Discontinuous-in-space explicit Runge-Kutta residual distribution schemes for time-dependent problems

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Abstract The Residual Distribution (RD) framework for multidimensional hyperbolic conservation laws can be illustrated by considering the scalar conservation law given by
\[ \nabla \cdot f = 0 \] (1)
on a domain \( \Omega \), with appropriate boundary conditions. The residual associated with a mesh cell \( E \) is defined to be
\[ \phi_E = \int_E \nabla \cdot f d\Omega, \] (2)
and this is then distributed among the vertices of \( E \). Assuming a piecewise linear representation of the approximate solution leads to the discrete system
\[ \sum_{E \in D_i} \beta^E_i \phi_E = 0 \quad \forall i \] (3)
where the \( \beta^E_i \) signify the proportion of the residual in cell \( E \) assigned to node \( i \) and \( D_i \) denotes the subset of triangles containing \( i \). System (3) is solved to find the approximate solution values at the mesh nodes, typically using a pseudo-time-stepping approach.

In the case of steady state problems, where \( f \) in (1) only has a spatial dependence, the residual distribution concept has already proven to be very successful. The RD approach, in a relatively natural manner, enables construction of positive, linearity preserving and conservative schemes able to carry out a truly multidimensional upwinding for both scalar and systems of hyperbolic conservation laws.

Extension to time-dependent problems is currently a subject of intensive ongoing research. It is possible to develop schemes of the form (3), as derived when the divergence in (1) includes the time variation, but solving the system (3) at each time-step is typically very cpu-intensive. To overcome this Abgrall and Ricchiuto in [1] proposed a framework for explicit, second order residual distribution schemes for transient problems. In this talk we will present their approach in conjunction with discontinuous-in-space data representation. This extends previous work on discontinuous residual distribution schemes for steady problems initiated in [2, 3]. It also extends work of Abgrall and Shu [4] in the sense that it reformulates the Runge-Kutta
Discontinuous Galerkin (DG) method in the framework of Runge-Kutta Residual Distribution schemes. This is, briefly speaking, done by considering flux differences (edge residuals in the RD framework) instead of the fluxes themselves.

Different types of cell– and edge–based distribution strategies can be applied and we will discuss the most interesting choices characteristic for either RD [2, 3] or DG type approaches [4, 5]. Numerical results for two-dimensional hyperbolic conservation laws on structured and unstructured triangular meshes will also be presented. A brief comparison with other recent developments in the discontinuous residual distribution framework (i.e. [6]) will also be made.

**Keywords:** Hyperbolic conservation laws, Time dependent problems, Explicit schemes, Residual distribution, Runge-Kutta time-stepping, Discontinuous Galerkin

**Mathematics Subject Classification 2000:** 65M99, 76M25

**References**


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