On the coupling of compressible and incompressible fluids.

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Abstract In some applications in multiphase flow, compressible effects cannot be neglected. In this case, one assumes in general, that the liquid phase as well as the gaseous phase are modelled by compressible equations such as the Euler equations

\[\begin{align*}
\partial_t \rho + \nabla \cdot (\rho \mathbf{v}) &= 0 \\
\partial_t (\rho \mathbf{v}) + \nabla \cdot (\mathbf{v} \otimes (\rho \mathbf{v})) + \nabla p_i &= 0 \\
\partial_t E + \nabla \cdot (\mathbf{v} (E + p_i)) &= 0
\end{align*}\] (1)

where as usual \(\rho\) denotes the density, \(\mathbf{v}\) the velocity, \(E = \rho e + \frac{1}{2} \rho |\mathbf{v}|^2\) the total energy and \(e\) the internal energy.

To model the different behaviours of the phases, each phase is described by a different equation of state \(p_i(\rho, e)\), which accounts especially for the different compressibilities. In the literature (see e.g. [1, 2]), it is for instance common to use the Tait equation of state

\[p_{\text{Tait}}(\rho) = k_0 \left( \left( \frac{\rho}{\rho_0} \right)^\gamma - 1 \right) + p_0\] (2)

to model the behaviour of very weakly compressible fluids such as liquids, where the constants are chosen to \(\gamma \approx 7\) and \(k_0 \approx 3000\). This leads to a very stiff pressure law and to a very large speed of sound in the liquid regions. Numerically, this results in severe time step restrictions due to the CFL-condition.

To circumvent this high restriction on the time step size, we propose a coupling of compressible and incompressible flows, where the flow in the liquid phase is given by the incompressible Euler equations

\[\begin{align*}
\nabla \cdot (\mathbf{v}) &= 0 \\
\partial_t \mathbf{v} + \nabla \cdot (\mathbf{v} \otimes \mathbf{v}) + \nabla p_i &= 0
\end{align*}\]

We show that the interface conditions for the coupling of Euler and incompressible Euler equations in one space dimension are given by an ordinary differential equation and discuss how the interface conditions can be solved numerically.

Unlike the purely compressible case, where a one-dimensional Riemann solver is sufficient to solve numerically also problems in higher space dimension, this holds not true for the coupling of compressible and incompressible flows. We show why this is the case and discuss the necessary extensions of the interface conditions for higher space dimensions.
References


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