Numerical modelling of nonlinear ellipse rotation

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ABSTRACT

Numerical modelling of nonlinear ellipse rotation has been made for an optical field propagating along a linear birefringent silica optical fiber. The kernel of this model comes from expressing the coupled field as a sum of two linearly polarized modes in the direction of the principal axes of the fiber. The developed model is suitable for the design of a big number of optoelectronic devices and let easily introduce the temporal dispersion effects.

Keywords: nonlinear rotation, Kerr effect, fiber optics

1. INTRODUCTION

Intensity-dependent changes of polarization along a single-mode birefringent fiber have been observed and used for nonlinear pulse reshaping[1-2], light modulation[3], intensity discrimination[4-5] and mode-locking of fiber lasers[6].

Likewise, we have revisited the theory of self-induced polarization changes in monomodal nontwisted[7] linear birefringent fibers. If we regard the literature, we note that the most usually adopted model neglects nonlinear ellipse rotation[1]. The assumption made is know as the Rotating Wave Approximation[6] (RWA) and it results as a consequence of neglecting the third order nonlinear couplage between the temporal phases of the main linear modes. As a result of this assumption self-induced polarizations changes can not occurs if the fiber axes are equally excited[1]. However, when the nonlinear phase couplage is not neglected, the most accurate model of Winful[8] demonstrated that polarization changes are possible even with equal excitation of the fiber principal axes.

In this work, taking a different way to the referenced Winful work, where the propagated optical field was written as a superposition of two orthogonal circular polarizations, we study a similar case regarding the propagated fundamental modes as a superposition of two linear polarization fields that propagate along the principal axes of the linear birefringent fiber.

As the more important benefits of regarding the problem from this point of view, we find that: the interaction between linear and nonlinear birefringence could be more clearly observed; with this formulation the temporal effects can be included for describing short pulse propagation, and the obtained expressions are more suitable for practical lab tasks.

The outline of this paper is as follows. Firstly, we describe the evolution expressions of the polarization state in terms of the difference in effective indexes for the principal linear polarization modes. Secondly, we use these expressions to perform an iterative method that observes the nonlinear ellipse rotation along a single mode birefringent fiber in some different significative situations. Finally, we expose the conclusions.

2. THEORETICAL MODEL EXPRESSIONS

A polarized ligthwave $\vec{E}$ is coupled into a birefringent fiber of length L . The complex electric field vector can be expressed as a superposition of two linear fields polarized in the direction of the fast and slow fiber axes, respectively:

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$$\vec{E}(r,t)=g(x,y)(E_x\hat{x}+E_y\hat{y})e^{-iwt} \quad (1)$$

where $g(x,y)$ is the transverse mode normalized pattern, $\hat{x}$ and $\hat{y}$ are the unitary vector of slow and fast axes respectively, w is the angular frequency of the optical field, and t is the temporal coordinate.

Furthermore, in the third order approximation for a weakly birefringent silica core fiber, the nonlinear polarization P^{NL} corresponds with[9]:

$$P_i^{NL}(z,t)=\frac{3\varepsilon_0}{2a_{eff}}\sum_j\left(\chi^{(3)}_{xxyy}E_iE_jE_j^*+\chi^{(3)}_{xyxy}E_jE_iE_j^*+\chi^{(3)}_{xyyx}E_jE_jE_i^*\right) \quad (2)$$

where $i, j= x$ or y being x, y the fast and slow transversal coordinate indices, E the complex field amplitude, a_{eff} the fiber effective mode area result of $g(x,y)$ integration across the fiber core, $\chi^{(3)}$ the third nonlinear order susceptibility and * denotes complex conjugation. The three independent components of $\chi^{(3)}$, are constrained by the relation $\chi^{(3)}_{xxxx}=\chi^{(3)}_{xxyy}+\chi^{(3)}_{xyxy}+\chi^{(3)}_{xyyx}$.

In the case of silica fibers the three components have nearly the same magnitude, and we can assumed to be identical for simplicity. In small nonlinear regime and with some straightforward calculations, it is possible to identify one expression of the nonlinear refraction index as function of the components $E_{\binom{x}{y}}$ of the electrical field:

$$n^{NL}_{\binom{x}{y}}=\frac{n_{2\binom{x}{y}}}{a_{eff}}\left[\left(|E_{\binom{x}{y}}|^2+\frac{2}{3}|E_{\binom{y}{x}}|^2\right)+\frac{1}{3}\frac{E^*_{\binom{x}{y}}E_{\binom{y}{x}}}{E_{\binom{x}{y}}}E_{\binom{y}{x}}\right] \quad (3)$$

where $n_{2\binom{x}{y}}=3\chi^{(3)}_{xxxx}/8n_{\binom{x}{y}}$ is the nonlinear index coefficient. In practice, as the birefringence is small in comparison with the refractive index value, the same value of n_2 can be assumed for both main axes of a silica fiber, so $n_2=n_{2(x)}=n_{2(y)}\cong 3.2\times10^{-20}m^2/W$.

Next, we can express the complex components of $\vec{E}$ in polar form:

$$E_x=|E_x|e^{i\varphi_x} \quad (4)$$

$$E_y=|E_y|e^{i\varphi_y} \quad (5)$$

where φ_x and φ_x denote the input temporal phases.

Then, by substituting Eqs.(4,5) in Eq.(3) we arrive to some other expressions for the nonlinear refractive index for each one of the principal axes x, y:

$$n_x^{NL}=\frac{n_2}{a_{eff}}\left[\left(|E_x|^2+\frac{2}{3}|E_y|^2\right)+\frac{\cos(2\varphi)}{3}|E_y|^2+i\frac{\sin(2\varphi)}{3}|E_y|^2\right] \quad (6a)$$

$$n_y^{NL}=\frac{n_2}{a_{eff}}\left[\left(|E_y|^2+\frac{2}{3}|E_x|^2\right)+\frac{\cos(2\varphi)}{3}|E_x|^2-i\frac{\sin(2\varphi)}{3}|E_x|^2\right] \quad (6b)$$

where $\varphi=\varphi_y-\varphi_x$ is phase difference between the polarization modes.

Regarding Eqs.(6) we note that the nonlinear index is a complex number, indeed is convenient to identify its imaginary term as an amplitude modulation coefficient before to define a nonlinear birefringence term:

$$\alpha_{\binom{x}{y}}=(\mp)\frac{2\pi n_2}{3\lambda a_{eff}}\sin(2\varphi)|E_{\binom{y}{x}}|^2 \tag{7}$$

Once made that, we can define a nonlinear birefringence strength as:

$$\Delta n^{NL}=\mathrm{Re}(n_y^{NL}-n_x^{NL})=\frac{2n_2}{3a_{eff}}\left[|E_y|^2-|E_x|^2\right]\sin^2\varphi \tag{8}$$

where Re denotes the real part.
We are now ready to express the optical field in function of the rest of influence parameters in its propagation along the fiber. The vectorial form of the optical field propagating in z is given by

$$\vec{E}(z,t)=|E_x|e^{\alpha_x^{NL}z}\hat{x}+|E_y|e^{\alpha_y^{NL}z}\exp\left[i\left(k(\Delta n^L+\Delta n^{NL})z+\varphi_0\right)\right]\hat{y} \tag{9}$$

where φ_0 is the difference of phase between the input field components, $\Delta n^L=n_y-n_x$ is the linear birefringence of the fiber and $k=2\pi/\lambda$ with λ the wavelength.

Δn^{NL} implicetely depends on z trough Δn^L so it will be necessary use numerical iterative methods to know the value of the electrical field in a particular z. Consequently, the value of the optical field for a particular distance in z will depend on the precedent way travelled by the wave.

Nevertheless, this aspect of the model that seems a disadvantage in front of the analytical expressions developed in Ref.[8], turns out however in a very important advantage in order to achieve a better understanding of the phenomenom of the nonlinear ellipse rotation, because lets the observation of the interaction between the linearly polarized basic states. Moreover it also makes easy the inclusion of temporal dispersion effect in the model through a typical split-step technique method[9].

3. NUMERICAL SIMULATION

Some numerical simulations to propagate an optical field into a linear birefringent single mode silica fiber were performed. The simulated set-up is shown in Figure 1. The input wave was linearly polarized to 45º degrees in respect to the fiber principal axes.

Firstly we proceed to simulate a similar situation to the exposed in Ref.[8], By considering different birefringent fibers, we propagate the wave along a fiber distance equal to a beat length[9] $L=\lambda/\Delta n^L$. In Figure 2(a) it is shown the corresponding azimut (Ψ) and ellipticity (ε) of the polarization ellipse in the output[10] versus the input power. The solid curves correspond with a wave (λ=1.550 μm) which travelled along a birefringent $\Delta n^L=10^{-6}$ fiber for L_A=1.55 m. The dashed and dotted curves correspond with the same input case for another fiber B ($\Delta n^L=5\times10^{-5}$) and C ($\Delta n^L=10^{-4}$) with a propagation length L_B=0.031 m and L_C=0.0155 m respectively.

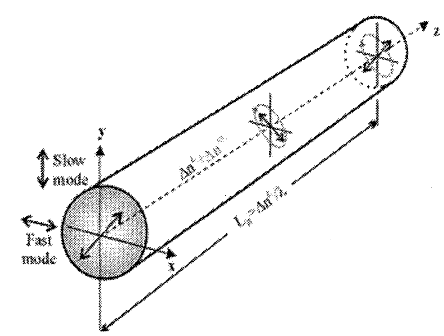

Figure 1.Diagram of the simulated propagation.

By direct comparison, the different resulting curves, corresponding to the propagation of a wave along the beat length of each birefringent fiber, we realise that the nonlinear ellipse rotation can be clearly observed for weakly birefringent fibers[9] $\Delta n^L=10^{-6}$ since it is necessary to have higher input power for achieving a similar behaviour as the birefringence of the fiber increases. This conclusions are in agreement with the results presented in Ref.[8]. We remark that in the balanced polarization configuration, it is possible to find an input power leading to a full polarization switching (see point F in Figure 2(a)).

Furthermore, if we have a birefringent fiber A with Δn_A^L and we observe a nonlinear rotation after a wave travelled along a distance L_A, we can make us the question, does it possible to find a length of another fiber D (Δn_D^L) which present a similar behaviour for identical input powers?

For answering that, we perform a second simulation. For the same linearly polarized input of the first one, we observe the nonlinear ellipse rotation behaviour of a birefringent the above fiber A, and then we try to find the length of another fiber D for which, after propagating the same input fields of the case A, we will obtain a similar output polarization state.

Regarding that $\Delta n_D^L > \Delta n_A^L$ it is clear that for the same input powers we need a fiber D longer than A to obtain that the propagated wave accumulate a similar magnitude of nonlinear phase.

The simulations show that for the studied case (see Figure 2(b)) a L_D 3.5 times longer than L_A is enough to achieve a similar behaviour at the output. Nevertheless, regarding the other superimposed curve, corresponding to another Δn_E^L=0.7×10^{-4} fiber. We note that achieving a full coincidence with the curve A is harder the bigger the difference between the two linear birefringence is. Additionally, the greater is the birefringence the longer is the required fiber length for achieving a similar nonlinear behaviour (L_E=22.26×L_A).

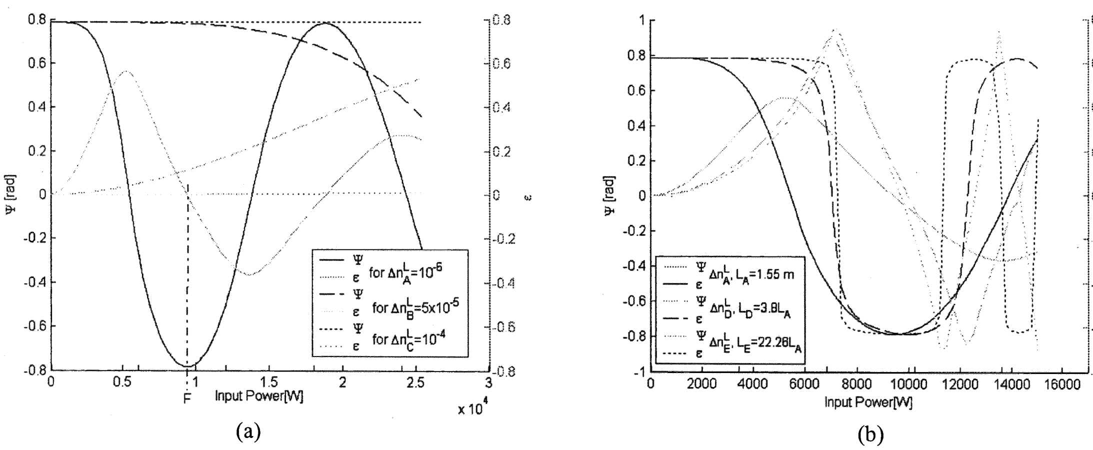

Figure 2. Polarization evolution curves of some simulated cases. (a) Different fibers A,B and C, all for a beat length. (b) Different fibers A, D and E, with length L_A, L_D and L_E respectively (see text for more details).

4. CONCLUSIONS

We have modelled the third order nonlinear interaction between the two main linear polarization modes during propagation along a linear birefringent silica optical fiber.

We have simulated the propagation of an input field linearly polarized to 45 ° with respect to the main axes of the fiber. For this case, similarly to Ref.[8], by comparing the propagation along a beat length of some different fibers, we concluded that the nonlinear ellipse rotation it is more important the smaller the linear birefringence of the fiber is.

Furthermore, by searching for more practical comparisons we have demonstrated that the nonlinear ellipse rotation could be eventually achieved for some different linear birefringent fibers if the wave propagates along a enough long fiber.

Likewise the length of the fiber is a very important parameter in order to decide if the nonlinear ellipse rotation terms must be considered in a numerical simulation.

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