Focusing of a chirped pulse train in phase opposition through linear chirped fiber grating (LCFG)

L. Chantada*[a], C. R. Fernández-Pousa[b], M. T. Flores-Arias[a], C. Bao[a], M. V. Pérez[a] and C. Gómez-Reino[a]
[a]Grupo de Óptica GRIN, Departamento de Física Aplicada, Universidade de Santiago de Compostela, Campus Sur, E15782 Santiago de Compostela, Spain.
[b]División de Óptica, Departamento de Ciencia y Tecnología de Materiales, Universidad Miguel Hernández, Avda. Ferrocarril s/n, E03202 Elche (Alicante) Spain.

ABSTRACT

There exists a well-known analogy between the paraxial or one-dimensional Fresnel diffraction and the propagation of pulses in linear dispersive medium with negligible attenuation. Under this analogy, the envelope of a pulse is equivalent to the distribution of complex amplitude of the light in diffraction. In this context, we study the propagation of a train of identical Gaussian chirped pulses arranged in time in the same way as the Fresnel zones of a phase zone plate, in a highly dispersive guiding medium. From this study we find that the input train focuses in an only pulse, for certain values of total dispersion. We establish the focusing condition and characterize the output signal trough its width and peak intensity.

Keywords: temporal Talbot effect, pulse propagation

1. INTRODUCTION

It exist a well-known analogy between the paraxial or one-dimensional Fresnel diffraction and the propagation of pulses in linear dispersive medium with negligible atenuation[1-2]. In this analogy, the envelope of a pulse is equivalent to the distribution of complex amplitude of the light in the diffraction. Through this analogy devices based on not only propagation of pulses but also of pulses trains[3] have been able to be proposed and demonstrated. Many phenomena have been leaded to temporal domain employing this analogy between diffraction and dispersion, as is the case of temporal Talbot effect[4], the Fourier transformation[5], the pinhole camera[6], the temporal lens[7], etc. Inside this space-temporal analogy our objective is the study of propagation of a train of identical Gaussian chirped pulses arranged in time in the same way as the Fresnel zones of a phase zone plate[8]. From this train of pulses we will find effects in temporal domain similar to diffractive effects that a zone plate produces. For certain values of total dispersion of the medium, a focusing single pulse is obtained. We will analyze the output pulse through the values of peak intensity and width, and we will show as this focusing effect can be used to get more intensive and narrower pulses.

2. FOCUSING CONDITION

We will consider a input signal composed of two trains of pulses in phase opposition. In each of theses trains, neighbour pulses are separated in a quantity proportional to square root of natural number

$$\psi_0(t) = f_0(t) + \sum_{n=1}^{M} f_0\left(t - t_0\sqrt{2n+1/2}\right) + \sum_{n=1}^{M} f_0\left(t + t_0\sqrt{2n+1/2}\right) - \sum_{n=1}^{M} f_0\left(t - t_0\sqrt{2n-1/2}\right) - \sum_{n=1}^{M} f_0\left(t + t_0\sqrt{2n-1/2}\right) \quad (1)$$

where t_0 is the time separation between different input pulses. The upper limit of the summation, M, gives the number of pulses of the train, N, through the expression $N=4M+1$, and the function f_0 expresses the initial pulse profile, which will be regarded as Gaussian with width t_p and chirp C,

$$f_0(t) = \exp\left(-(1+iC)t^2/2t_p^2\right) \quad (2)$$

5th Iberoamerican Meeting on Optics and 8th Latin American Meeting on Optics, Lasers, and Their Applications, edited by A. Marcano O., J. L.Paz, Proc. of SPIE Vol. 5622 (SPIE, Bellingham, WA, 2004) · 0277-786X/04/$15 · doi: 10.1117/12.591724

We will assume that the width of the pulses t_p is much smaller than the separation parameter t_0, or equivalently the linewidth $1/\tau=(1+C^2)^{1/2}/t_p$ is small compared with $1/t_0$. The pulses in the input signal are arranged in time like the Fresnel zones in a zone plate. The principal difference is the width of the pulses in the temporal signal is the same, whereas the width of the Fresnel zones is proportional to the square root of natural numbers.

The input signal (1) is propagated in a linear dispersion medium with negligible attenuation whose impulse-response is equivalent to the one-dimensional Fresnel diffraction kernel. In a dispersive medium of length L, this kernel is

$$h_\xi(t) = (-2\pi i\xi)^{-1/2} \exp\left[-it^2/2\xi\right] \tag{3}$$

where $\xi=\beta_2 L$ is the lineal dispersion coefficient and t is the proper time. General dispersive media with first-order dispersion, like linear chirped fiber Bragg gratings, are characterized by the parameter ξ that represents the total dispersion of the device. In our analysis we will assume that the total bandwidth of the train of pulses lie inside a passband around the carrier where ξ can be assumed constant.

The output signal of this dispersive device is given by

$$\begin{aligned}\psi_\xi(t) = \frac{t_p \exp\left(-\kappa t^2/2\right)}{\left(t_p^2+\xi C - i\xi\right)^{1/2}} \Bigg[& 1+2\exp\left(-\kappa t_0^2/4\right)\sum_{n=1}^{M} \exp\left(-n\kappa t_0^2\right)\cosh\left(\kappa t_0 t\sqrt{2n+1/2}\right) \\ & -2\exp\left(\kappa t_0^2/4\right)\sum_{n=1}^{M}\exp\left(-n\kappa t_0^2\right)\cosh\left(\kappa t_0 t\sqrt{2n-1/2}\right)\Bigg]\end{aligned} \tag{4}$$

where we have introduced the complex parameter κ,

$$\kappa = \frac{1}{t_p^2+2\xi C+\xi^2/\tau^2} + i\frac{\xi/\tau^2+C}{t_p^2+2\xi C+\xi^2/\tau^2} \tag{5}$$

The real part of κ is simply the inverse of the squared width of the dispersed pulse; whereas its imaginary part accounts for the phase of a single pulse after dispersion. Note that for $\xi >> \tau^2$ i.e., for high dispersion, κ reduces to $(\tau/\xi)^2+i/\xi$ and therefore the dependence of the dispersed signal on the sign of chirp is subleading.

To find the condition focusing we must analyze the relative phase between the dispersed pulses. When these pulses are in phase a constructive interference will take place, obtaining an output singal formed by an one only pulse. The values of dispersion that cause the constructive interference, can be straigthforward deduced from (4) taking account that $\tau << t_0$,

$$\xi_m \approx \frac{t_0^2}{2\pi}\frac{1}{2m-1} = \frac{\xi_1}{2m-1} \tag{6}$$

where m is an integer number that represents the dispersion order, in analogy with the diffraction orders in a zone plate, and $\sqrt{\xi_1} = t_0/\sqrt{2\pi}$ is the principal focal time. If we consider the positive branch with positive dispersion orders, and the negative branch with dispersion orders m=0 or negative, both set of solutions are equivalent, since the resulting values or dispersion differ only in sign. Notice finally that, since these values of dispersion are of the order of t_0^2, the high dispersion limit $\xi >> \tau^2$ is equivalent to the small linewidth limit, $t_0 >> \tau$.

The output relative intensity for these values of dispersion is depicted in figure 1. The parameters of input train are N=41, t_0/t_p=30, C=0 and m=1, 2 and 3. The pulse centered in t=0 carries most part of the energy of the total input signal. At both sides of the central pulse, there are a series of symmetric lobes generated by destructive interference between the dispersed pulses, which carry the energy to be focused at the other diffraction orders, in analogy with the spatial zones plates. The general analysis of the dispersed signal as a function of the parameters that define the train is presented in the following Sections.

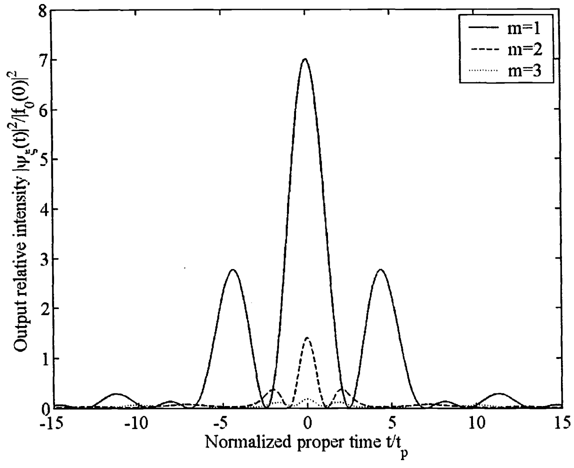

Figure 1: The output relative intensity in the three first dispersion order when the input signal is formed by N=41 pulses with C=0 and $t_0=30t_p$.

A general analysis of dispersed signal can be carried out by a set of magnitudes characterizing a train of pulses. In particular, two of them have importance of knowing, that is, peak intensity and width of the output pulse.

3. PEAK INTENSITY OF THE OUTPUT PULSE

The relationship between the peak intensity of the output pulses and the peak intensity of the individual pulses in the original train can be obtained from (4),

$$I_\xi(0)=\frac{q_\xi}{q_0}\left|1+i(-1)^m\frac{\exp\left(-q_\xi^2/2\right)}{\sinh\left(q_\xi^2/4\right)}\left[1-\exp\left(-Mq_\xi^2\right)\right]\right|^2, \tag{7}$$

where we have introduced the parameters:

$$q_0=\frac{t_0}{t_p}, \quad \text{and} \quad q_\xi=\frac{t_p t_0}{\sqrt{\left(t_p^2+C\xi_m\right)^2+\xi_m^2}}. \tag{8}$$

These parameters are a measure of the relative dimensions of the separation parameter t_0 in relation to the individual width of the pulses, for the input, q_0, and the dispersed pulses, q_ξ, respectively. In the limit of high dispersion, q_ξ, can be approximated as q_ξ, $\cong 2\pi(2m-1)\tau/t_0$. From (7) we can see as the peak intensity of the output pulse grows with the number of pulses through M, and then it saturates for a certain number of pulses, reaching the maximum value of peak intensity

$$I_\xi(0)\Big|_{\max}\approx\frac{q_\xi}{q_0}\left(1+\frac{\exp\left(-q_\xi^2\right)}{\sinh^2\left(q_\xi^2/4\right)}\right) \tag{9}$$

From (7) we can straightforward deduced the number of pulses for saturation

$$N_{SAT} \sim q_{\xi}^{-2} \propto \frac{1}{(2m-1)^2}\frac{t_0^2}{\tau^2} \qquad (10)$$

From (10) we can see as N_{SAT} increases with the separation parameter and with the chip parameter through the linewidth and decreases with the dispersion order.

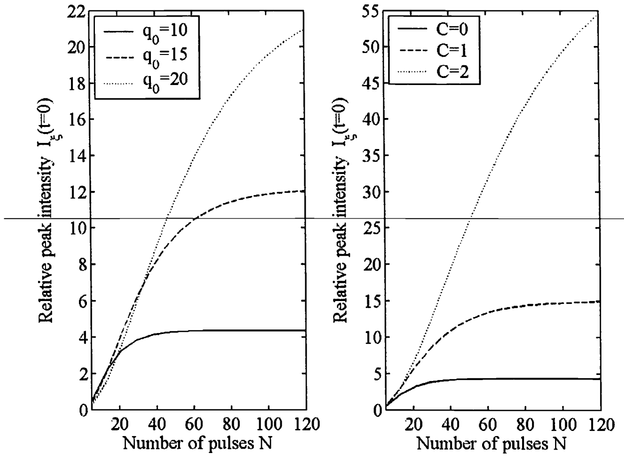

Figure 2: Peak intensity versus number of pulses for the first dispersion order. The parameter of left figure are: C=0 and q_0=10, 15 and 20 and of the right one are: q_0=10 and C=0, 1 and 2

In figure 2 the relative peak intensity of the output signal, eq. (7), in the dispersive orders m=1 is depicted as a function of the total number of pulses in the zone plate train, N. On the left picture, the input pulses are unchirped and q_0 =10, 15 and 20 so that q_ξ=0.62, 0.42 and 0.32 respectively, and on the right one q_0=10 and C=0, 1 and 2, so q_ξ=0.62, 0.44 and 0.28. On the left figure, the parabolic behaviour with N is followed by the saturation is apparent. Also, we can see as the number of pulses for saturation rises with the parameter q_0, as eq. (10) predicts. Moreover, the maximum values of peak intensity, given by eq. (9) (4.4, 12.2, and 23.3 for q_0=10, 15 and 20 respectively), are in accordance with our observations after figure 2. Therefore, if the entrance is only composed of 21 pulses, we can obtain an output signal formed by a single pulse with a relative intensity is in the order of 4. But if we are able to produce trains consist of more pulses, then the ratio of multiplication will be much higher. When the input pulses are chirped the saturation phenomenon appears for higher values of N as we can see on the right figure. In this way, we are able to reach much higher values of peak intensity. Typically we get relative output peak intensity about 9 from an input chirped signal with only 21 pulses and this multiplication factor of intensity can be much higher if N or q_0 increases. So, we can conclude that the focusing process is much more efficient if the input pulses are chirped.

4. WIDTH OF THE OUTPUT PULSE

From (4) we can deduce the width of the output pulse after some calculations

$$\Delta t \sim \frac{1}{2m-1}\frac{t_0}{\sqrt{N}} \qquad (11)$$

Equation (11) shows that the width increases linearly with the separation parameter t_0 (and therefore with q_0), and is inversely proportional to the dispersion order and to the square root of the number of pulses.

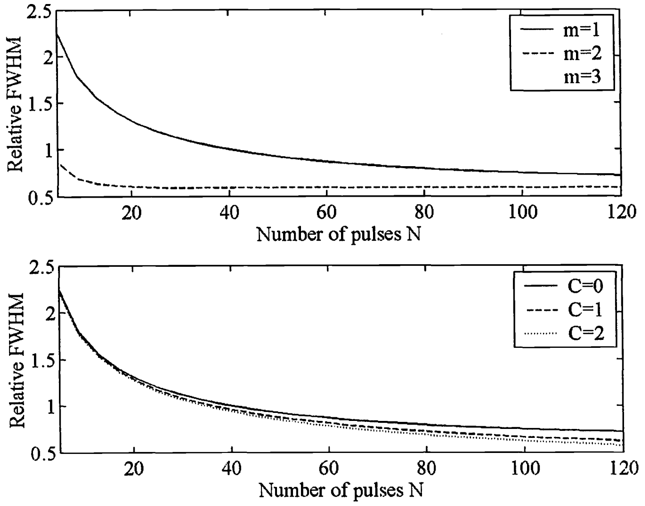

Figure 3: Relative FWHM versus number of pulses. The parameters of the above figure are C=0, q_0=20 and m=1 and 2, and of the below one are: =1 and q_0=20 and C=0, 1and 2.

In figure 3 we represent the numerically computed full width-half maximum (FWHM) of the output signal, normalized by FWHM of the input signal, versus number of pulses. In the top figure the parameters are C=0, q_0=20 and m=1 and 2, and in the bottom one q_0=20, m=1and C=0, 1 and 2. In both figures we observe as the width of the output pulse decreases with number of pulses as eq. (11) indicates, and then it saturates for N_{SAT} like the peak intensity. In the above figure we can check as the relative width is inversely proportional to the dispersion order, in accordance with eq. (11). On the other hand, the width of the output pulse does not depend on the chip parameter up to N_{SAT}. The number of pulses for saturation increases with the chirp parameter, so if the input is chirped, the width of the output pulse stops diminishing for a higher value of N, reaching slightly lower values of FWHM.

5. CONCLUSIONS

In this work we have studied the propagation of two trains of chirped Gaussian pulses in phase opposition through a linear dispersive device. This study has been performed in temporal domain establishing certain analogy with a phase zone plate. We have seen as the output signal is formed by only one pulse, for certain values of the total dispersion. We have established the condition focusing and have characterized the output pulse, through its width and peak intensity. Besides we have shown, as the output pulse can be more intensive and narrower than input pulses for certain conditions of the input signal.

ACKNOWLEDGEMENTS

This work was supported by the Ministerio de Educación y Cultura, Spain, under contract TIC03-03041. The research of Laura Chantada Santodomingo was supported by the government of Spain through a Formación Profesorado Universitario (FPU) grant.

REFERENCES

1. G.P. Agrawal, "Nonlinear fiber optics", 3^a ed, Academic Press, San Diego 2001.
2. A. Papoulis, "Pulse compression, fiber communications, and diffraction: a unified approach", J. Opt. Soc. Am. **A11**, 3-13 (1994) .
3. M. A. Muriel y J. Capmany, "Optical pulse sequence transmission through single-mode fibers under second and third order despersion" Electron. Lett. **24**, 1252-1253 (1988).

4. J. Azaña y M. A. Muriel "Temporal Talbot effect in fiber gratings and its applications" Appl. Opt. **38**, 6700-6704 (1999).
5. M. A. Muriel, J. Azaña, y A. Carballar "Real-time Fourier transformer based en fiber gratings" Opt. Lett., **24**, 1-3 (1999).
6. B.H.Kolner "The pinhole time camera" J. Opt. Soc. Am. A **14**, 3349-3357 (1997)
7. B.H.Kolner "Temporal imaging with a time lens", Opt. Lett. **14**, 630-632 (1989).
8. J. Ojeda-Castañeda and C. Gómez-Reino, eds., *Selected Papers on Zone Plates.* Vol. MS 128 of SPIE Mileston Series (SPIE Press, Bellingham, Wash., 1996) and references.