Fractional Talbot effect in a Selfoc gradient-index lens

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The fractional Talbot effect is demonstrated inside a standard 0.25-pitch Selfoc gradient-index lens under uniform illumination. Comparisons with theoretical expressions of positions and magnification of fractional Talbot images are given.

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The Talbot effect[1] is one of the basic diffractive phenomena in Gaussian systems. On conventional scales and in homogeneous media, the effect has several applications, for instance, in imaging for increasing or decreasing spatial frequency, integrated optics for optical coupling, Talbot microscopy, image restoration, and Talbot cavity design.[2] It is therefore interesting to explore this effect at different scales and in different media. In this context, the Talbot effect has been generalized and demonstrated in waveguides,[3] and it is also used in the design of integrated optics components for wavelength-division, multiplexing applications.[4] Recently,[5–7] Talbot images produced by a periodic object in an inhomogeneous medium were described theoretically. One of the natural media in which these images can be studied is conventional gradient-index (GRIN) microlenses, thus giving insight into this effect at micrometer scale. In this Letter we report on an experimental verification of the Talbot effect inside a Selfoc GRIN lens with a period of 0.25.

The optical setup can be described as follows (Fig. 1): A grating of period $p_0 = 432 \pm 3\ \mu\text{m}$ is illuminated by a He–Ne laser of wavelength $\lambda_0 = 632.8$ nm. An image of the grating is formed inside a $0.25p$ Selfoc lens (from NSG, Somerset, N.J.) with a 10×0.35 numerical-aperture (NA) microscope objective. Approximately 15 periods of the grating are focused along the diameter of the spot. The radial refractive-index distribution of the Selfoc lens is given by

$$n^2(x, y) = {n_0}^2[1 - g^2(x^2 + y^2)]. \tag{1}$$

The parameters of this lens[8] are, for the value of the index along the z axis, $n_0 = 1.6073$, and, for the gradient, $g = 0.304\ \text{mm}^{-1}$, for a total length of 5.168 mm and diameter of 2 mm.

To visualize this image and to probe the diffractive fields after it, we position a microscope after the GRIN lens. The objective of this microscope has a 3.5×0.10 NA. There, L is the distance between an arbitrary object plane π_0 inside the microlens and the exit plane of the microlens π_E, and d is the distance from the microscope objective to the exit plane. The objective forms the image in a plane π_I, located at a fixed distance d' after it. The $ABCD$ matrix relating π_O and π_I is

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & d' \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1/f & 1 \end{bmatrix} \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ 0 & n_0 \end{bmatrix} \begin{bmatrix} \cos(gL) & \sin(gL)/g \\ -g\sin(gL) & \cos(gL) \end{bmatrix}, \tag{2}$$

where the last matrix is the well-known ray-transfer matrix in a Selfoc medium[5] and f is the objective focal length. Then, imposing the imaging condition $B = 0$, we can easily obtain the following relation between distances inside (L) and outside (d) the microlens:

$$\tan(gL) = n_0 g\left(\frac{f}{d' - f}\, d' - d\right). \tag{3}$$

When $L = 0$, the exit plane is focused, corresponding to the Gauss law value $d_E = d'f/(d' - f)$. We can simplify Eq. (3), using the difference from the point where the exit plane is brought to focus and the measured distance inside (d), $\Delta = d_E - d$, so that

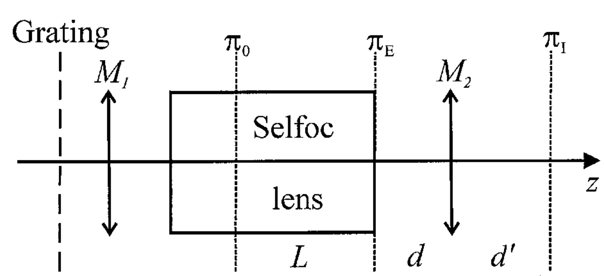

Fig. 1. Geometry of the input and readout system: M_1, entrance objective; π_0, object plane; π_E, exit plane of the microlens; π_I, image plane; L, distance from π_0 to π_E; d, distance from π_E to the microscope objective; M_2, exit objective of focal length f; d', length from the exit objective to π_I.

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$$\tan(gL) = n_0 g\Delta\,. \tag{4}$$

In this way we can probe the planes formed in the Selfoc lens up to $gL = \tan^{-1}(n_0 g d_E)$, a value that is slightly smaller than $0.25p$. Thus with this setup we can focus almost all the planes along the whole length of the lens. However, the transverse magnification of the observation system, $M_{tS} = A$, in the planes focused by Eq. (3) is given by

$$M_{tS}(\Delta) = M_0[1 + (n_0 g\Delta)^2]^{1/2}, \tag{5}$$

where M_0 is the magnification of the objective. Then, we can measure the period p of a periodic object formed in any plane inside the GRIN lens, characterized by Δ, by measuring the apparent period, p_{ap}, in the image plane π_I so that $p(\Delta) = p_{ap}(\Delta)/M_{tS}(\Delta)$.

The image of the original grating is focused for a certain value $\Delta = \Delta_G$, corresponding to $L = L_G$. We checked that it is a conventional image of the grating and not a Talbot field by imaging an arbitrary transparency. We obtained $\Delta_G = 9.81 \pm 0.04$ mm so that $L_G = 4.49 \pm 0.06$ mm and $M_{tS}(\Delta_G) = 17.11 \pm 0.06$. Then, since the apparent period as measured through the microscope is $p_{ap}(\Delta_G) = 600 \pm 4\ \mu$m, the resulting period of the image of the grating inside the microlens is $p(\Delta_G) = 35.1 \pm 0.4\ \mu$m.

After the periodic object formed inside the GRIN lens, fractional Talbot images of order β/α are formed. Photographs of these planes compared with the same Talbot planes in air are shown in Fig. 2. We focus in the fractional planes $\beta/\alpha = 1/4, 1/3, 1/2$. The original image corresponds to the value $\beta/\alpha = 0$. We choose these values since they produce recognizable patterns and they cover the whole length between the grating plane and the exit plane. The last Talbot plane $\beta/\alpha = 1/2$ is formed in the exit plane of lens π_E. We can easily accomplish **this** experimentally by changing the position of the grating with respect to the lens. The measured positions $z_{\beta/\alpha} = L_G - L_{\beta/\alpha}$ and the transverse magnifications are shown in Fig. 3.

The theoretical expression for the location of these planes and their transverse magnifications can be obtained as follows: First, we form an equivalent system composed of a grating in the input face of a GRIN medium under uniform illumination. Notice that since the original illumination of the system is planar, the Gaussian system producing the grating inside the microlens will also produce an image under uniform illumination. This equivalent system is shown in Fig. 4. The grating has period p, and the distance from the origin of the illumination to the grating is d_0. The remaining notation is as in Fig. 1. Then, the axial positions of the planes are given by[5–7]

$$\tan(gz_{\beta/\alpha}) = \frac{\beta p^2 n_0 g d_0}{\alpha\lambda_0 d_0 - \beta p^2}\,, \tag{6}$$

and the transverse magnifications are

$$M_t = \cos(gz_{\beta/\alpha}) + \sin(gz_{\beta/\alpha})/n_0 d_0 g\,. \tag{7}$$

To check our results it is necessary to obtain an independent measure of the equivalent uniform illumination focus, d_0, since all the remaining parameters in Eqs. (6) and (7) are known. Since the last Talbot plane inside the microlens is 1/2, the first integer Talbot plane, whose form is easily recognizable, must be formed in air after the microlens. The location of this plane must

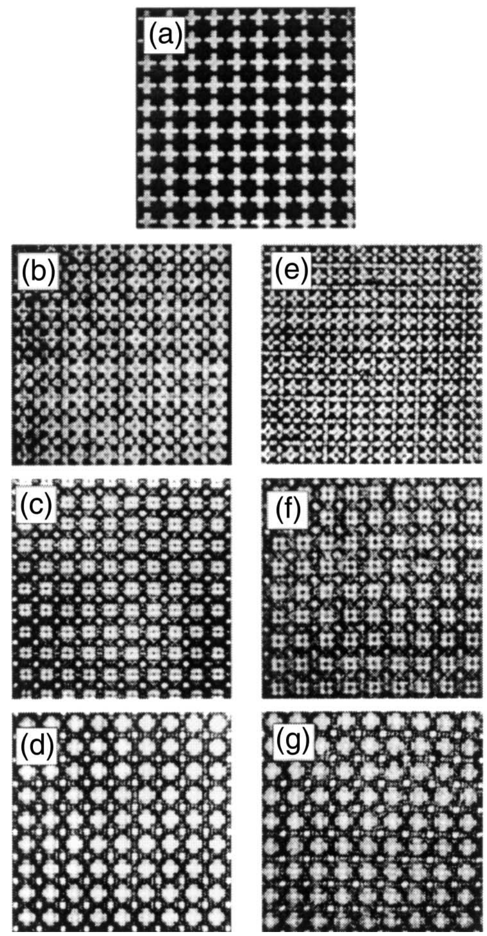

Fig. 2. (a) Periodic object; (b), (c), (d) fractional Talbot planes $z_{1/2}$, $z_{1/3}$, and $z_{1/4}$, respectively, in air; (e)–(g) the same planes inside the Selfoc GRIN lens.

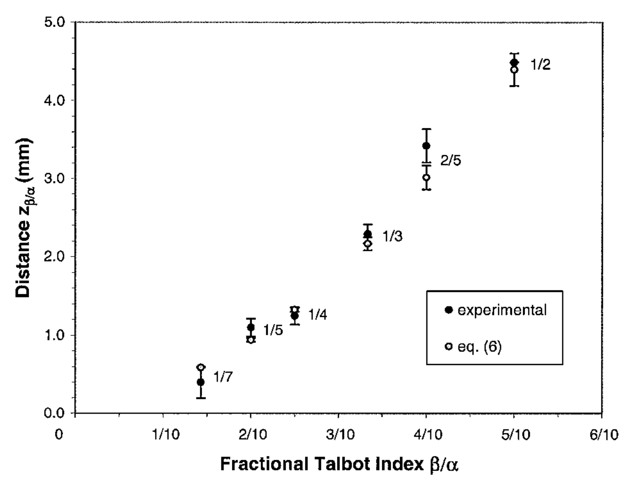

Fig. 3. Measured positions versus the fractional Talbot index. Positions as predicted from Eq. (6) are also given. Fractions besides the points indicate the exact fractional Talbot indices.

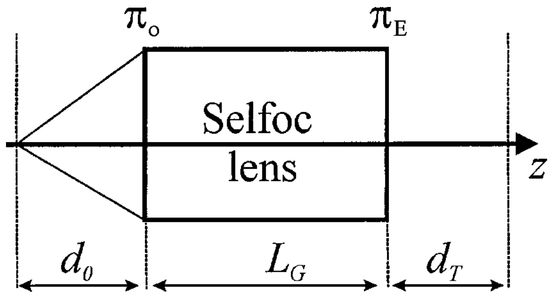

Fig. 4. Equivalent system under uniform illumination: d_0, distance from the point source to the plane where the image of the grating is formed; L_G, length of the lens after the image of the grating; d_T, distance after the microlens where the first integer Talbot image is formed.

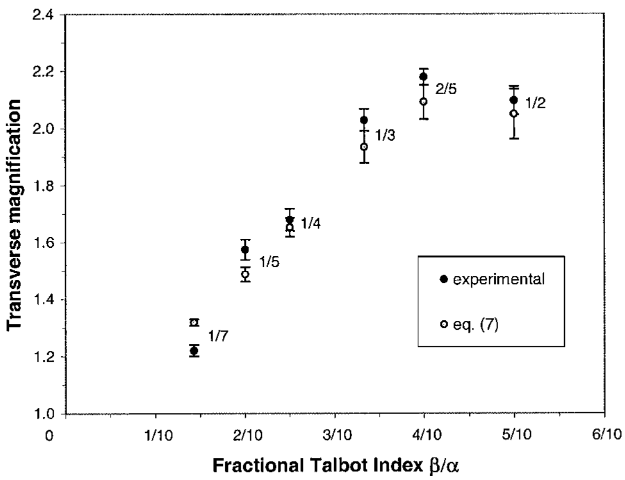

Fig. 5. Measured transverse magnification versus fractional Talbot index. Magnifications as predicted from Eq. (7) are also given. The fractions beside the points indicate the exact fractional Talbot indices.

depend on d_0, so a measure of its position can provide the desired information. To get a theoretical formula for this relation we recall the result[9] that in a general Gaussian system under uniform illumination, described by its $ABCD$ matrix, β/α fractional Talbot planes are formed whenever this relation holds:

$$\frac{B}{A} = \frac{\beta}{\alpha}\frac{p^2}{\lambda_0}. \tag{8}$$

Then, if we apply this formula to the hybrid system composed of a point source illumination at a distance d_0, GRIN lens of length L_G, and free propagation along a distance d_T, it is possible to obtain the positions of Talbot planes formed after the lens. The $ABCD$ matrix for this system is

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & d_T \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & n_0 \end{bmatrix} \times \begin{bmatrix} \cos(gL) & \sin(gL)/g \\ -g\sin(gL) & \cos(gL) \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ 0 & 1/n_0 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1/d_0 & 1 \end{bmatrix}, \tag{9}$$

where the last matrix accounts for the divergent illumination originated in the point source located at d_0. Then, using Eqs. (8) and (9) for $\beta/\alpha = 1$, we get

$$\frac{1}{d_0} = \frac{\lambda_0}{p^2} + \frac{(n_0 g)^2 d_T \Delta_G - 1}{d_T + \Delta_G}. \tag{10}$$

With this procedure we found that $d_0 = 1.10 \pm 0.03$ mm. Now, Eq. (7) can be checked. Transverse magnification variation with the Talbot index is shown in Fig. 5. Agreement is good within experimental errors.

In conclusion, we have experimentally demonstrated the Talbot effect in a Selfoc GRIN lens. This is to our knowledge the first report of this effect in an inhomogeneous medium.

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