr>
<tr>
<td>P(m)</td>
<td>8.28</td>
<td>0.19</td>
<td>0.27</td>
</tr>
<tr>
<td>P(k)</td>
<td>9.48</td>
<td>0.13</td>
<td>0.18</td>
</tr>
<tr>
<td>W(m)</td>
<td>0.007</td>
<td>0.99</td>
<td>0.99</td>
</tr>
<tr>
<td>W(k)</td>
<td>0.637</td>
<td>0.98</td>
<td>0.99</td>
</tr>
<tr>
<td>P(m)</td>
<td>7.76</td>
<td>0.19</td>
<td>0.27</td>
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<tr>
<td>P(k)</td>
<td>9.47</td>
<td>0.12</td>
<td>0.16</td>
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<tr>
<td>W(m)</td>
<td>0.005</td>
<td>0.99</td>
<td>0.99</td>
</tr>
<tr>
<td>W(k)</td>
<td>0.637</td>
<td>0.98</td>
<td>0.99</td>
</tr>
<tr>
<td>P(m)</td>
<td>-0.72</td>
<td>0.92</td>
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<tr>
<td>P(k)</td>
<td>0.63</td>
<td>0.65</td>
<td>0.93</td>
</tr>
<tr>
<td>W(m)</td>
<td>-0.32</td>
<td>0.86</td>
<td>0.99</td>
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<tr>
<td>W(k)</td>
<td>0.43</td>
<td>0.82</td>
<td>0.99</td>
</tr>
<tr>
<td>P(m)</td>
<td>2.78</td>
<td>0.11</td>
<td>0.23</td>
</tr>
<tr>
<td>P(k)</td>
<td>3.01</td>
<td>0.051</td>
<td>0.09</td>
</tr>
<tr>
<td>W(m)</td>
<td>-0.04</td>
<td>0.88</td>
<td>0.99</td>
</tr>
<tr>
<td>W(k)</td>
<td>0.67</td>
<td>0.77</td>
<td>0.99</td>
</tr>
</tbody>
</table>

Bootstrap rep.: 1000, P: k= 4, m=2; W: k=5, m=2
At this point, both of the series are accepted to contain unit root in both regimes and the analysis is continued with the cointegration analysis.

3.3 TAR Cointegration Results. We obtained the cointegration relation \( v_t = P_{t-1} - 0.83W_t \) following the minimization of the Likelihood function. The estimated threshold value is \( \hat{\gamma} = 0.719 \). As a result, the first regime is accepted to be governing where the wages are more than 70 percent higher than the productivity. Consequently, the first regime we obtained in the analysis dominated a major part (typical regime) ; 95 % of the whole period whereas the second regime corresponds only to 5 %. A similar result is obtained by the Hansen and Seo (2002) study in which the regime covering 8% of the period is defined as the “extreme regime”. It is concluded that, similar to the results of Hansen and Seo (2002) the extreme regime that corresponds to 5% of the period occurs only if the wages are 72 percent less than productivity. The first regime is achieved so that, (typical regime) \( P_t \leq 0.83W_t + 0.719 \) whereas the second regime, (extreme regime) is dominant if \( P_t > 0.83W_t + 0.719 \). The estimated VAR model is given below.

The Eicker-White standard errors are given in parentheses for the estimated Threshold VAR model:

\[
\Delta P_t = \begin{cases} 
-0.011 + 0.004v_{t-1} - 0.14\Delta P_{t-1} - 0.01\Delta W_{t-1} + u_{t1}, & v_{t-1} \leq 0.719 \\
0.007 + 0.016v_{t-1} - 0.004\Delta P_{t-1} - 0.48\Delta W_{t-1} + u_{t1}, & v_{t-1} > 0.719
\end{cases}
\]

\[
\begin{pmatrix}
(0.018) & (0.02) & (0.173) & (0.56) \\
(0.002) & (0.002) & (0.006) & (0.09)
\end{pmatrix}
\]

\[
\Delta W_t = \begin{cases} 
-0.24 + 0.16v_{t-1} + 0.387\Delta P_{t-1} - 1.946\Delta W_{t-1} + u_{t2}, & v_{t-1} \leq -0.719 \\
-0.049 + 0.048v_{t-1} + 0.03\Delta P_{t-1} + 1.6\Delta W_{t-1} + u_{t2}, & v_{t-1} > -0.719
\end{cases}
\]

\[
\begin{pmatrix}
(0.155) & (0.155) & (0.119) & (1.78) \\
(0.02) & (0.025) & (0.04) & (1.06)
\end{pmatrix}
\]
4 Conclusion

In the study, we analyzed the relationship between wages and productivity in the 1990:01 – 2007:03 period in accordance with the proposed threshold cointegration tests. During the analysis process, we followed the traditional unit root tests at the first stage and developed the analysis followed by unit root test procedure proposed by Caner and Hansen (2001). Our results suggest that, both according to the traditional unit root tests and to the TAR unit root tests, the productivity and wages series follow I(1) process.

At the second stage, we covered the main analysis of our study at which we discussed the long run nonlinear relationship between wages and productivity by employing the TAR cointegration analysis developed by Hansen and Seo (2002). According to the results we obtained, the long run linear relation between wages and productivity is rejected against the nonlinear long run relation in the light of the empirical evidence.

The nonlinear long run relation between wages and productivity is estimated as

\[ v_t = P_{t-1} - 0.83W_t \]

for the period investigated. In addition to the obtained long run nonlinear relationship, the estimated threshold 0.72 implies the fact that, the first and second regimes occurs as \( P_t \leq 0.83W_t + 0.719 \) and \( P_t > 0.83W_t + 0.719 \) respectively. Another way to interpret the results is that, first there are two long run cointegration relations for the first and the second regimes defined as the dominant and the extreme for the former and the latter, hence as long as the increase in productivity passes the 0.70 threshold the first regime dominated the economy for the period investigated.

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