Influence of the off-axis illumination and the finite object dimensions in the Talbot effect in a tapered GRIN media

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ABSTRACT

The irradiance at Talbot images in tapered gradient-index (GRIN) media for ideal periodic objects is presented. The effects on Talbot images due to off-axis illumination and finite object dimension are considered. Results have been applied to a hybrid structure composed by a divergent linear tapered GRIN medium and a sinusoidal amplitude grating to show transverse shift and finite-apperture diffractive effect on the Talbot images as well as the walk-off effect.

Keywords: GRIN optics, Talbot effect, Image formation

1. INTRODUCTION

The integer and fractional Talbot effect in a homogeneous medium is well known in optics and has received wide attention[1-3]. Talbot effect has been studied also in transverse quadratic index media[4-5], and the authors have described integer and fractional Talbot effect in a tapered gradient-index (GRIN) media[6-7]. In these papers, theoretical aspects of the effect were investigated assuming strictly periodic objects of infinite dimensions illuminated by coherent uniform and non uniform beams. The aim of this work is to study the influence of the displacement of the source and of the finite object dimensions on the integer and fractional Talbot images in GRIN media in order to get a more realistic description of the phenomenon.

The study will be restricted to the one-dimensional transverse case, but extension to the two-dimensional case is straightforward.

2. IRRADIANCE DISTRIBUTION AT TALBOT IMAGES FOR AN IDEAL PERIODIC OBJECT

Let is consider a tapered GRIN medium characterized by a transverse parabolic refractive index modulated by an axial index and whose refractive index is given by

$$n^2(x,z) = n_0^2\{1 - g^2(z)x^2\} \tag{1}$$

where n_0 is the index at the *z* optical axis and *g*(*z*) the taper function that describes the evolution of the transverse index along the *z* axis.

The transmission function of the one-dimensional periodic object of infinite dimension located at the input of GRIN medium is

$$T(x_0) = \sum_{m=-\infty}^{+\infty} a_m \exp\left[-i\frac{2\pi m x_0}{p}\right] \tag{2}$$

where *p* is the spatial period and a_m is the amplitude of the *m*th harmonic.

When the hybrid optical structure formed by the periodic object and the tapered GRIN medium is illuminated by a coherent uniform beam (fig.1), the complex amplitude distribution $z>0$ is given by the integral equation

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$$\phi(x;z)=\int_{-\infty}^{+\infty}\phi(x_0)K(x,x_0;z)dx_0 \tag{3}$$

being

$$\phi(x_0)=T(x_0)\frac{1}{\sqrt{d}}\exp\left(i\frac{\pi x_0^2}{\lambda d}\right) \tag{4}$$

and K the one-dimensional optical propagator of this medium expressed as

$$K(x,x_0;z)=\left[\frac{n_0}{i\lambda H_1(z)}\right]\exp\{ikn_0 z\}\exp\left\{i\frac{kn_0}{2H_1(z)}[x^2\dot{H}_1(z)+x_0^2H_2(z)-2xx_0\right\} \tag{4}$$

where H_1, H_2 and $\dot{H}_1$, $\dot{H}_2$ are the position and the slope of the axial and field rays at z, respectively, dot being the derivative with respect to z[8].

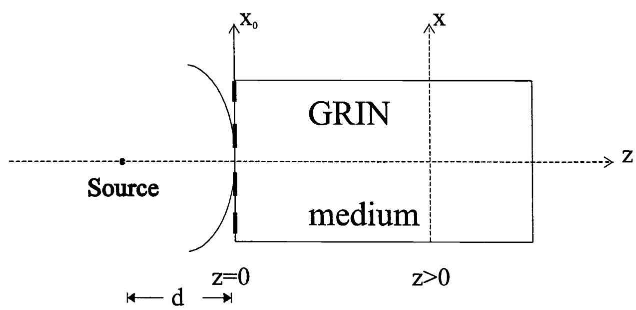

Fig.1: Geometry for the evaluation of the complex amplitude distribution in a tapered GRIN medium due to a periodic object located at z=0 and illuminated by a cylindrical uniform wavefront.

After a simple calculation, the complex amplitude distribution becomes

$$\phi(x;z_{\beta/\alpha})=\frac{1}{\sqrt{dF(z_{\beta/\alpha})}}\exp[i\varphi(z_{\beta/\alpha})]\exp\left[i\frac{kn_0\dot{F}(z_{\beta/\alpha})}{2F(z_{\beta/\alpha})}x^2\right]\sum_{m=-\infty}^{+\infty}a_m\exp\left[-i\frac{2\pi mx}{pF(z_{\beta/\alpha})}\right]\exp\left[-i\frac{\pi m^2\beta}{\alpha}\right] \tag{5}$$

with

$$\overset{(\cdot)}{F}=\overset{(\cdot)}{H}_2(z)+\frac{\overset{(\cdot)}{H}_1(z)}{n_0 d} \tag{6}$$

For the Talbot condition given by[7]

$$\frac{\lambda H_1(z_{\beta/\alpha})}{n_0p^2F(z_{\beta/\alpha})}=\frac{\beta}{\alpha} \tag{7}$$

where α and β are integer, and β/α is referred to as the self-image number. The fractional Talbot effect is obtained for α and β coprime integers, that is for β/α fractional number, and, on the contrary, the integer Talbot effect is achieved for $\beta/\alpha=\nu$, ν being an integer.

From eq.(5) it follows that the irradiance at Talbot distance can be written as

$$I(x;z_{\beta/\alpha})=\left[dF(z_{\beta/\alpha})\right]^{-1}\sum_{m,l=-\infty}^{+\infty}a_m a_l^*\exp\left[-i\frac{2\pi(m-l)x}{pF(z_{\beta/\alpha})}\right]\exp\left[-i\frac{\pi\beta(m^2-l^2)}{\alpha}\right] \tag{8}$$

In order to study the integer and fractional Talbot images, we consider a hybrid system formed by a planar GRIN medium with a divergent linear taper function and a sinusoidal amplitude object.
The linear taper function of the planar GRIN medium is given by

$$g(z) = \frac{g_0}{1 + z/L} \tag{9}$$

L being the distance from z=0 to the common apex of the equi-index lines[7].
The transmission function of the sinusoidal amplitude object is expressed as

$$T(x_0) = A + 2B\cos\left(\frac{2\pi x_0}{p}\right) \tag{10}$$

where A and B are the parameters of amplitude modulation.

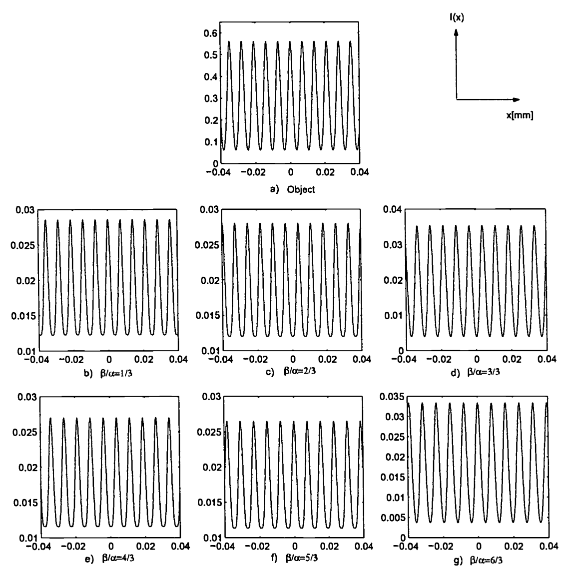

Fig. 2. Irradiance distribution at (a) the object; (b,c, and e,f) some fractional Talbot images and (d and g) the first two integer Talbot images. Calculations have been made for object parameters A=1/2, B=1/8 and p=7μm; GRIN parameters n_0=1.5, g_0=0.01 mm^{-1} and L=1mm; illumination parameters d=15mm and λ=0.7μm; and α=3. For β/α=3/3 and 6/3, the first two integer self-images are obtained.

In this case, the irradiance at Talbot distances are given by

$$I(x; z_{\beta/\alpha}) = 2\left[dF(z_{\beta/\alpha})\right]^{-1}\left\{\frac{A^2}{2} + 2B^2\cos^2\left(\frac{2\pi x}{pF(z_{\beta/\alpha})}\right) + 2AB\cos\left(\frac{2\pi x}{pF(z_{\beta/\alpha})}\right)\cos\left(\frac{\pi\beta}{\alpha}\right)\right\} \tag{11}$$

From eq.(11) it follows that at Talbot distances, irradiance patterns with period pF are observed.
Figure 2 shows the irradiance distribution at (a) the periodic object; (b,c, and e,f) some fractional Talbot images; and (d and g) the first two integer Talbot planes. The maximum contrast, equal to the object contrast, coincides with integer Talbot images. However, for fractional images contrast decreases to a half value from that in the periodic object.

3. TALBOT EFECT FOR OFF-AXIS ILLUMINATION

We suppose an off-axis source located at a distance ζ in transverse direction from axis. The complex amplitude distribution on the periodic object is now given by

$$\phi(x_0)=\frac{1}{\sqrt{d}}\exp\left(i\frac{\pi(x_0-\zeta)^2}{\lambda d}\right)T(x_0) \tag{12}$$

Tacking into accounts eq. (12) and eq.(3), the irradiance at Talbot distances can be written as

$$I(x;z_{\beta/\alpha})=2\left[dF(z_{\beta/\alpha})\right]^{-1}\sum_{m,l=-\infty}^{+\infty}a_m a_l^*\exp\left[-i\frac{2\pi(m-l)}{pF(z_{\beta/\alpha})}\left(x+\frac{H_l(z_{\beta/\alpha})\zeta}{n_0 d}\right)\right]\exp\left[-i\frac{\pi\beta(m^2-l^2)}{\alpha}\right] \tag{13}$$

In order to analyze the effects arising the displacement of the source, we consider above hybrid system. For this case, eq.(13) becomes at Talbot images

$$I(x;z_{\beta/\alpha})=2\left[dF(z_{\beta/\alpha})\right]^{-1}\left\{\frac{A^2}{2}+2B^2\cos^2\left(\frac{2\pi}{pF(z_{\beta/\alpha})}\Delta\right)+2AB\cos\left(\frac{2\pi}{pF(z_{\beta/\alpha})}\Delta\right)\cos\left(\frac{\pi\beta}{\alpha}\right)\right. \tag{14}$$

where $\Delta = x+\frac{H_1(z_{\beta/\alpha})}{n_0 d}$

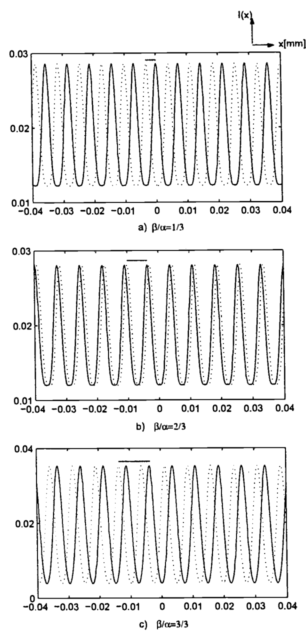

Fig.3 Irradiance distribution for on-axis (solid curve) and off-axis illumination (doted curve) at some Talbot images. Calculations have been made for parameters of fig.3 and ζ=2mm

Comparing eq.(14) and eq(11) it follows that an example of the difference in behaviour between on-axis and off-axis illumination concerns the lateral shift of irradiance pattern[1,9] , since this pattern is now centred along a geometrical trajectory proportional to the axial ray which is given by

$$x(z)=-\dot{x}_0 H_1(z) \tag{15}$$

where $\dot{x}_0=\zeta/n_0 d$ is the slope of the refracted ray at the input.

Figure 3 shows for comparison, the irradiance distribution at the two first fractional images and the first integer image of figure 2 for off-axis (dot line) and on-axis (solid line) illuminations. A lateral shift in reverse sense between both patterns is observed for ζ=+2mm. On the contrary, lateral shift in same sense would be obtained for ζ=-2mm.

4. INFLUENCE OF FINITE OBJECT DIMENSION

We consider a hybrid system of finite dimensions with aperture $2a$. In this case, the complex amplitude distribution in the tapered GRIN medium at z>0 becomes

$$\phi(x;z)=\int_{-a}^{+a}\phi(x_0)K(x,x_0;z)dx_0 \tag{16}$$

In this case, making calculations analogous to the above section, we arrive to the following equation for the irradiance at Talbot planes

$$I(x;z_{\beta/\alpha})=\left[2df(z_{\beta/\alpha})\right]^{-1}\sum_{m,l=-\infty}^{+\infty}a_m a_l^*\exp\left[-i\frac{2\pi(m-l)x}{pF(z_{\beta/\alpha})}\right]\exp\left[-i\frac{\pi\beta(m^2-l^2)}{\alpha}\right]$$
$$\times\left[Q(\vartheta_m^+)-Q(\vartheta_m^-)\right]\left[Q^*(\vartheta_l^+)-Q^*(\vartheta_m^-)\right] \tag{17}$$

where Q is the complex Fresnel integral and

$$\vartheta_m^\pm=\sqrt{\frac{2\alpha}{\beta p^2}}\left[\pm a-\left(\frac{x}{F(z_{\beta/\alpha})}+\frac{m\beta p}{\alpha}\right)\right] \tag{18}$$

are the arguments of Q evaluated at Talbot condition.

Eq.(17) applied to the hybrid system formed by the tapered GRIN medium and the amplitude grating gives

$$I(x;z_{\beta/\alpha}) = \left[dF(z_{\beta/\alpha})\right]^{-1}\left\{A^2\left[|Q(\vartheta_0^+)|^2 + |Q(\vartheta_0^-)|^2 - 2\,\mathrm{Re}(|Q(\vartheta_0^+)Q^*(\vartheta_0^-)\right] + \right.$$
$$+B^2\left[|Q(\vartheta_1^+)|^2 + |Q(\vartheta_1^-)|^2 + |Q(\vartheta_{-1}^+)|^2 + |Q(\vartheta_{-1}^-)|^2 - 2\,\mathrm{Re}(Q(\vartheta_1^+)Q^*(\vartheta_1^-)) - 2\,\mathrm{Re}(Q(\vartheta_{-1}^+)Q^*(\vartheta_{-1}^-)\right] +$$
$$2AB\,\mathrm{Re}\left[\exp\left(i\frac{\pi\beta}{\alpha}\right)\left[Q(\vartheta_0^+) - Q(\vartheta_0^-)\right]\left(\exp\left[i\frac{2\pi x}{pF(z_{\beta/\alpha})}\right]\left[Q^*(\vartheta_1^+) - Q^*(\vartheta_1^-)\right] + \right.\right. \quad (19)$$
$$\left.\left.\left. + \exp\left[-i\frac{2\pi x}{pF(z_{\beta/\alpha})}\right]\left[Q^*(\vartheta_{-1}^+) - Q^*(\vartheta_{-1}^-)\right]\right)\right]\right\}$$

where 0, ±1 subindices correspond to 0, ±1 harmonics of the periodic object respectively, and Re denoting real part.

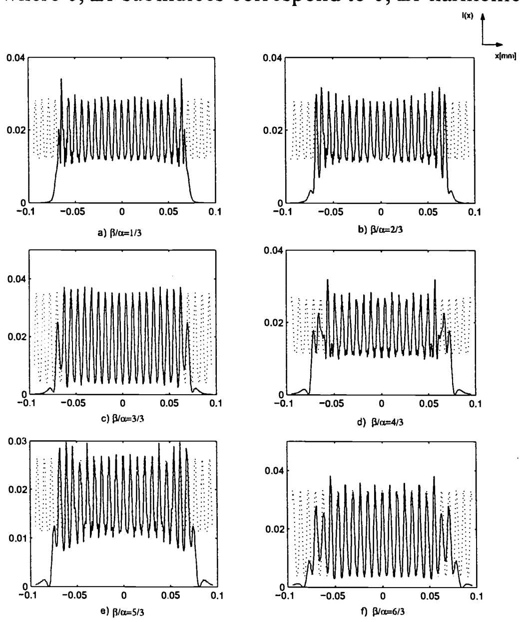

Fig.4 Irradiance distribution at the cases depicted in fig.2. Calculations have been made for parameters of fig.3 and aperture 140µm. The dotted curves denote those without the effects of the aperture size.

Figure 4 depicts the effects on fractional and integer Talbot images of Fig.2 which arise as a result of finite dimension. At the center of the images, quality of the gratings decreases with z from that in the case of an infinite object. Diffraction around the edge of geometrical shadow ($x=aF$) of the periodic object is observed. Likewise, because of finite object dimension, the ±1 harmonics are gradually and laterally displaced of the zero-order harmonic (walk-off effect[1,10]) and do not take part in the image formation when using an object of only a few unit cells.

5. CONCLUSIONS

The effects on Talbot images in GRIN media due to off-axis illumination and finite object dimension have been considered. Results have been applied to a hybrid structure formed by a GRIN medium with a divergent linear taper function and a sinusoidal amplitude grating to show the transverse shift and the diffractional effect on the Talbot images as well as the walk-off effect.

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