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Talbot effect in selfoc microlens: application in manufacturing

M V Pérez, C Bao, M T Flores-Arias, A Castelo and C Gómez-Reino

Grupo de Óptica GRIN, Departamento de Física Aplicada, Escola Universitaria de Óptica e
Optometría and Facultade de Física, Universidade de Santiago de Compostela,
Campus Sur s/n, E15782 Santiago de Compostela (A Coruña), Spain[1]

E-mail: fatoni@usc.es

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Abstract
We present a hybrid optical device for the measurement of axial refractive index and gradient parameter variations in a selfoc microlens by the Talbot effect. The intrinsic nature of the device, that combines a linear grating and an inhomogeneous medium, permits us to measure changes of the parameters as functions of the position shift of the first self-image. The potential accuracy of the measurement of these parameters is estimated.

Keywords: Talbot effect, selfoc microlens, GRIN optics, optical testing

1. Introduction

The Talbot effect [1–4], also called the self-imaging effect, is a well known phenomenon in optics. This effect consists in the reproduction of a transverse periodic field distribution at periodical spatial intervals along the longitudinal direction as a periodic object, in a homogeneous medium, is illuminated by coherent light. Fundamental properties and many practical applications have been considered [5]. The self-imaging phenomenon has also been studied in inhomogeneous media and recently authors have described the Talbot effect in a tapered gradient-index (GRIN) medium [6–8]. On the other hand, GRIN optics is an active area of research in optical communications and optical sensing. Many devices for fibre-optic sensors include GRIN-rod microlenses or GRIN-fibre microlenses [8]. The purpose of this paper is to present applications of hybrid devices formed by a periodic object and a selfoc microlens, in order to detect small changes of the manufacturing characteristics of selfoc microlenses.

The plan of the paper is as follows. In section 2, we establish the statement of the problem and the results concerning the Talbot effect in selfoc microlenses for uniform illumination. In section 3, we present the sensitivity and accuracy of this device in order to know the limitations for which a position shift of the self-images is transformed into axial refractive index and gradient parameter detectable changes of selfoc microlenses. In section 4, the conclusions are given.

2. Statement of the problem

Let us consider a selfoc microlens characterized by a transverse parabolic refractive index given by

$$n^2(x) = n_0^2(1 - g_0^2 x^2) \tag{1}$$

where n_0 is the index along the direction of propagation and g_0 is the gradient parameter that describes the evolution of the parabolic transverse index along the z axis. The study will be restricted to the one-dimensional case, but extension to the two-dimensional case is straightforward.

We assume that a one-dimensional periodic object located at the input of the GRIN microlens is represented by

$$T(x_0) = \sum_{m=-\infty}^{\infty} a_m \exp\left(-\mathrm{i}\frac{2\pi m x_0}{p}\right) \tag{2}$$

where p is the period and a_m is the amplitude of the mth harmonic.

When the hybrid structure is illuminated by a uniform plane wave of amplitude A and wavelength λ,

$$\psi(x_0) = A\exp(-\mathrm{i}kz) \tag{3}$$

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[1] http://www.usc.es/grinteam

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the complex amplitude distribution on the periodic object located at $z = 0$ is written as

$$\phi(x_0) = AT(x_0). \tag{4}$$

The complex amplitude distribution in the selfoc microlens, that is, at $z > 0$, is given by the integral equation

$$\phi(x) = \int_{-\infty}^{\infty} \phi(x_0) K(x, x_0; z)\, \mathrm{d}x_0. \tag{5}$$

K is the one-dimensional optical propagator of the GRIN medium

$$K(x, x_0; z) = \left[\frac{kn_0}{2\pi \mathrm{i} H_1(z)}\right]^{1/2} \exp(\mathrm{i}kn_0 z) \times \exp\left[\frac{\mathrm{i}kn_0}{2H_1(z)}(x^2 \dot{H}_1(z) + x_0^2 H_2(z) - 2xx_0)\right] \tag{6}$$

where H_1, H_2 and $\dot{H}_1$, $\dot{H}_2$ are the positions and slopes of the axial and field rays at z, respectively, the dot indicating the derivative with respect to z [8].

Substituting equations (2)–(4) and (6) in equation (5) and integrating, the complex amplitude distribution, apart from constant factors, is given by

$$\phi(x, z) = \left(\frac{1}{H_2(z)}\right)^{1/2} \exp\left[\mathrm{i}\frac{\pi n_0 \dot{H}_2(z)}{\lambda H_2(z)} x^2\right] \times \sum_m a_m \exp\left[-\mathrm{i}\frac{\lambda \pi m^2 H_1(z)}{n_0 p^2 H_2(z)}\right] \exp\left[-\mathrm{i}\frac{2m\pi}{pH_2(z)} x\right]. \tag{7}$$

The first exponential term of the summation in equation (7) is of basic importance to self-imaging and it represents the phase changes of the diffractive orders with the axial distance z. When the phase changes are discrete and multiples of π, this term becomes ± 1 for the relation

$$\frac{H_1(z_\nu)}{n_0 H_2(z_\nu)} = \frac{\nu p^2}{\lambda} \tag{8}$$

where ν is an integer indicating the self-image number. Equation (8) is called the integer Talbot condition. The self-image distances are given by [6]

$$z_\nu = \frac{1}{g_0} \tan^{-1}\left[\frac{\nu n_0 g_0 p^2}{\lambda}\right]. \tag{9}$$

The z_νs are the distances for which the input complex amplitude distribution is periodically repeated along the z axis of the microlens. Equation (9) shows the dependence of the self-image distances on the object period, the gradient parameter and the axial refractive index of the selfoc microlens and the wavelength of illumination. Figure 1 represents the position of the self-images versus the self-image number. The interval between consecutive self-image distances decreases with the self-image number. We have chosen a commercial selfoc microlens, with values $n_0 = 1.6073$ for the axial refractive index and $g_0 = 0.154$ mm^{-1} for the gradient parameter. The period of the linear grating is $p = 432$ μm and the device is illuminated by a He–Ne laser of wavelength $\lambda = 632.8$ nm.

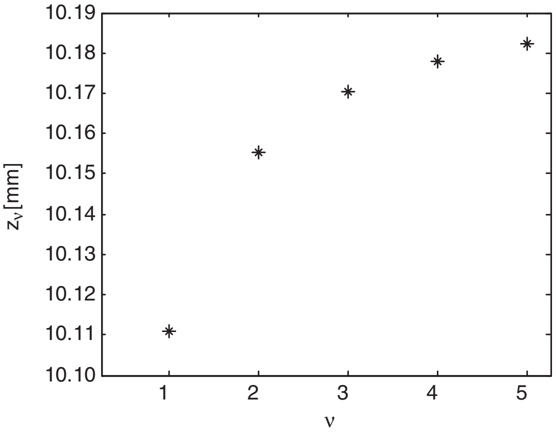

Figure 1. Self-image distances versus self-image number for the first five integer Talbot images. Calculations have been made with $p = 432$ μm, $\lambda = 632.8$ nm, $n_0 = 1.6073$ and $g_0 = 0.154$ mm^{-1}.

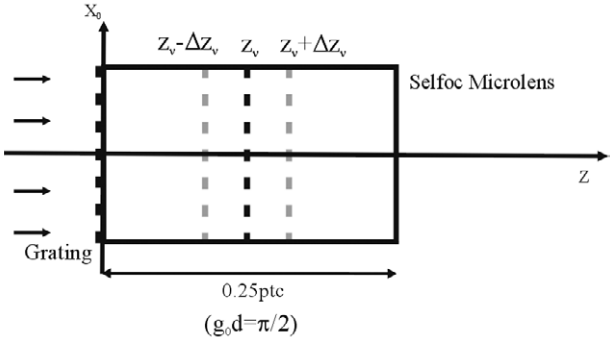

Figure 2. A hybrid device for obtaining the Talbot effect in a quarter-pitch selfoc microlens.

3. Talbot effect in selfoc microlens for quality control

The Talbot effect in GRIN media can be used in applications to control the quality of manufacturing technology of microlenses by the study of position shift of the self-images induced by changes on the parameters characterizing selfoc microlenses.

We consider a quarter-pitch selfoc microlens for which $g_0 d = \pi/2$, d being the thickness of the lens. The hybrid device used to obtain the Talbot effect is shown in figure 2 and it is formed by a quarter-pitch microlens (0.25 ptc) and a grating located at the input face of the microlens. The axial position z_ν of any integer Talbot image varies if the parameters of the microlens suffer small changes during performance or the fabrication process [8]. Hence, any mechanism that induces a change of the parameters in the selfoc microlens can be measured as a shift in the self-image positions. Figure 3 presents the position of the self-images versus axial refractive index in a selfoc microlens. The Talbot position moves away from or moves back to the input face of the hybrid device as the axial refractive index increase or decreases, respectively. The sensitivity of the device to the axial refractive index arises from the position change of the Talbot images. With the aid of equation (9) the axial refractive index change Δn_0 for a

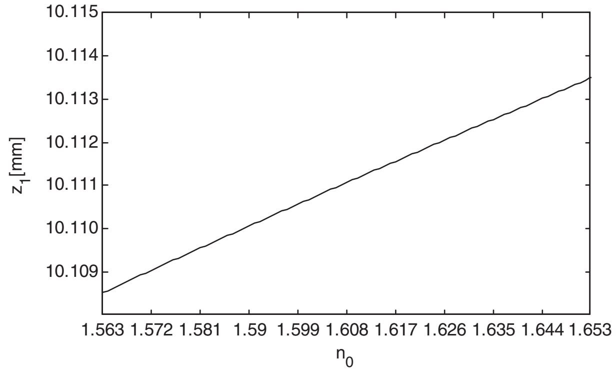

Figure 3. Position of the first self-image versus refractive index along the z axis in a selfoc microlens. Calculations have been made with $p = 432\ \mu$m, $\lambda = 632.8$ nm, and $g_0 = 0.154$ mm^{-1}.

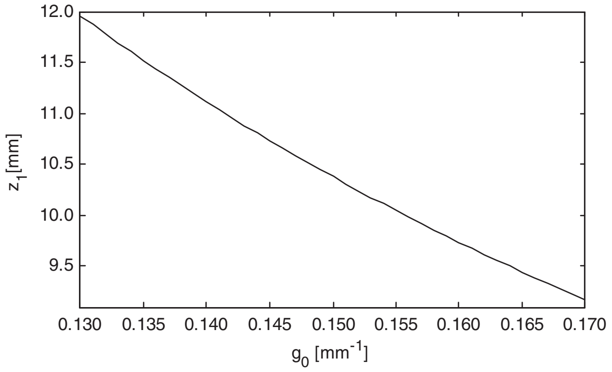

Figure 4. Position of the first self-image versus gradient parameter in a selfoc microlens. Calculations have been made with $\lambda = 632.8$ nm, $p = 432\ \mu$m and $n_0 = 1.6073$.

position shift Δz_ν is given by

$$\Delta n_0 = \frac{\partial n_0}{\partial z_\nu}\Delta z_\nu = \frac{\lambda \sec^2(g_0 z_\nu)}{p^2 \nu}\Delta z_\nu. \tag{10}$$

For the first Talbot image ($\nu = 1$), in which the highest image contrast is obtained, equation (10) becomes

$$\Delta n_0 = \frac{\lambda \sec^2(g_0 z_1)}{p^2}\Delta z_1. \tag{11}$$

The accuracy of the measurement of the position shift of the Talbot images is determined by the depth of focus (DOF) of the microscope objective used to evaluate the axial location of these images [9]. For a 0.62 NA (40×) microscope objective, the DOF is approximately 0.75 μm at $\lambda = 632.8$ nm. It can be read from figure 3 that for $\Delta n_0 = 0.095$ (quite a significant change) the shift of the first self-image position is as small as 5.2 μm and the DOF is almost an order of magnitude smaller than this displacement. This means that a moderate precision of n $_0$ measurement can be obtained. At the first Talbot image, the error is around 15% for the shift measurement.

Another interesting application is the measurement of the gradient parameter of the selfoc microlens since it is a basic magnitude for optical characterization of the parabolic shape of the refractive index profile [10]. When the gradient parameter suffers any change, the Talbot images vary their axial positions. From equation (9), the sensitivity of the device for the gradient parameter can be expressed as

$$\Delta g_0 = \frac{\partial g_0}{\partial z_\nu}\Delta z_\nu = \frac{g_0}{\frac{\nu n_0 p^2}{\lambda}\cos^2(g_0 z_\nu) - z_\nu}\Delta z_\nu \tag{12}$$

that becomes

$$\Delta g_0 = \frac{g_0}{\frac{n_0 p^2}{\lambda}\cos^2(g_0 z_1) - z_1}\Delta z_1 \tag{13}$$

for the first self-image.

Figure 4 depicts the position shift of the first Talbot image with the variation of g_0 around a typical value of 0.154 mm^{-1}. It can be seen that for $\Delta g_0 = 0.04$ mm^{-1} the longitudinal shift of the first self-image is approximately 2.8 mm. In this case, the DOF of the microscope objective is a very small fraction of these 2.8 mm and then a good precision of g_0 measurement is achieved. At the first self-image, observed through the microscope objective, the error is around 0.03% for the axial displacement caused by gradient parameter variations.

From the two applications suggested in the paper, the most interesting may be the measurement, with a good accuracy, of the gradient parameter concerning the control of refractive index profile in selfoc microlenses.

4. Conclusions

A hybrid optical device to control refractive index distribution variation in a selfoc microlens by the Talbot effect has been proposed. Changes of the parameters characterizing the selfoc microlens such as axial refractive index and gradient parameter are measured as longitudinal shifts in the first self-image positions. The precision of measurement of the position shift is dependent on the parameter that causes it. Precision is smaller for axial refractive index variation than for gradient parameter change in the selfoc microlens. Future work will more thoroughly analyse the performance of this device.

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