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Approximation of the Navier-Stokes system by multilayer models

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Abstract Due to computational issues associated with the free surface Navier-Stokes or Euler equations, the simulations of geophysical flows are often carried out with shallow water type models of reduced complexity. Thus the hydrostatic assumption for shallow water flows leads to non-linear hyperbolic systems of Saint-Venant type [1]. For these vertically averaged models, efficient and robust numerical techniques [2, 3] are available and avoid to deal with moving meshes.

Here, we are interested in flows where the horizontal velocity can hardly be approximated – as in the Saint-Venant system – by a vertically constant velocity (large bottom friction, significant water depth, wind effects, . . .). We are also interested in lakes and estuarine waters, which typically exhibit a significant density stratification related to vertical variations of temperature and chemical composition. In these water bodies effects related to small density gradients may strongly affect the hydrodynamics. Density stratification processes are therefore often important in environmental flows.

To model and simulate such complex flows, multilayer Saint-Venant models are often used but the proposed models do not allow mass exchanges between neighboring layers and make a close relation to models for non-miscible fluids (see [4, 5] and the references therein). Here, we derive another and simpler multilayer model where we prescribe the vertical discretization of the layers taking into account the (unknown) total height of water. In our approach [6, 7] the layer partition is merely a discretization artefact, and it is not physical. Therefore, the internal layer boundaries do not necessarily correspond to isopycnic surfaces. A critical distinguishing feature of our model is that it allows fluid circulation between layers. This changes dramatically the properties of the model and its ability to describe flow configurations that are crucial for the foreseen applications, such as recirculation zones.

The numerical solution of the multilayer system we propose is based on a kinetic interpretation of the model and uses a finite volume kinetic scheme with an extended hydrostatic reconstruction technique [8]. In contrast with Navier–Stokes solvers, our discretization technique allows easily satisfying properties such as conservation, positivity, well-balancing of source terms (cf. [2]), and handling robustly wet/dry interfaces over variable bottom topography.
The presentation is organized as follows. We start giving the cornerstones of the derivation of the multilayer model starting from the Navier-Stokes system. Then we propose a numerical scheme for the solution of the obtained model. A special emphasis is placed on the discretization of the source terms. Finally, some numerical experiments in 3d including comparisons with analytical solutions and confrontations with experimental data are presented.

References


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