Zone-plate diffraction patterns in gradient-index media

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Abstract: Diffraction patterns inside gradient-index media in which a complex zone plate is placed at a Fourier transform plane are studied by making use of the optical propagator. The results are illustrated with an example corresponding to a selfoc medium and a Fresnel zone plate of the amplitude and of the phase.

OCIS codes: (110.2760) Gradient-index lenses; (350.5500) Propagation

References and links

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1. Introduction

Fresnel zones and zone plates are classic topics in optics and have received widespread attention regarding the study of the propagation and diffraction of light through homogeneous media. Fundamental properties and many practical applications, like image formation, synthesis of holograms, coherence measurements, spectrometry and optical testing, to name but a few, have been considered [1]. Nevertheless, except for a short study by Kravtsov and Orlov [2], we have not found any research on zone plates inside inhomogeneous media, even when inhomogeneous media attract a lot of attention from researchers around the world.

Recently Rivas-Moscoso *et al.* studied the free propagation of a wave front in tapered gradient-index media with parabolic refractive index and the division of the wave front into Fresnel zones, thereby obtaining the zone contribution to the disturbance at observation points at the optical axis and the total disturbance produced by the summation of a number of such zones [3]. The results showed that the contributions of successive zones were alternately positive and negative and, as a consequence, an even number of zones gave a minimum of irradiance and an odd number gave a maximum for on-axis points. This meant that the zone-plate construction would constitute an adequate method to easily focus light. After proving that the planar zone-plate ring radii coincided with the radii of the Fresnel zones into which the wave front was divided, the irradiance along the axis for various examples of zone-plate transmission functions and GRIN media was calculated, whereby it could be stated that for selfoc media [4] and zone plates of the amplitude the diffractive order 0 was always placed at the image planes, whether the wave front was on or off a Fourier transform plane, and that positive and negative orders were symmetric around order 0.

The aim of this paper is to extend the study of the diffraction patterns produced by a zone plate in a GRIN medium to off-axis points. For the sake of simplicity, we will only take into account wave fronts at a Fourier transform plane.

2. Statement of the problem

Let us begin by considering a tapered GRIN medium characterized by a transverse parabolic refractive index modulated by an axial index and whose refractive index profile is given by

$$n^2(r;z) = n_0^2\left[1 - g^2(z)r^2\right]; \quad r^2 = x^2 + y^2, \tag{1}$$

where n_0 is the index at the z optical axis and $g(z)$ is the taper function that describes the evolution of the transverse parabolic index distribution along the z axis.

The disturbance produced at a point r_0 on a Fourier transform plane at $z' = z$ by an on-axis point source at $z' = 0$ can be expressed, in parabolic approximation, by the following plane wave front

$$\psi(r_0;z) = \frac{kn_0}{2\pi \mathrm{i} H_1(z)} \exp(\mathrm{i}kn_0 z), \tag{2}$$

where $H_1(z)$ is the position of the axial ray [4].

A zone plate of infinite dimension is placed at the Fourier transform plane. This zone plate will be represented as

$$T(r_0) = \sum_{m=-\infty}^{\infty} a_m \exp\left\{-\mathrm{i}\frac{2\pi m}{p} r_0^2\right\}, \tag{3}$$

where p is the spatial period and a_m is the amplitude of the mth harmonic. Period p assumes the value given by $2h_1^2$, where h_1 is the radius of the first Fresnel zone for a wave front at the Fourier transform plane [3].

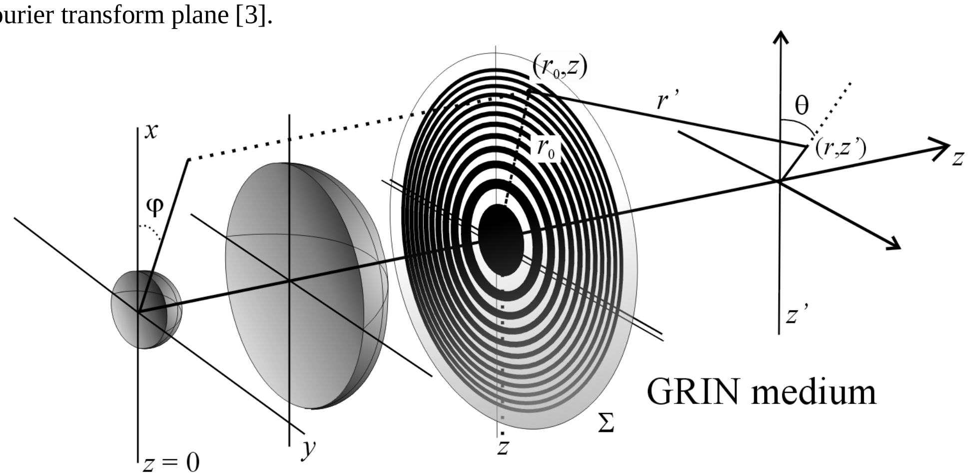

Fig. 1. Geometry for the propagation of a plane wave front at a Fourier transform plane through a zone plate inside a GRIN medium.

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The complex amplitude distribution at a point (r, z') inside the medium after the zone plate (see Fig. 1) may be calculated from the wave front in Eq. (2) and the zone-plate transmission function in Eq. (3) by solving the diffraction integral [5]

$$\psi(r;z') = \int_{\Sigma} K(r,r_0;z')T(r_0)\psi(r_0)r_0\,\mathrm{d}r_0 d\varphi\,, \tag{4}$$

where $K(r, r_0; z')$ is the kernel or propagator for GRIN media in cylindrical coordinates

$$K(r,r_0;z') = \frac{-ikn_0}{2\pi H_1(z')}\exp[\mathrm{i}kn_0(z'-z)]$$
$$\times\exp\left\{\frac{\mathrm{i}kn_0}{2H_1(z')}\left[\dot{H}_1(z')r^2 + H_2(z')r_0^2 - 2rr_0\cos(\varphi-\theta)\right]\right\} \tag{5}$$

and $H_1(z')$, $H_2(z')$, $\dot{H}_1(z')$ and $\dot{H}_2(z')$ are, respectively, the position and the slope of the axial and field rays at z' after propagating from z. This involves changing the integration limits in the argument of the sine and cosine functions in the expressions of $H_1(z)$, $H_2(z)$, $\dot{H}_1(z)$ and $\dot{H}_2(z)$ given in Ref. 4, which now go from z to z'.

Substitution of Eqs. (2), (3) and (5) into Eq. (4) and integration in φ provides

$$\psi(r;z') = \frac{-k^2n_0^2}{2\pi H_1(z')H_1(z)}\exp(\mathrm{i}kn_0z')\exp\left[\frac{\mathrm{i}kn_0}{2}\frac{\dot{H}_1(z')}{H_1(z')}r^2\right]$$

$$\times\sum_{m=-\infty}^{\infty}\int_0^{\infty} a_m\exp\left\{\mathrm{i}\left[\frac{kn_0H_2(z')}{2H_1(z')} - \frac{\pi m}{h_1^2}\right]r_0^2\right\}\mathrm{J}_0\left[\frac{kn_0rr_0}{H_1(z')}\right]r_0\mathrm{d}r_0\,, \tag{6}$$

regardless of the value of θ, where J_0 is the zero-order Bessel function of the first kind.

For $r = 0$, carrying out the integral in Eq. (6) gives an amplitude distribution of the form

$$\psi(z') = -\frac{\mathrm{i}k^2n_0^2\exp(\mathrm{i}kn_0z')}{2\pi H_1(z')H_1(z)}\sum_{m=-\infty}^{\infty}a_m\left[\frac{kn_0H_2(z')}{H_1(z')} - \frac{2\pi m}{h_1^2}\right]^{-1}, \tag{7}$$

from where it follows that the irradiance distribution along the optical axis can be written as

$$I(z') = \frac{k^4n_0^4}{4\pi^2H_1^2(z')H_1^2(z)}\sum_{m,l=-\infty}^{\infty}a_ma_l^*\left\{\left[\frac{kn_0H_2(z')}{H_1(z')} - \frac{2\pi m}{h_1^2}\right]\left[\frac{kn_0H_2(z')}{H_1(z')} - \frac{2\pi l}{h_1^2}\right]\right\}^{-1}. \tag{8}$$

Foci will be at positions f_m where the denominator in Eq. (8) cancels, and therefore they can be obtained by solving the equation

$$\tan\left[\int_z^{f_m} g(\bar{z})\,\mathrm{d}\bar{z}\right] = \frac{kn_0g_0h_1^2}{2\pi m}. \tag{9}$$

This equation for a selfoc medium reduces to

$$f_m = (1/g_0)\tan^{-1}\left[kn_0g_0h_1^2/(2\pi m)\right] + z\,. \tag{10}$$

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Allowing for the oscillatory function in Eq. (10), in a selfoc medium, foci replicate periodically at distances multiple of π/g_0 from their first occurrence after the zone plate.

For $r \neq 0$, integration of Eq. (6) provides [6]

$$\psi(r;z') = -\frac{\mathrm{i}\,k^2 n_0^2 \exp(ikn_0 z')}{2\pi H_1(z')H_1(z)} \exp\left(\mathrm{i}\frac{kn_0}{2H_1(z')}\dot{H}_1(z')r^2\right) \sum_{m=-\infty}^{\infty} \frac{a_m}{2}$$
$$\times \left[\frac{kn_0}{2H_1(z')}H_2(z') - \frac{2\pi\, m}{p}\right]^{-1} \exp\left\{-\mathrm{i}\frac{k^2 n_0^2 r^2}{4H_1^2(z')}\left[\frac{kn_0}{2H_1(z')}H_2(z') - \frac{2\pi\, m}{p}\right]^{-1}\right\}, \qquad (11)$$

and the irradiance can be written as

$$I(z') = \frac{k^4 n_0^4}{4\pi^2 H_1^2(z')H_1^2(z)} \sum_{m,l=-\infty}^{\infty} a_m a_l^* \left\{\left[\frac{kn_0 H_2(z')}{H_1(z')} - \frac{4\pi\, m}{p}\right]\left[\frac{kn_0 H_2(z')}{H_1(z')} - \frac{4\pi\, l}{p}\right]\right\}^{-1}$$
$$\times \exp\left\{-\mathrm{i}\frac{k^2 n_0^2 r^2}{4H_1^2(z')}\left[\left(\frac{kn_0}{2H_1(z')}H_2(z') - \frac{2\pi\, m}{p}\right)^{-1} - \left(\frac{kn_0}{2H_1(z')}H_2(z') - \frac{2\pi\, l}{p}\right)^{-1}\right]\right\}. \qquad (12)$$

3. Discussion

To illustrate the extent of this calculus, let us apply these results for Fresnel zone plates of the amplitude and of the phase [1] (with m in the summation going from –1 to +1, which is equivalent to having a sinusoidal zone plate) and selfoc media. These simplifications do not restrict the study; rather the conclusions drawn may be straightforwardly extended to other types of media and zone-plate transmission functions. Calculations are made for refractive index at the optical axis $n_0 = 1.5$, gradient parameter $g(0) = g_0 = 0.1\ \mathrm{mm}^{-1}$, wave-front position $z = \pi/(2g_0)$ and zone-plate period $p = 0.1216\ \mathrm{mm}^2$.

As can be observed in Fig. 2, along the optical axis, in the portion of the medium between two Fourier transform planes, there are three peaks of irradiance for the zone plate of the amplitude, which correspond to diffractive orders 0 and ±1, and two peaks, corresponding to orders +1 and –1, for the zone plate of the phase.

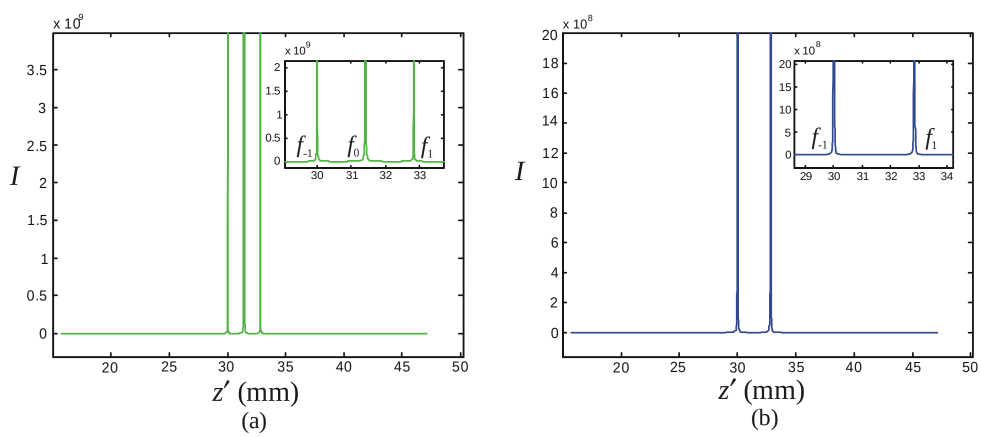

Fig. 2. Irradiance along the optical axis for (a) a Fresnel zone plate of the amplitude and (b) of the phase. Shown are enlargements of the foci regions in both cases.

z = 30.38 mm

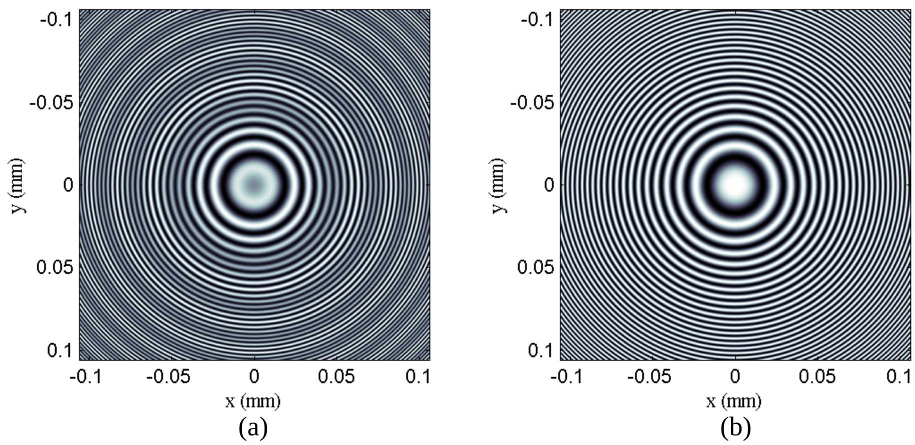

Fig. 3. (both 1.48 MB) Evolution of the zone-plate diffraction patterns in a selfoc medium with (a) a Fresnel zone plate of the amplitude and (b) of the phase (pace: every 0.1 mm).

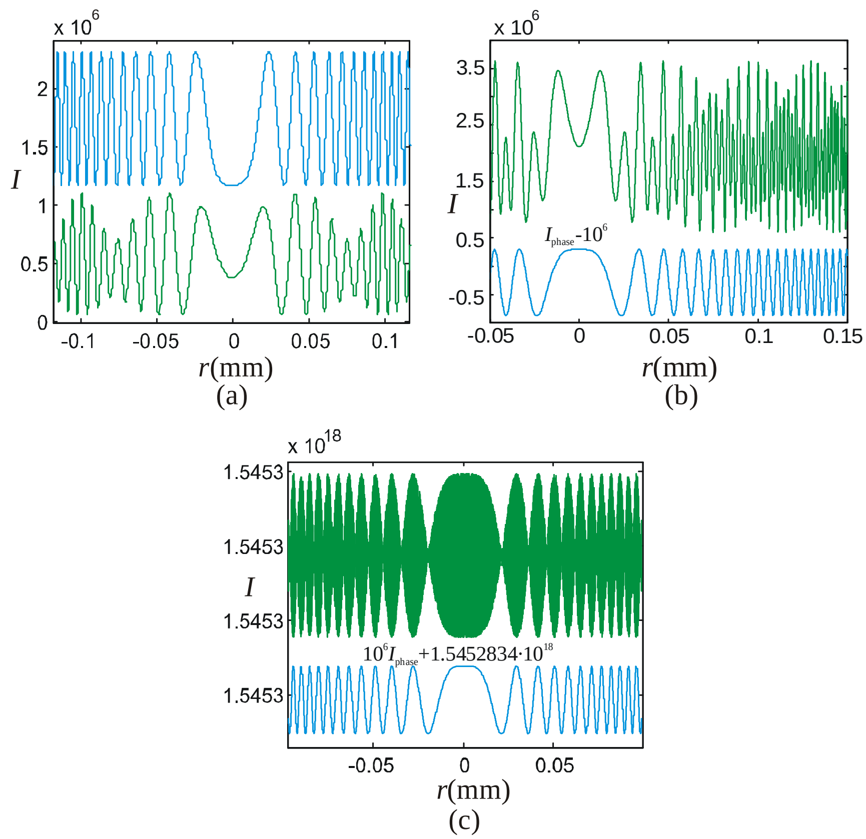

Fig. 4. Irradiance at transverse planes (a) z' = 29.4409 mm, (b) z' = 30.9840 mm and (c) z' = 31.4159 mm for Fresnel zone plates of the amplitude (green line) and of the phase (blue line) with period p = 0.1216 mm^2 in a selfoc medium with parameters n_0 = 1.5 and g_0 = 0.1 mm^{-1}. For the sake of reproducibility, in graphs (b) and (c) the irradiance for the zone plate of the phase was shifted upwards and/or enlarged by the amounts shown in the respective graphs.

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From Eq. (7), by making $m = 0$ in the summation, which is tantamount to removing the zone plate, we can see that the free propagation of a plane wave front at the Fourier transform plane would produce a singularity in the irradiance distribution at the image plane (position f_0) and therefore there would be a focus even without the zone plate. This singularity disappears when placing a zone plate of the phase, as can be stated from the inspection of Fig. 2(b).

Now the foci have been located, we move on to analyze the diffraction patterns across the medium. Figure 3 shows the transverse irradiance evolution with the propagation length. To achieve a better knowledge of the phenomena going on, let us home in on a few representative planes. In Fig. 4(a) we show the transverse irradiance distribution profile along the radial direction at a plane prior to f_{-1} for a zone plate of the amplitude (green-line graph) and of the phase (blue-line graph). The irradiance distribution consists of a series of maxima and minima of irradiance, which gives place to bright and dark rings, whose brightness varies according to a pattern (observe the graph for the zone plate of the amplitude in Fig. 3). Figure 4(b) shows the transverse irradiance distribution at a plane halfway between foci –1 and +1. By comparison with Fig. 4(a), we observe that there is a central minimum of irradiance before focus –1 for the zone plate of the phase and this minimum switches to a maximum at this plane. Likewise, the reverse transition, i.e. from maximum to minimum, occurs at f_{+1}. For the zone plate of the amplitude, there is no such change, the irradiance presenting a central minimum all along. On the other hand, in Figure 4(b) the irradiance pattern for the zone plate of the amplitude looks as though alternate maxima (and minima) were modulated by different enveloping waves. This happens at planes different from the ones that correspond to diffractive orders 0 and ± 1. At these planes maxima of irradiance smash together and cannot be told apart. Of the three of them, the order zero is especially interesting. To show what happens at this plane, Fig. 4(c) is provided. For a zone plate of the amplitude the undistinguishable maxima form a figure as if "modulated" by the irradiance pattern for a zone plate of the phase. Similarly, at foci ± 1, apart from the change for the zone plate of the phase mentioned above, the irradiance for the zone plate of the amplitude is "modulated" by the square modulus of the contributions of orders 0 and ∓ 1 to the complex amplitude distribution.

4. Conclusion

We extended the study of the zone-plate diffraction patterns in a gradient-index medium for off-axis observation points and presented an equation for the complex amplitude and the irradiance when the wave front was placed at a Fourier transform plane. We applied this study to the well-known selfoc media and the Fresnel zone plates of the amplitude and of the phase to show that the irradiance on transverse planes have maxima and minima that converge or diverge as we probe the medium to approach the planes where there is a focus on the optical axis or move away from them. Furthermore, we saw that for the zone plate of the phase there is a central maximum or minimum of irradiance that switches to a minimum or maximum, respectively, every time we pass the planes where there is a focus on the optical axis that corresponds to positive or negative diffractive orders.

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