Talbot effect in a tapered gradient-index medium for nonuniform and uniform illumination

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A generalization of the Talbot effect to the case of a tapered gradient-index medium for nonuniform and uniform illumination is considered. Self-image positions are changed by a function depending on the taper profile, the illumination, and the periodic object. An analogy with the conventional lens-imaging formula for both types of illumination is presented. © 1999 Optical Society of America [S0740-3232(99)01010-8]

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1. INTRODUCTION

The Talbot effect is well known in optics and has received wide attention. Fundamental properties and many practical applications have been considered.[1] Recently the Talbot effect has also been studied in atomic optics[2–3] and in transverse quadratic index media.[4–6]

The purpose of this paper is to generalize the self-imaging phenomenon to a tapered gradient-index (GRIN) medium when a periodic object located at the input of the medium is illuminated by a coherent nonuniform light beam and a uniform light beam. The study will be restricted to the one-dimensional transverse case, but extension of the analysis to the two-dimensional case is straightforward. The results on the generalization of the Talbot effect to a tapered GRIN medium can have important and wide applicability.

The plan of this paper is as follows. In Section 2 we obtain the central result for the propagation of the object diffraction orders. In Section 3 we discuss the imaging condition and the Talbot effect for tapered GRIN media illuminated by a Gaussian beam; in that section we establish an analogy between the Talbot effect and the conventional lens-imaging formula and we derive the results for uniform illumination as a special case of nonuniform illumination. In Section 4 we determine the self-image distances for both types of illumination. In section 5 we apply the results on the Talbot effect to a particular case of a tapered GRIN medium with a divergent linear taper function and we analyze the dependence of the self-image distances on the taper function, the illumination, the wavelength, and the periodic object as well as the variation of transverse magnification with the axial localization of self-images. In Section 6 we present our conclusions.

2. EVALUATION OF THE COMPLEX AMPLITUDE DISTRIBUTION IN A TAPERED GRADIENT-INDEX MEDIUM FOR A PERIODIC OBJECT AND NONUNIFORM ILLUMINATION

Let us consider a tapered GRIN medium characterized by a transverse parabolic refractive index modulated by an axial index and whose refractive index is given by

$$n^2(x, z) = {n_0}^2[1 - g^2(z)x^2], \tag{1}$$

where n_0 is the index at the z optical axis and $g(z)$ is the taper function that describes the evolution of the transverse index along the z axis.

To evaluate the complex amplitude distribution in the tapered GRIN medium, we assume that a one-dimensional periodic object of infinite dimension is located at the input of this medium and is illuminated by a coherent nonuniform beam (Fig. 1). The one-dimensional periodic object will be represented as

$$T(x_0) = \sum_m a_m \exp\left(-i\frac{2\pi m x_0}{p}\right), \tag{2}$$

where p is the spatial period and a_m is the amplitude of the mth harmonic.

When the hybrid structure formed by the periodic object and the tapered GRIN medium is illuminated by a coherent nonuniform beam, the complex amplitude distribution on the periodic object located at $z = 0$ can be written as

$$\phi(x_0) = T(x_0)\psi_0(x_0), \tag{3}$$

where

$$\psi_0(x_0) = \left[\frac{w_0}{w(0)}\right]^{1/2} \exp[i\varphi(0)]\psi[x_0, U(0)] \tag{4}$$

is the complex amplitude distribution due to a Gaussian illumination of wavelength λ and

$$\psi[x_0, U(0)] = \exp\left[i\frac{\pi U(0){x_0}^2}{\lambda}\right] \tag{5}$$

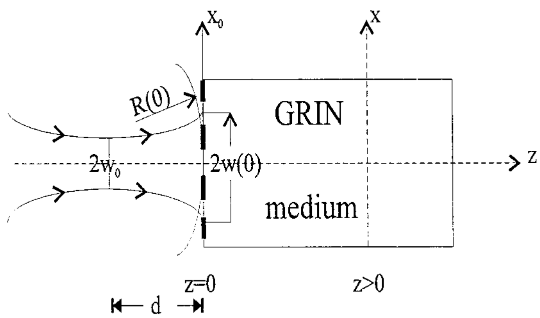

Fig. 1. Geometry for the evaluation of the complex amplitude distribution in a tapered GRIN medium due to a periodic object located at $z = 0$ and illuminated by a Gaussian beam.

is the quadratic phase factor of the cylindrical Gaussian beam. The beam parameters at a distance d from the waist plane of diameter $2w_0$ are given by the complex curvature

$$U(0) = \frac{1}{R(0)} + i\frac{\lambda}{\pi w^2(0)}, \tag{6}$$

$$\varphi(0) = \tan^{-1}(\lambda d/\pi w_0{}^2), \tag{7}$$

$R(0)$ and $w(0)$ being the radius of curvature and the beam half-width at $z = 0$, respectively.

The complex amplitude distribution $\phi(x; z)$ in the tapered GRIN medium, that is, at $z > 0$, is given by the integral equation[7]

$$\phi(x; z) = \int_{-\infty}^{+\infty} \phi(x_0) K(x, x_0; z) \mathrm{d}x_0, \tag{8}$$

K being the one-dimensional optical propagator of this medium expressed as

$$K(x, x_0; z) = \left[\frac{n_0}{i\lambda H_1(z)}\right]^{1/2} \exp(ikn_0 z) \exp\left\{i\frac{kn_0}{2H_1(z)}[x^2\dot{H}_1(z) + x_0{}^2 H_2(z) - 2xx_0]\right\}, \tag{9}$$

where H_1, H_2 and $\dot{H}_1$, $\dot{H}_2$ are the position and the slope of the axial and field rays at z, respectively, dot being the derivative with respect to z.[8]

Substituting Eqs. (3)–(5) and (9) into Eq. (8) and integrating, we arrive at the following complex amplitude distribution:

$$\phi(x; z) = \left[\frac{w_0}{w(0)F(z)}\right]^{1/2} \exp[i\varphi(z)] \times \exp\left[i\frac{\pi U(z)}{\lambda}x^2\right] \sum_m a_m \exp\left[-i\frac{2\pi m}{pF(z)}x\right] \times \exp\left[-i\frac{\pi\lambda m^2 H_1(z)}{n_0 p^2 F(z)}\right], \tag{10}$$

where $U(z)$ denotes the complex curvature of the Gaussian beam at z given by

$$U(z) = n_0 \dot{F}(z) F^{-1}(z) = n_0 \frac{\mathrm{d}\ln F(z)}{\mathrm{d}z}. \tag{11}$$

$F(z)$ and $\dot{F}(z)$ being

$$F(z) = \frac{U(0)H_1(z)}{n_0} + H_2(z), \tag{12}$$

$$\dot{F}(z) = \frac{U(0)\dot{H}_1(z)}{n_0} + \dot{H}_2(z), \tag{13}$$

$$\varphi(z) = \varphi(0) + kn_0 z. \tag{14}$$

Equation (10) is the central result of the present analysis; it enables us to discuss self-imaging phenomenon and represents the propagation of the object diffraction orders through a tapered GRIN medium.

3. SELF-IMAGING PHENOMENON: IMAGING CONDITION AND THE TALBOT EFFECT

The second exponential term of the summation in Eq. (10) is of basic importance to self-imaging. Now we analyze the imaging condition that this term cancels.

Let us consider axial distances $z = z_q$ in a tapered GRIN medium, where q is an integer, such that $H_1(z_q) = 0$. Then the phase term becomes unity, and Eq. (10) reduces to

$$\phi(x; z_q) = \left[\frac{w_0}{w(z_q)}\right]^{1/2} \exp[i\varphi(z_q)] \times \exp\left[i\frac{\pi U(z_q)}{\lambda}x^2\right] \sum_m a_m \exp\left[-i\frac{2\pi m}{p(z_q)}x\right], \tag{15}$$

where the beam half-width, complex curvature, and object period at z_q are given, respectively, by

$$w(z_q) = w(0)H_2(z_q), \tag{16a}$$

$$U(z_q) = U(0)\frac{\dot{H}_1(z_q)}{H_2(z_q)}, \tag{16b}$$

$$p(z_q) = pH_2(z_q). \tag{16c}$$

When Eq. (15) is compared with Eq. (3) and Eqs. (4)–(5) are taken into account, it follows that the complex amplitude distribution at distances z_q is a replica of the complex amplitude distribution at $z = 0$, as the well-known imaging condition $H_1(z_q) = 0$ for a GRIN medium is satisfied.[7]

On the other hand, in addition to this self-imaging phenomenon, periodic repetition along the optical z axis of lateral complex amplitude distribution occurs for the Talbot condition. The summation in Eq. (10) can be rewritten as

$$\sum_m a_m \exp\left\{-\frac{\pi\lambda m^2 \operatorname{Im}[F(z)]}{n_0 p^2 |F(z)|^2} H_1(z)\right\} \times \exp\left\{-\frac{2\pi m \operatorname{Im}[F(z)]}{p|F(z)|^2} x\right\} \times \exp\left\{-i\frac{\pi\lambda m^2 \operatorname{Re}[F(z)]}{n_0 p^2 |F(z)|^2} H_1(z)\right\} \times \exp\left\{-i\frac{2\pi m \operatorname{Re}[F(z)]}{p|F(z)|^2} x\right\}, \tag{17}$$

where $\operatorname{Re}[F(z)]$ and $\operatorname{Im}[F(z)]$ are the real and imaginary parts of $F(z)$:

$$\operatorname{Re}[F(z)] = \frac{H_1(z)}{n_0 R(0)} + H_2(z), \tag{18a}$$

$$\operatorname{Im}[F(z)] = \frac{H_1(z)}{z_R}, \tag{18b}$$

where z_R is the Rayleigh range that characterizes the Lorentzian profile of Gaussian beam intensity along the z axis of the GRIN medium, expressed as

$$z_R = \frac{\pi n_0 w^2(0)}{\lambda}, \tag{19a}$$

and $|F(z)|$ is the modulus of $F(z)$; that is,

$$|F(z)| = \left\{\left[\frac{H_1(z)}{n_0 R(0)} + H_2(z)\right]^2 + \frac{H_1{}^2(z)}{z_R{}^2}\right\}^{1/2}, \tag{19b}$$

which relates the beam half-width at $z > 0$ to the beam half-width at $z = 0$ as follows:

$$w(z) = w(0)|F(z)|. \tag{19c}$$

The first two exponential terms and the next two phase terms of Eq. (17) describe amplitude and phase changes of the diffraction orders along the axial and the transverse directions, respectively. In particular, the third term represents the phase changes of the diffraction orders with axial distance z. When this term becomes unity, all diffraction orders are cophasial and are reinforced at distances z_ν, fulfilling the relation

$$\frac{\lambda \operatorname{Re}[F(z_\nu)]}{n_0 p^2 |F(z_\nu)|^2} H_1(z_\nu) = 2\nu$$

or

$$\frac{2\nu p^2}{\lambda} = \frac{\operatorname{Re}[F(z_\nu)]}{n_0 |F(z_\nu)|^2} H_1(z_\nu), \tag{20}$$

called the Talbot condition, where ν is an integer referred to as the self-image number.

In this case, the last term of Eq. (17), which describes the phase changes of the diffraction orders along the lateral direction, can be expressed as

$$\exp\left[-i\frac{2\pi m}{p(z_\nu)} x\right], \tag{21}$$

where the period of self-images

$$p(z_\nu) = p M_t{}^g(z_\nu) \tag{22}$$

carries information about the transverse magnification of a periodic object due to Gaussian illumination, given by

$$M_t{}^g(z_\nu) = \frac{|F(z_\nu)|^2}{\operatorname{Re}[F(z_\nu)]} = \frac{w^2(z_\nu)}{w^2(0)\operatorname{Re}[F(z_\nu)]}, \tag{23}$$

where Eq. (19c) has been used.

Note that the factor of 2 in Eq. (20) can be omitted. In such a case, when ν is an odd integer, images present a transverse shift with respect to the object.

Equation (20), which determines the Talbot condition for a tapered GRIN medium, may also be expressed as

$$\frac{2\nu p^2}{\lambda d_0{}^2} = \frac{1}{R(0)} - \frac{1}{z'_\nu}, \tag{24}$$

where

$$z'_\nu = R(0) + z_\nu = \frac{|F(z_\nu)|^2}{\operatorname{Re}[F(z_\nu)]} R(0), \tag{25a}$$

$$z_\nu = \frac{|F(z_\nu)|^2 - \operatorname{Re}[F(z_\nu)]}{\operatorname{Re}[F(z_\nu)]} R(0), \tag{25b}$$

$$d_0{}^2 = \frac{H_1(z_\nu)}{n_0 z_\nu} R^2(0). \tag{25c}$$

Therefore the Talbot effect can be considered the equivalent effect of an apparent lens of multifocal length

$$f_\nu = \frac{\lambda d_0{}^2}{2\nu p^2}, \tag{26}$$

situated at the curvature center of the Gaussian beam that illuminates a periodic object, as shown in Fig. 2(a). The transverse magnification of the lens is now given by the ratio of the distances of the observation and the object planes from the curvature center of the beam; that is,

$$\frac{z'_\nu}{R(0)} = \frac{|F(z_\nu)|^2}{\operatorname{Re}[F(z_\nu)]}, \tag{27}$$

which coincides with Eq. (23).

If the incident Gaussian beam has its waist located on the input, in other words, if the periodic object is illuminated by a plane Gaussian beam $[R(0) \to \infty]$, Eq. (12) becomes

$$F^{\text{pg}}(z_\nu) = H_2(z_\nu) + i\frac{H_1(z_\nu)}{z_R^{\text{pg}}}, \tag{28}$$

where

$$z_R^{\text{pg}} = \frac{\pi n_0 w_0{}^2}{\lambda}. \tag{29}$$

Inserting Eq. (28) into Eq. (20), the Talbot condition for plane Gaussian illumination reduces to

$$\frac{2\nu p^2}{\lambda} = \frac{H_1(z_\nu) H_2(z_\nu)}{n_0 |F^{\text{pg}}(z_\nu)|^2} \tag{30}$$

and transverse magnification Eq. (23) becomes

$$M_t^{\text{pg}}(z_\nu) = \frac{|F^{\text{pg}}(z_\nu)|^2}{H_2(z_\nu)}. \tag{31}$$

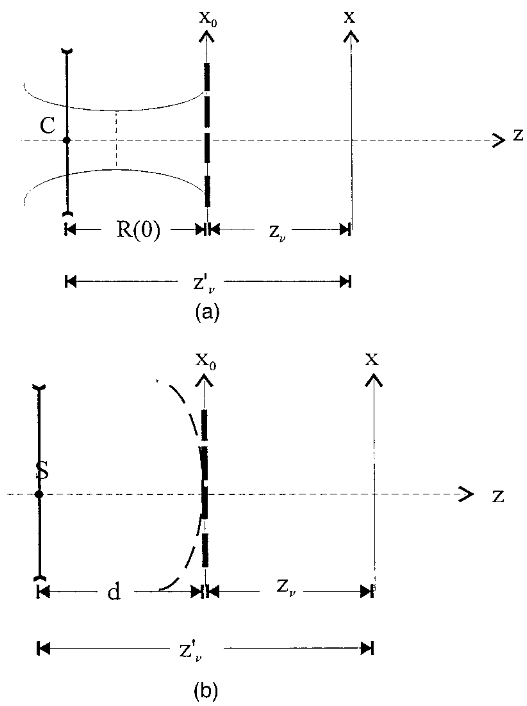

Fig. 2. Equivalent optical system for the Talbot effect in a tapered GRIN medium under divergent (a) Gaussian illumination and (b) uniform illumination.

Likewise, Eq. (17) includes uniform illumination as a special case. For this illumination $R(0) \to d$, $w(0) \to \infty$, and $z_R \to \infty$. Under these conditions, $F(z)$ becomes real:

$$F^u(z) = H_2(z) + \frac{H_1(z)}{n_0 d}. \tag{32}$$

The Talbot condition and the equivalent lens equation are now given by

$$\frac{2\nu p^2}{\lambda} = \frac{d H_1(z_\nu)}{n_0 d H_2(z_\nu) + H_1(z_\nu)}, \tag{33}$$

$$\frac{2\nu p^2}{\lambda d_{u0}^2} = \frac{1}{d} - \frac{1}{z'_\nu}, \tag{34}$$

where

$$z'_\nu = d + z_\nu = F^u(z_\nu) d, \tag{35a}$$

$$z_\nu = [F^u(z_\nu) - 1] d, \tag{35b}$$

$$d_{u0}^2 = \frac{H_1(z_\nu)}{n_0 z_\nu} d^2. \tag{35c}$$

Equation (34) can be regarded as the equation of a lens of multifocal distance $f_\nu{}^u = \lambda d_{u0}^2 / 2\nu p^2$ located at the linear source illuminating the object as shown in Fig. 2(b) and whose lateral magnification is written as

$$M_t{}^u(z_\nu) = \frac{z'_\nu}{d} = F^u(z_\nu), \tag{36}$$

which coincides with Eq. (32). This interpretation is in full agreement with the geometrical shadow principle.[9]

The Talbot condition and transverse magnification for uniform plane illumination are obtained from Eqs. (32) and (33) by putting $d \to \infty$; that is,

$$\frac{2\nu p^2}{\lambda} = \frac{H_1(z_\nu)}{n_0 H_2(z_\nu)}, \tag{37}$$

$$M^{\mathrm{pu}}(z_\nu) = H_2(z_\nu). \tag{38}$$

When Eq. (38) is compared with Eq. (16c), it follows that the transverse magnification is the same as for nonuniform illumination when the imaging condition is fulfilled.

Finally, for free space where $g(z) \to 0$, $n_0 = 1$, $H_1(z) \to z$, and $H_2(z) \to 1$, the Talbot condition and the equivalent lens equation become, respectively,

$$\frac{2\nu p^2}{\lambda} = \frac{z_\nu z'_\nu}{R(0)} \left[\frac{w(0)}{w(z_\nu)} \right]^2, \tag{39}$$

$$\frac{2\nu p^2}{\lambda R^2(0)} = \frac{1}{R(0)} - \frac{1}{z'_\nu}, \tag{40}$$

with $z'_\nu = R(0) + z_\nu$ for Gaussian illumination, and

$$\frac{2\nu p^2}{\lambda} = \frac{z_\nu d}{z'_\nu}, \tag{41}$$

$$\frac{2\nu p^2}{\lambda d^2} = \frac{1}{d} - \frac{1}{z'_\nu}, \tag{42}$$

with $z'_\nu = d + z_\nu$ for uniform illumination.[9]

4. SELF-IMAGE DISTANCES

The self-image distances correspond to the lengths z_ν of a tapered GRIN medium for which the input complex amplitude distribution is periodically repeated along the optical z axis of this medium.

The axial localization of self-images can be obtained from the Talbot condition [Eq. (20)], if we take into account that the position of the axial and field rays are given by[7]

$$H_1(z) = [g_0 g(z)]^{-1/2} \sin\left[\int_0^z g(z')\mathrm{d}z' \right], \tag{43}$$

$$H_2(z) = \left[\frac{g_0}{g(z)} \right]^{1/2} \cos\left[\int_0^z g(z')\mathrm{d}z' \right], \tag{44}$$

g_0 being the value of $g(z)$ at $z = 0$.

Equations (43) and (44) can be rewritten as

$$H_1(z) = \frac{u(z)}{\{g_0 g(z)[1 + u^2(z)]\}^{1/2}}, \tag{45}$$

$$H_2(z) = \left\{ \frac{g_0}{g(z)[1 + u^2(z)]} \right\}^{1/2}, \tag{46}$$

where

$$u(z) = \tan\left[\int_0^z g(z')\mathrm{d}z' \right]. \tag{47}$$

Substituting Eqs. (45), (46), (19b), and (18a) into Eq. (20), we have the following second-order equation:

$$a u^2(z_\nu) + b u(z_\nu) + c = 0, \tag{48}$$

with

$$a = \frac{1}{g_0}\left[2\nu p^2 n_0\left(\frac{1}{n_0{}^2 R^2(0)} + \frac{1}{z_R{}^2}\right) - \frac{\lambda}{n_0 R(0)}\right], \tag{49}$$

$$b = \frac{4\nu p^2}{R(0)} - \lambda, \tag{50}$$

$$c = 2\nu p^2 n_0 g_0. \tag{51}$$

Solution of Eq. (48) is given by

$$u(z_\nu) = \frac{n_0 g_0 R^2(0) z_R{}^2 \lambda}{4\nu p^2[z_R{}^2 + n_0{}^2 R^2(0)] - 2\lambda R(0) z_R{}^2} \times \left[1 - \frac{4\nu p^2}{R(0)\lambda} - \sqrt{1 - \left(\frac{4 n_0 \nu p^2}{z_R \lambda}\right)^2}\right]. \tag{52}$$

Note that negative sign for the square root in Eq. (52) has been taken for recovering the initial condition $u(z_\nu) = 0$ for $\nu = 0$, which indicates the position of the periodic object at the input.

From Eq. (52) it follows that real values of $u(z_\nu)$ are achieved as the requirement

$$\frac{4 n_0 \nu p^2}{z_R \lambda} \leqslant 1 \quad \text{or} \quad \nu \leqslant \frac{\pi}{4}\left[\frac{w(0)}{p}\right]^2 \tag{53}$$

is fulfilled, where Eq. (19a) has been used.

Then the integer ν is limited by a square relationship between the beam half-width at $z = 0$ and the period of the object. One or more self-images are obtained as $w(0)/p \geqslant \sqrt{4/\pi}$.

If the periodic object is illuminated by a plane Gaussian beam, Eq. (52) reduces to

$$u^{\mathrm{pg}}(z_\nu) = \frac{g_0 \lambda (z_R^{\mathrm{pg}})^2}{4\nu p^2 n_0}\left[1 - \sqrt{1 - \left(\frac{4 n_0 \nu p^2}{z_R^{\mathrm{pg}} \lambda}\right)^2}\right], \tag{54}$$

with the requirement

$$\nu \leqslant \frac{\pi}{4}\left(\frac{w_0}{p}\right)^2, \tag{55}$$

where Eq. (29) has been used.

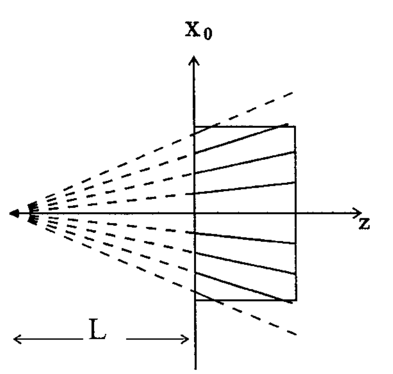

Fig. 3. Equi-index lines for a divergent linear tapered GRIN medium.

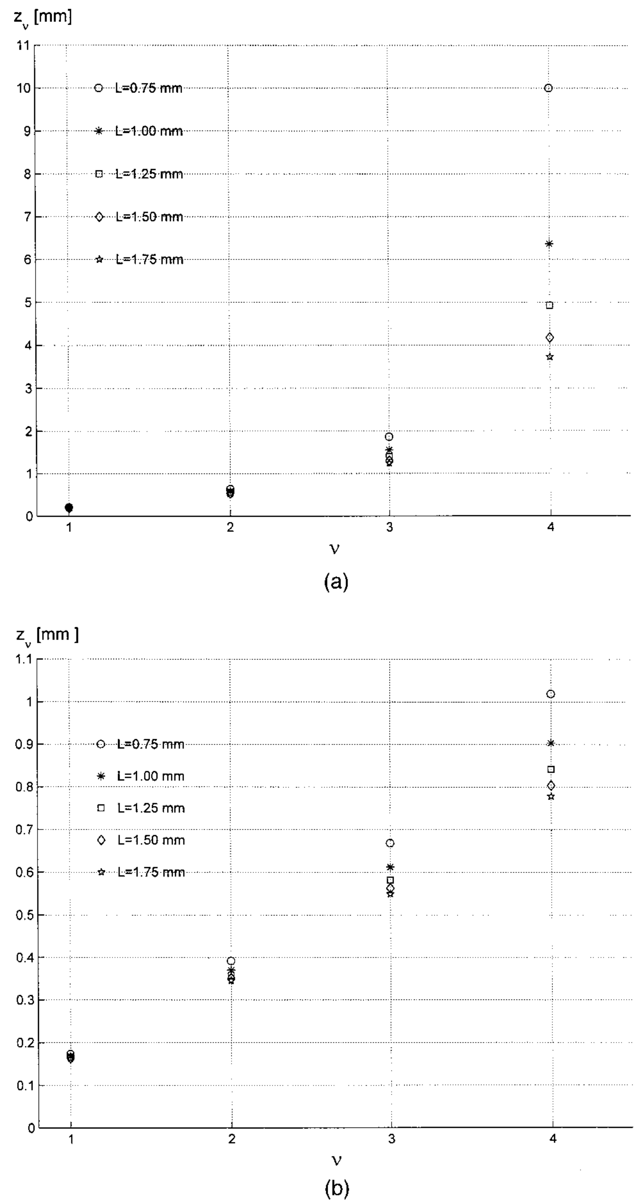

Fig. 4. Dependence of the self-image distances on L for (a) non-uniform illumination and (b) uniform illumination. Calculations have been made for $\lambda = 0.7\,\mu\mathrm{m}$, $p = 6\,\mu\mathrm{m}$, $d = 10\,\mathrm{mm}$, $R(0) = 0.65\,\mathrm{mm}$, and $w_0 = 7\,\mu\mathrm{m}$.

For uniform illumination Eq. (52) becomes[10]

$$u^{u}(z_\nu) = \frac{2\nu p^2 n_0 g_0 d}{d\lambda - 2\nu p^2} \tag{56}$$

and in this case no requirement on ν is obtained, since $w(0) \rightarrow \infty$.

Likewise, Eq. (56) reduces to

$$u^{\mathrm{pu}}(z_\nu) = \frac{2\nu p^2 n_0 g_0}{\lambda} \tag{57}$$

for uniform plane illumination.

Finally, in a transverse quadratic medium for which $g(z)$ is a constant g_0, the axial location of self-images for uniform plane illumination is given by

$$z_\nu = \frac{1}{g_0}\tan^{-1}\left(\frac{2\nu p^2 n_0 g_0}{\lambda}\right), \tag{58}$$

where Eqs. (47) and (57) have been used. Equation (58) coincides with the result obtained by Agarwal.[4]

5. PARTICULAR CASE: A GRADIENT-INDEX MEDIUM WITH A DIVERGENT LINEAR TAPER FUNCTION

Now we study self-image locations for a particular tapered GRIN medium with a divergent linear taper function given by

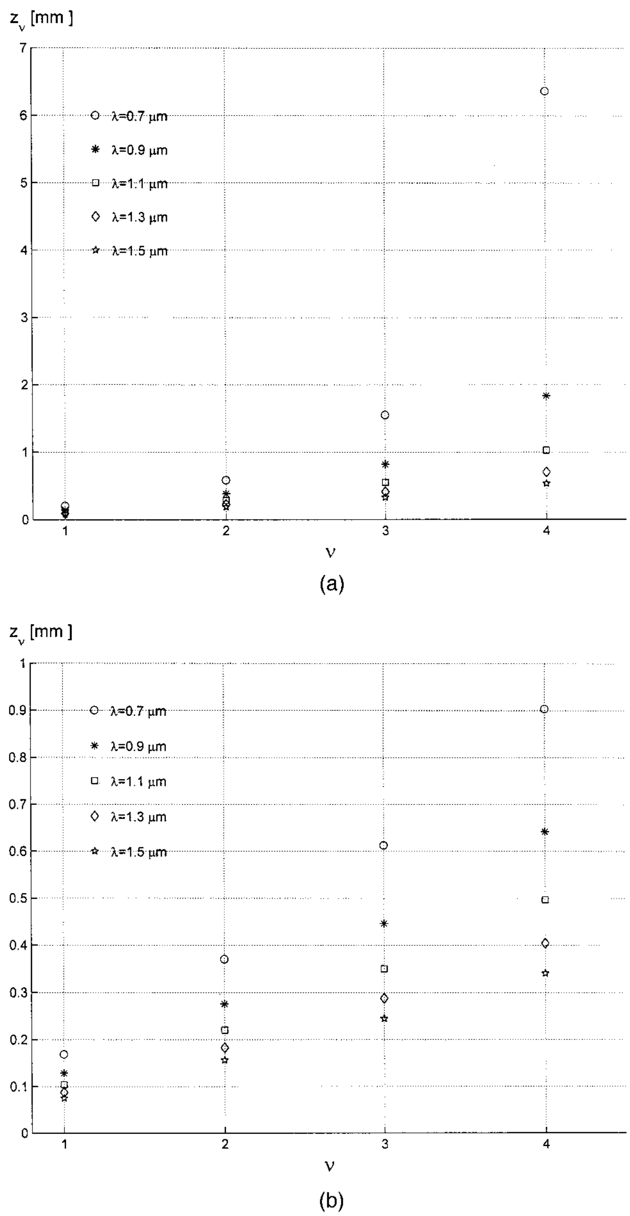

Fig. 5. Dependence of the self-image distances on λ for (a) nonuniform illumination and (b) uniform illumination. Calculations have been made for $p = 6\ \mu\text{m}$, $L = 1$ mm, $d = 10$ mm, $R(0) = 0.65$ mm, and $w_0 = 7\ \mu\text{m}$.

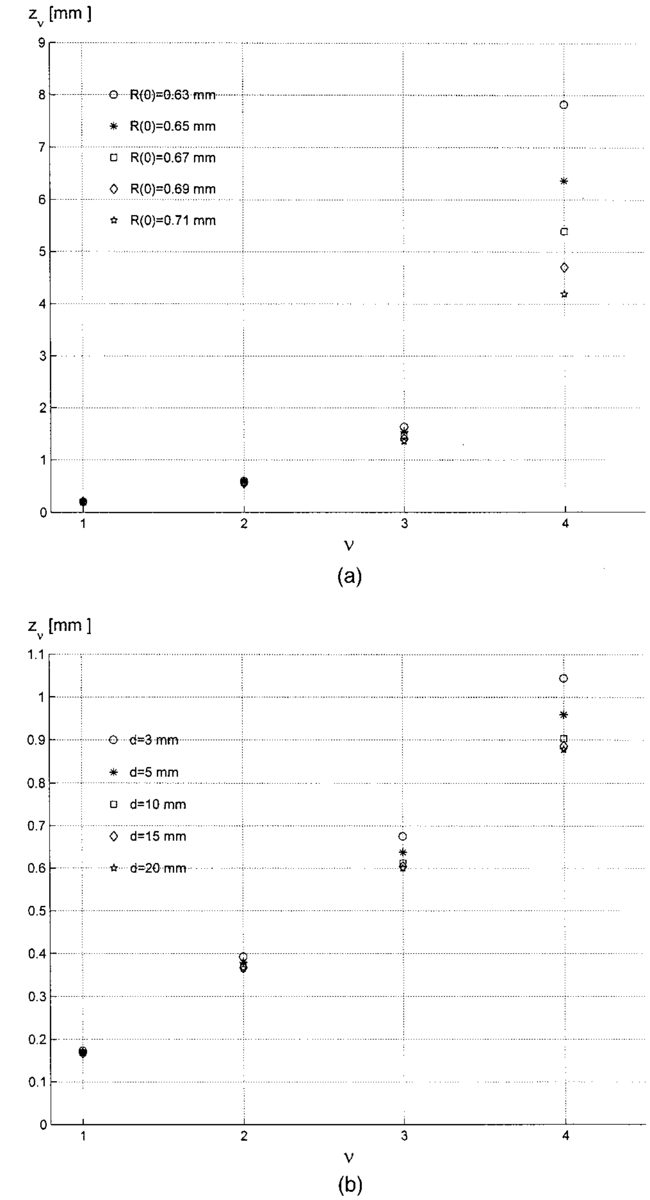

Fig. 6. Dependence of the self-image distances on the curvature radius $R(0)$ and d for (a) nonuniform illumination and (b) uniform illumination, respectively. Calculations have been made for $\lambda = 0.7\ \mu\text{m}$, $L = 1$ mm, $p = 6\ \mu\text{m}$, and $w_0 = 7\ \mu\text{m}$.

$$g(z) = \frac{g_0}{1 + (z/L)}, \tag{59}$$

L being the distance from $z = 0$ to the common apex of the equi-index lines (Fig. 3).

For this case the axial location of self-images becomes

$$z_\nu = L\left[\exp\left(\frac{1}{g_0 L}\tan^{-1}\left\{\frac{n_0 g_0 R^2(0) z_R{}^2 \lambda}{4\nu p^2[z_R{}^2 + n_0{}^2 R^2(0)] - 2\lambda R(0) z_R{}^2} \times \left[1 - \frac{4\nu p^2}{R(0)\lambda} - \sqrt{1 - \left(\frac{4 n_0 \nu p^2}{z_R \lambda}\right)^2}\right]\right\}\right) - 1\right] \tag{60}$$

for nonuniform illumination and

$$z_\nu = L\left\{\exp\left[\frac{1}{g_0 L}\tan^{-1}\left(\frac{2\nu p^2 n_0 g_0 d}{\lambda d - 2\nu p^2}\right)\right] - 1\right\} \qquad (61)$$

for uniform illumination, where Eqs. (52) and (56) have been used.

Equations (60) and (61) represent the discrete axial distances for which the phase coincidence between the interfering diffraction orders occurs.

We will analyze the dependence of the self-image distances on the taper function, the illumination, the wavelength, and the periodic object, in this kind of tapered GRIN medium.

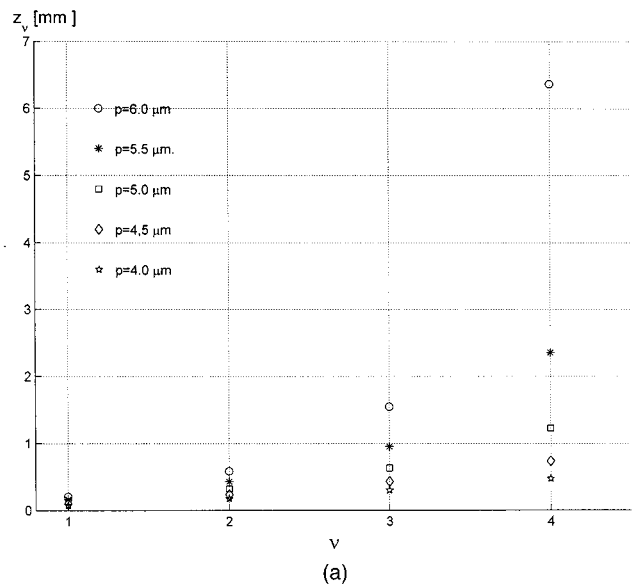

Fig. 7. Dependence of the self-image distances on p for (a) nonuniform illumination and (b) uniform illumination. Calculations have been made for $\lambda = 0.7\,\mu\text{m}$, $L = 1$ mm, $d = 10$ mm, $R(0) = 0.65$ mm, and $w_0 = 7\,\mu\text{m}$.

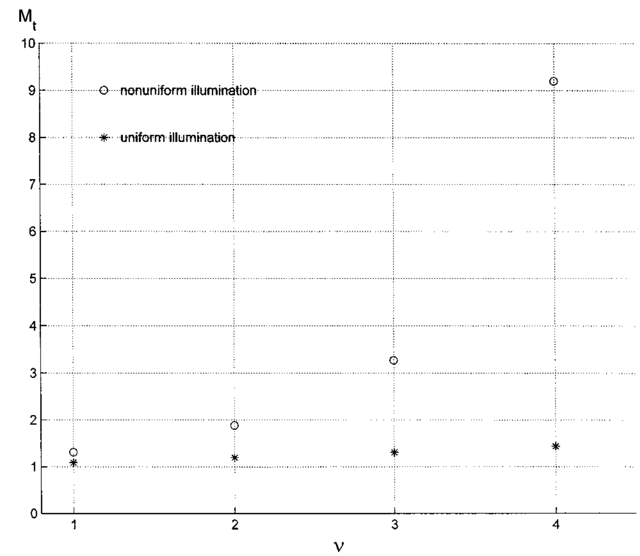

Fig. 8. Transverse magnification versus self-image number for nonuniform illumination and uniform illumination. Calculations have been made for $n_0 = 1.5$, $g_0 = 0.01\,\text{mm}^{-1}$, $\lambda = 0.7\,\mu\text{m}$, $L = 1$ mm, $p = 6\,\mu\text{m}$, $d = 10$ mm, $R(0) = 0.65$ mm, and $w_0 = 7\,\mu\text{m}$.

Figures 4–7 show the axial location of self-images versus self-image number for different values of taper parameter, wavelength, curvature radius of illumination, and period of the object.

Figure 4 shows dependence on L for (a) nonuniform illumination and (b) uniform illumination. For a given L the interval between consecutive self-images increases with ν, and, in contrast, self-image locations decrease with L for a given ν.

A similar behavior is shown in Figs. 5 and 6 for the dependence of the self-image distances on the wavelength and the curvature radius of the beam for (a) nonuniform illumination and (b) uniform illumination, respectively.

Figure 7 represents the self-image evolution for several values of the object period. For a given p the interval between consecutive self-images increases with ν, and, likewise, self-image locations increase with p for a given ν.

Figure 8 shows variation of transverse magnification with self-image number for nonuniform and uniform illumination. The first four self-image positions are obtained at $z_1 = 0.202$, $z_2 = 0.584$, $z_3 = 1.549$, and $z_4 = 6.365$ mm for Gaussian illumination and at $z_1 = 0.169$, $z_2 = 0.370$, $z_3 = 0.612$, and $z_4 = 0.903$ mm for uniform illumination. A very slow dependence on self-image number is obtained for uniform illumination; on the other hand, for Gaussian illumination it is possible to obtain a pseudolens that yields images with significant lateral magnification of the periodic object for an object–image distance of ~6 mm.[11]

Figures 4–8 indicate that the dependence of self-image locations on taper profile, wavelength, curvature radius, and period of the object as well as variation of transverse magnification with self-image number for nonuniform illumination is higher than those for uniform illumination. In all cases, calculations have been made for $n_0 = 1.5$ and $g_0 = 0.01\,\text{mm}^{-1}$.

Finally, it is easy to prove that the reverse behavior occurs in a convergent linear tapered GRIN medium whose taper function is given by

$$g(z) = \frac{g_0}{1 - (z/L)}. \tag{62}$$

6. CONCLUSIONS

A generalization of the Talbot effect to the case of a tapered GRIN medium for nonuniform and uniform illumination has been considered, and an analogy between the self-imaging phenomenon and the conventional lens-imaging formula has been derived. Self-image distances for both types of illumination have been evaluated. Results on the Talbot effect have been applied to a particular case of a tapered GRIN medium with a divergent linear taper function to show the dependence of self-image distances on taper function, illumination, wavelength, and periodic object as well as the variation of transverse magnification with self-image number.

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