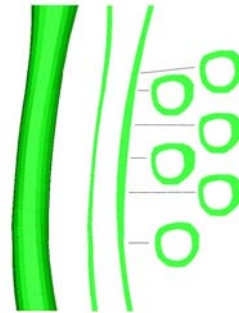


GEMM

Group of Structural Mechanics
and Material Modelling

MODELADO MATEMÁTICO DE LA MECANOBIOLOGÍA ÓSEA

José Manuel García Aznar
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instituto de investigación
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Universidad de Zaragoza

Santiago de Compostela, Julio 2010

I3A - Definition

WHAT IS THE I3A?

- An **Interdisciplinary University Research Institute** with wide-ranging activities in various branches of engineering.
- We cover the **complete research cycle** from the basics to prototype design.
- The **generation of new knowledge** is one of our essential activities.



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I3A - Definition

What is our purpose?

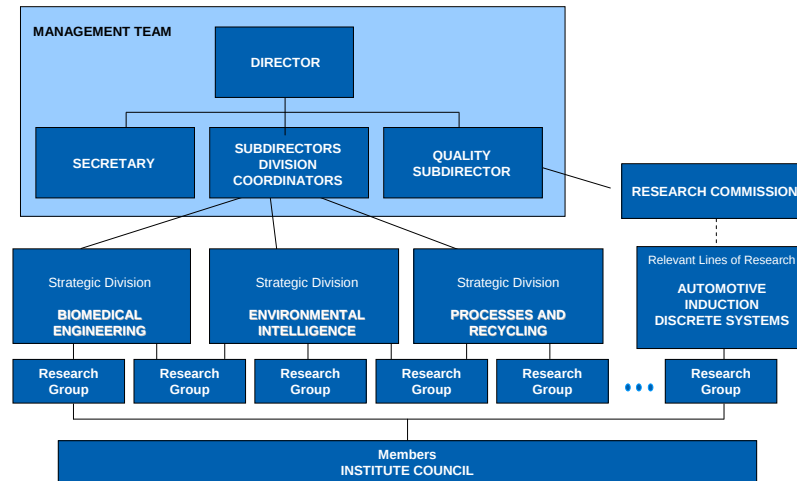
- Promoting and improving **scientific research** in the continuing search for excellence.
- Supporting economic development through **technology transfer** to the industrial sector.
- High quality **training** at postgraduate and doctoral level.
- **Promoting** science and technology in society.

I3A - Composition

RESEARCH PERSONNEL

- 287 PhD members
- 300 associate members (research fellows, hired staff and graduate lectures)
- 28 research groups recognised by the Government of Aragon
 - 6 excellence level
 - 13 consolidated level
 - 7 consolidated for applied research
 - 2 emergent level

I3A – Structure



I3A – DIVISIONS

IB



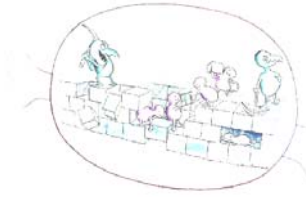
BIOMEDICAL ENGINEERING DIVISION

Technology applications for health management and improvement

1. Tissue engineering and biomaterials
2. Biological modelling and biomechanics
3. Imaging, signal and biomedical instruments
4. Healthcare and prevention technologies

OVERVIEW

- ***Bone properties***
- Role of mechanical factors: bone mechanobiology
- Methods of Science: computer simulation
- Modelling Bone Mechanobiology:
 - Bone Remodelling
 - Bone Healing
 - Bone Distraction
 - Bone Tissue Engineering
- Final conclusions



BONE PROPERTIES

- Different functions

FUNCTIONS OF BONE

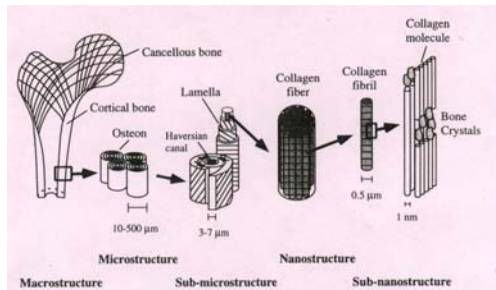


- **Structural support**
- **Protection** (e.g. skull encloses brain)
- **Movement** (transmission of forces, in fact, muscles use bones as levers)
- **Mineral storage** (calcium, phosphate)
- **Blood cell production** (in bone marrow)

BONE PROPERTIES

- Different functions
- Bone is a **hierarchical structural composite** material:

BONE TISSUE: A COMPLEX MATERIAL

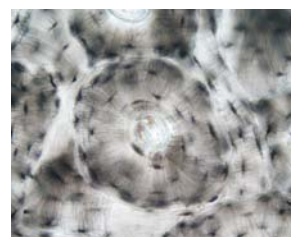


(Rho et al., 1998)

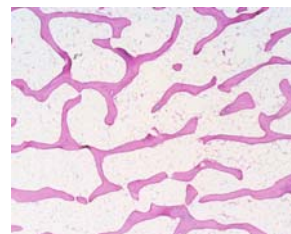
- Materials with structural hierarchy
- Porous, heterogeneous and anisotropic material
- Ideal material properties: high stiffness, strength and fracture toughness and low weight
- It has been designed, as all living tissues, by the blind force of natural selection

CORTICAL AND TRABECULAR BONE

- Two different bone tissues: cortical and trabecular
- Both have the same composition and structure
- Cortical bone has lower porosity (10 %) than trabeculae bone (50-90 %)
- Different distribution implies different mechanical properties



Cortical bone



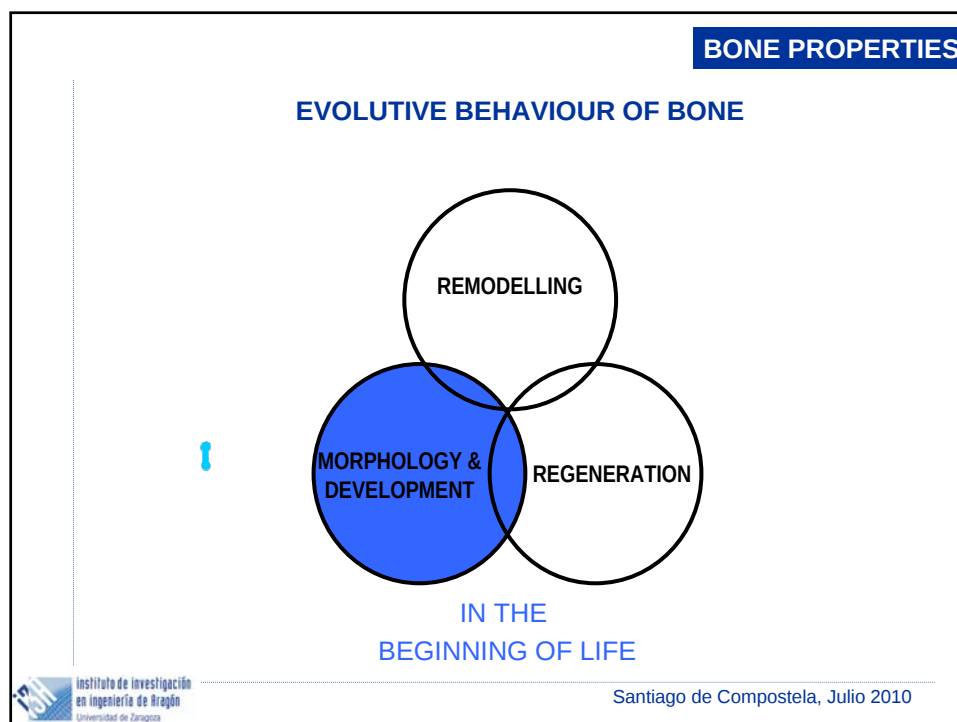
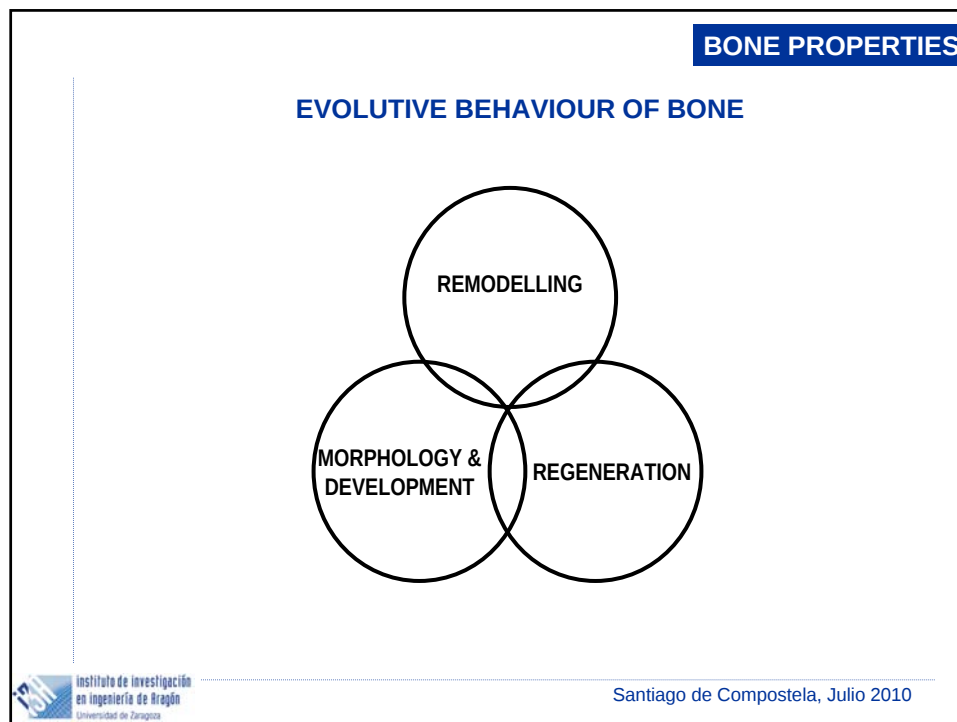
Trabecular bone

BONE COMPOSITION

- Matrix:
 - It is composed with an organic and inorganic component with water
 - Organic part (mainly collagen) defines the bone shape and tensile strength
 - Inorganic part (mineral) defines the compressive strength
- Cells: responsible of the modification of the structure and properties of tissues (2% of the bone mass)
 - Osteoblasts (surface)
 - Osteoclasts (surface)
 - Bone lining cells (surface)
 - Osteocytes (inside)

BONE PROPERTIES

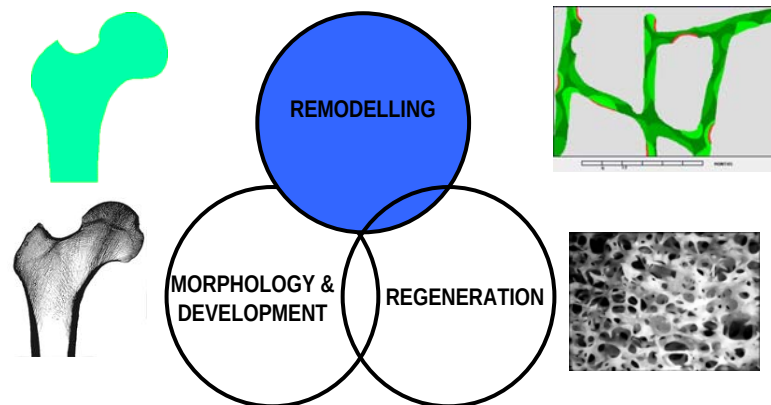
- Different functions
- Bone is a **hierarchical structural composite** material:
- It is an **evolutive (adaptative)** material able to modify its internal properties, size and shape in function of the environment that is supporting.



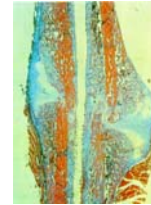
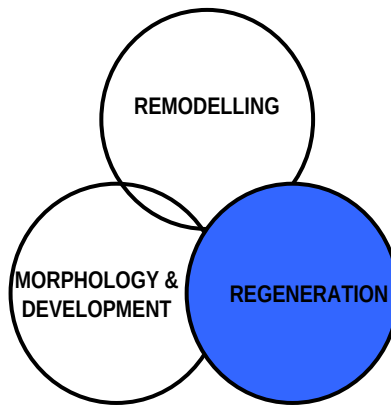
- Construction and reconstruction of tissues is the combined result of two factors: phylogenetic and ontogenetic processes.
- Phylogenetic processes (species evolution) involve random genetic variation and natural selection of the best fitted.
- Ontogenetic (individual) adaptation is accomplished by appropriate mechano-regulatory rules.
- The mechano-regulatory process can be evolved themselves, so epigenetic (environmental) factors can have also important influence in long-term evolution of species. (Carter, D.R., Mikic, B. and Padian, K. "Epigenetic mechanical factors in the evolution of long bone epiphyses". *Zool. J. Linnean Soc.* 123 (1998) 163-178)
- These processes are controlled by a combination of genetic and epigenetic factors (Edelman, *Topobiology. An Introduction to Molecular Embriology*. Basic Books, 1988).
- Genes direct the formation of the basic building blocks, including proteins, extracellular matrix and adhesion molecules.
- Epigenetic factors, including chemical agents, and mechanical strain influence which genes are expressed and how these blocks are assembled into tissues.

BONE PROPERTIES

EVOLUTIVE BEHAVIOUR OF BONE



ALL THE LIFETIME

EVOLUTIVE BEHAVIOUR OF BONE

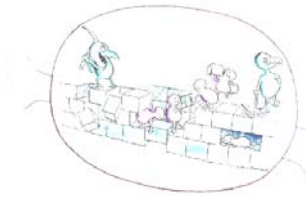
AFTER A FRACTURE

BONE PROPERTIES

- Different functions
- Bone is a **hierarchical structural composite** material:
- It is an **evolutive (adaptative)** material able to modify the properties and geometry in function of the environment that is supporting.
- Although the **biology** of these evolutive processes characteristic of these living tissues is mainly controlled by **biochemical** factors, the **mechanical** effects can also regulate them.

OVERVIEW

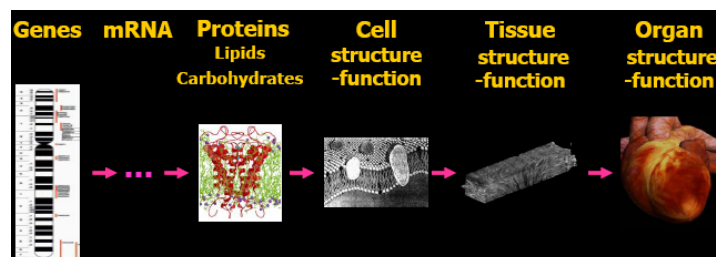
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BONE MECHANOBIOLOGY

MECHANICAL FACTORS CAN REGULATE BIOLOGY

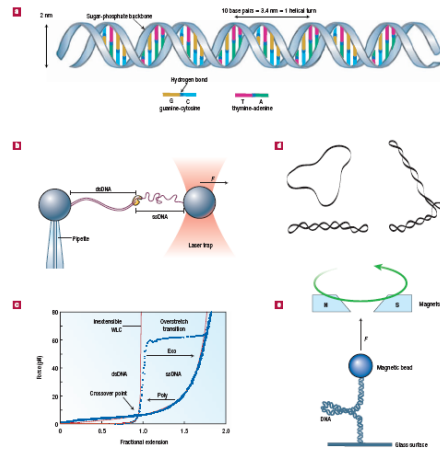
- Mechanical forces modulate morphological and structural fitness of the skeletal tissues (bone, cartilage, ligament and tendon) (van der Meulen and Huiskes, 2002)
- Mechanical forces may act within tissues to regulate biological processes at different spatial scales



(Hunter and Borg, 2003)

BONE MECHANOBIOLOGY

At molecular level, e.g., DNA mechanics

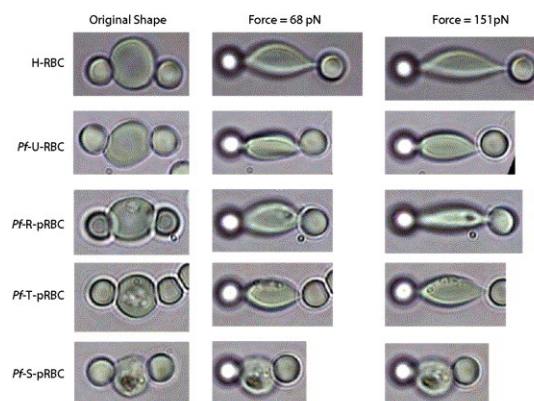


Bao and Suresh, *Nature Materials* 2003

Santiago de Compostela, Julio 2010

BONE MECHANOBIOLOGY

At cellular level, e.g., mechanical properties of healthy and infected/abnormal cells



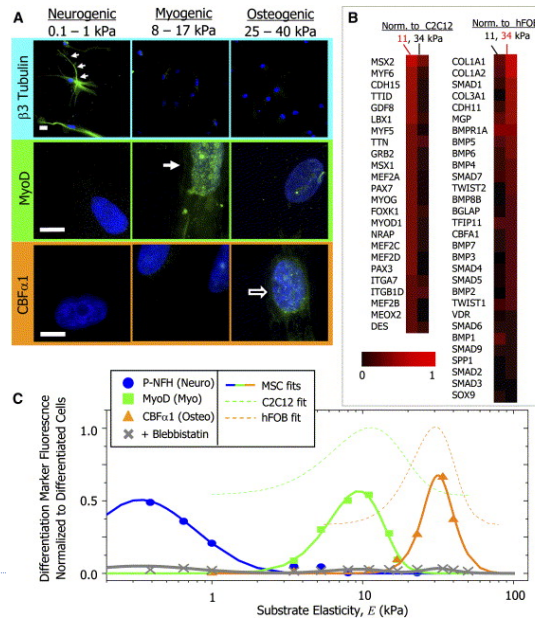
Suresh et al., *Acta Biomaterialia* 2005

Santiago de Compostela, Julio 2010

BONE MECHANOBIOLOGY

Cell-extracellular
matrix interaction,
e.g., cellular
differentiation
dependent on the
matrix elasticity

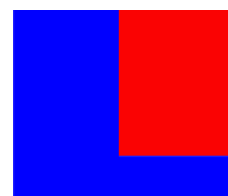
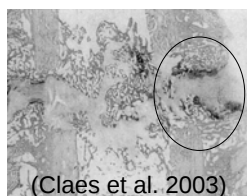
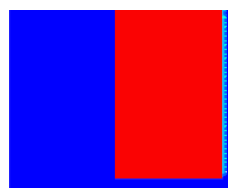
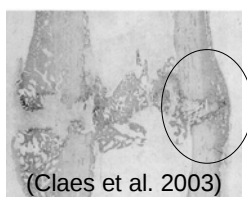
Engler et al., *Cell* 2006



BONE MECHANOBIOLOGY

At tissue level, e.g., bone healing

0 días



(Gómez-Benito et al, 2005)

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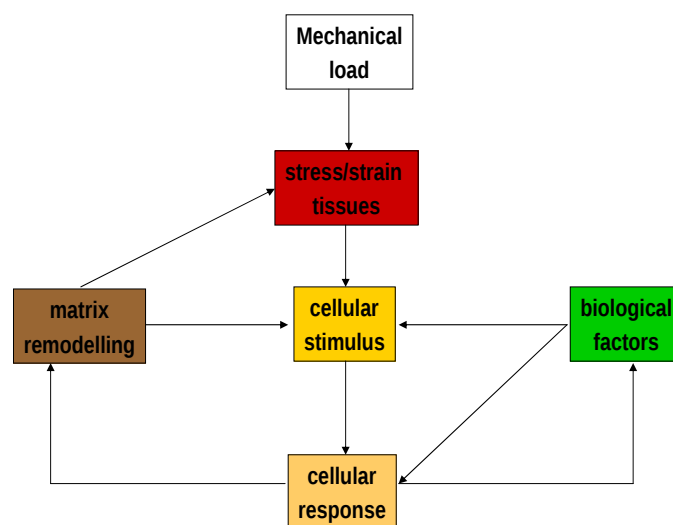
At organ level, e.g., mouse femora



(Chalmers & Ray, 1962)

Figure 1 Two mouse femora illustrate the effects of genetics and the effects of normal mechanical loading and genetics. (a) Normal morphology of a mouse femur developed within a normal functional loading environment. (b) Morphology of a mouse femur deprived of its normal mechanical loading. This femur was produced by transplanting the primordial template to the spleen and allowing it to develop there. The experiment thus provides insight into the nature of the primordial template for the mouse femur. From Chalmers & Ray (9).

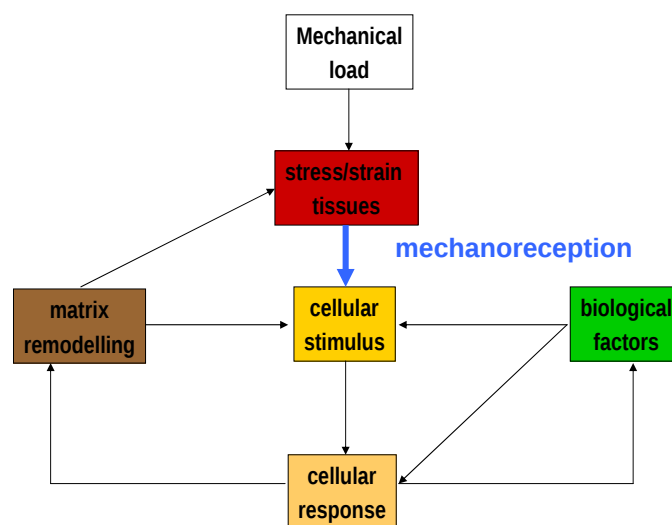
HOW CAN BONE CELLS SENSE THE MECHANICAL LOAD?



HOW CAN BONE CELLS SENSE THE MECHANICAL LOAD?

- The mechanosensory mechanisms in bone include:
 - The mechanoreception system that identifies the process that transmits the informational content of an extracellular mechanical stimulus to a receptor cell.

HOW CAN BONE CELLS SENSE THE MECHANICAL LOAD?



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- 
- instituto de investigación
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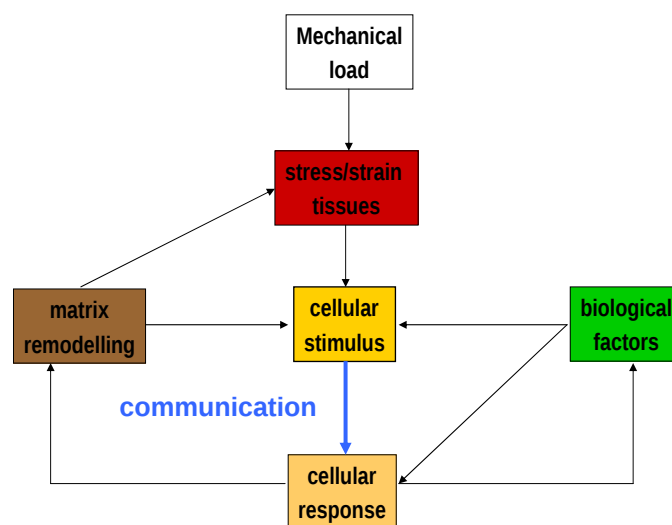
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HOW CAN BONE CELLS SENSE THE MECHANICAL LOAD?

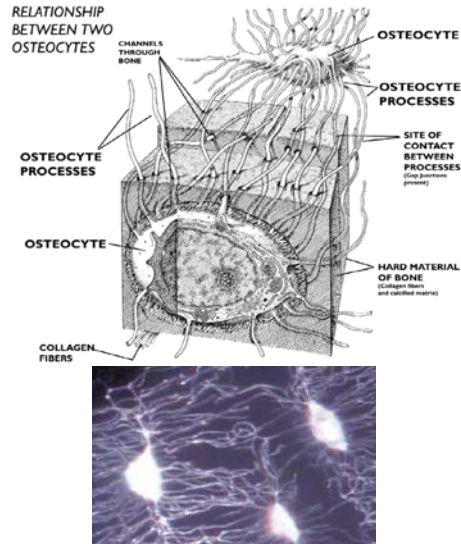
- The mechanosensory mechanisms in bone include:
 - The mechanoreception system that identifies the process that transmits the informational content of an extracellular mechanical stimulus to a receptor cell.
 - The mechanotransduction system that describes the process that transforms the mechanical stimulus content into an intracellular signal.
 - The communication system that transmits that signal to the effector cells (osteoblasts and osteoclasts).

HOW CAN BONE CELLS SENSE THE MECHANICAL LOAD?



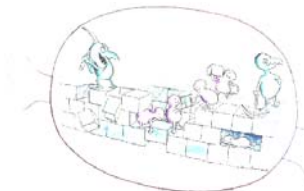
COMMUNICATION SYSTEM

- All bone cells except osteoclasts are extensively interconnected by the cell process of the osteocytes forming a syncytium or connected cellular network (CNN). This syncytium provides a route for rapid passage of ions and signal molecules. (Cowin, S.C. and Moss, M.L. in *Textbook of Tissue Engineering*. Lanza, R., Langer, R., Chick, W. (eds.). Academic Press, 723-738, 2000).



OVERVIEW

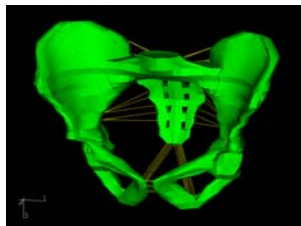
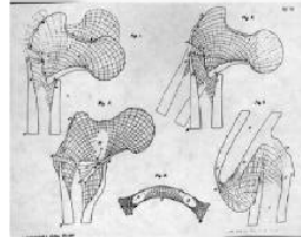
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- **Final conclusions**



METHODS OF SCIENCE

- Theory
- Empiry
- Computer Simulation

J. Kelly, "The Third Culture", Science 1998



METHODS OF SCIENCE: Computer Simulation

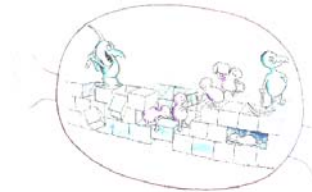
- Computational Modelling:
“A computational model is a simplified and mathematical representation of a system for the purposes of analysing the behaviour of this system under different conditions”

METHODS OF SCIENCE: Computer Simulation

- Why computational modelling in mechanobiology?
 - Great difficulty of many experimental tests and practical impossibility of their customisation.
 - Possibility of comparing many different conditions and factors: mechanical, biological, pharmacological, etc.
 - Simulations closer to reality in some cases due to the possibility of considering and controlling factors that cannot be controlled or measure in experimental essays.
 - Economical impact: low cost.
 - Reduction of animal experiments

OVERVIEW

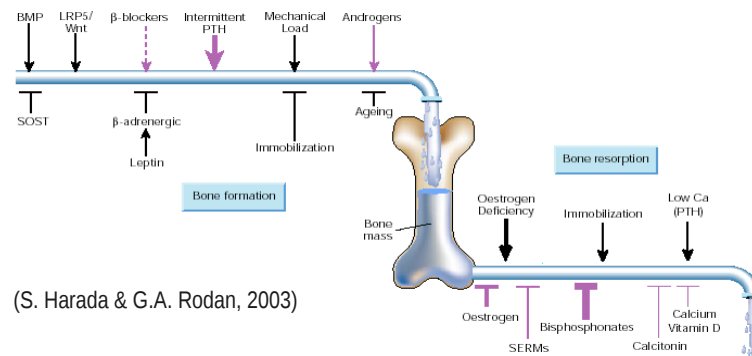
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BONE REMODELLING

OBJECTIVES?

- Mechanical factors
- Biochemical factors

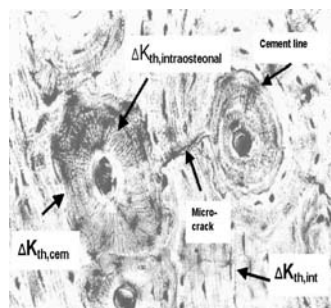


(S. Harada & G.A. Rodan, 2003)

BONE REMODELLING

OBJECTIVES?

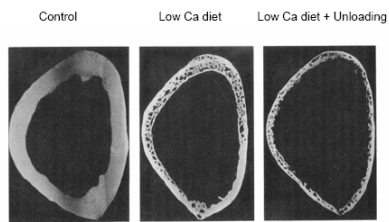
- **Mechanical factors**
 - A mechanism to repair damage, reducing the risk for fracture (internal remodelling)
 - Optimize the stiffness and strength with minimum weight (external remodelling)
- Biochemical factors



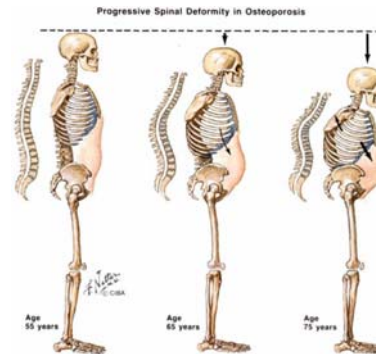
BONE REMODELLING

OBJECTIVES?

- Mechanical factors
- **Biochemical factors**
 - Regulate minerals and hormones
 - Nutrition
 - Osteoporosis



Lanyon et al. 1986



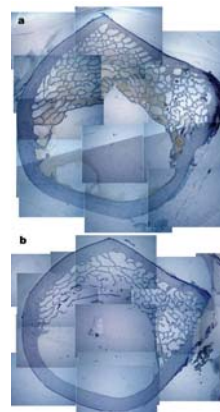
BONE REMODELLING

INTERNAL AND EXTERNAL REMODELLING

- Occur at bone surfaces where are located the bone lining cells
- Two types:
 - *Internal remodelling* or *remodelling*: in the trabeculae surfaces
 - *External remodelling* or *modelling*: in the periosteum and endosteum



Internal remodelling or remodelling

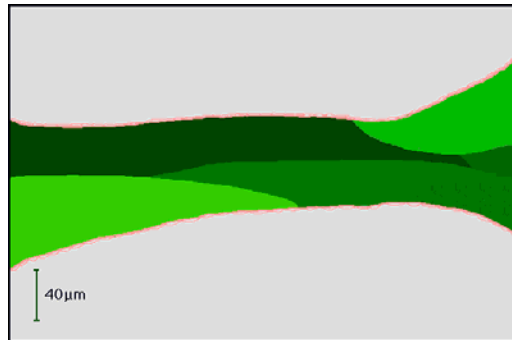


external remodelling or modelling

BONE REMODELLING

INTERNAL BONE REMODELLING: BMUs

- It is accomplished by packets of cells, (Basic Multicellular Units, **BMUs**) composed by osteoclasts and osteoblasts.
- It always follows the same sequence: Activation, Resorption, Reversal, Formation

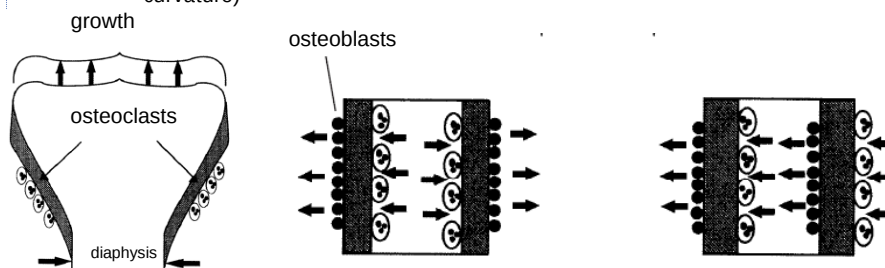


<http://uwcme.org/site/courses/legacy/bonephys/opgallery.php>

BONE REMODELLING

EXTERNAL BONE REMODELING

- External bone remodelling (or modelling) may be resorptive or formative, there is an uncoupled process
- It could appear in the following:
 - Resorptive modelling beneath the growth plate to form the diaphysis from the metaphysis
 - Formative (periosteal surface) and resorptive (endosteal surface) modelling to enlarge the diaphysis
 - Modelling to “drift” the diaphysis to the left (thereby altering diaphyseal curvature)



FIRST THEORIES

- **Wolff's law:** "dependence between bone structure and the load that it supports".
- **Roux's theory:** "bone adapted itself in order to support stresses in an optimal way with minimum mass"



(Cowin, S.C. "The false premise in Wolff's law", in *Bone Mechanics*. 2nd Ed. (Cowin, S.C. ed.), 30;1-15)

PHENOMENOLOGICAL VS MECHANOBIOLOGICAL MODELS

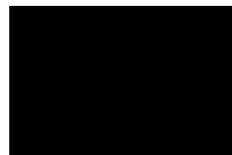
Phenomenological

- predict bone remodelling: direct relationships between mechanical stimulus and bone response
- follow experimental evidences

Mechanobiological

- Mechanobiology (Carter et al., 1998): "the study of how mechanical or physical conditions can regulate biologic processes"
- follow mechanobiological facts

Mechanical
stimulus



Bone remodelling

PHENOMENOLOGICAL MODEL

PHENOMENOLOGICAL VS MECHANOBIOLOGICAL MODELS

Phenomenological

- predict bone remodelling: direct relationships between mechanical stimulus and bone response
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Mechanobiological

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BONE REMODELLING

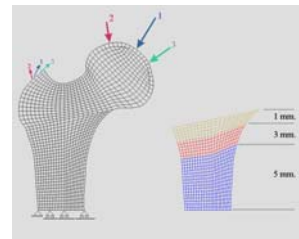
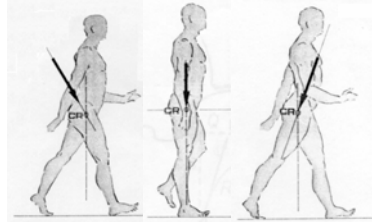
SOME PHENOMENOLOGICAL MODELS

- Models based on global optimality criteria: Rodrigues, 1998; Fernandes, Rodrigues and Jacobs, 1998; Terrier and Rakotomanana, 1997,..Pedersen, 1993).
- Models based on achieving a homeostatic state of strain or stress: Pauwels and Kummer 1965, 1972; Cowin and Hegedus, 1977; Carter et al., 1987; Huiskes et al. (1987), Beaupré et al, 1990a, 1990b; Cowin et al. (1992), Jacobs, 1994, 1997; Fyhrie y Schaffler, 1995; Terrier et al, 1997a; Mikic y Carter, 1995; Petermann et al, 1997; Stölplner et al, 1997; Turner et al., 1997; Luo et al., 1995, **Doblaré and García, 2001**, and many others.
- Models based on damage repairing: Prendergast and Taylor, 1994; Ramtani, 2000,2001.

BONE REMODELLING EXAMPLES

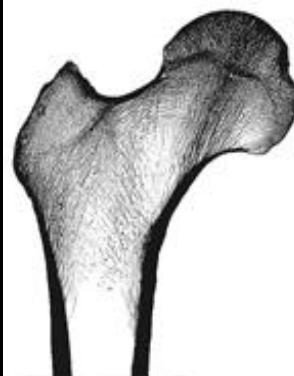
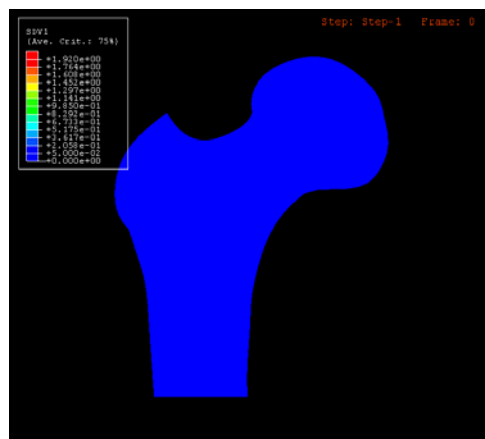
2D SIMULATION OF THE REMODELLING PROCESS OF THE PROXIMAL FEMUR

Load cases	Cycles per day	Load over the femoral head	Reaction at the abductor	Orientation (°)
		Magnitude (N)	Magnitude (N)	
1	6000	2317	703	28
2	2000	1158	351	-8
3	2000	1548	468	35



BONE REMODELLING EXAMPLES

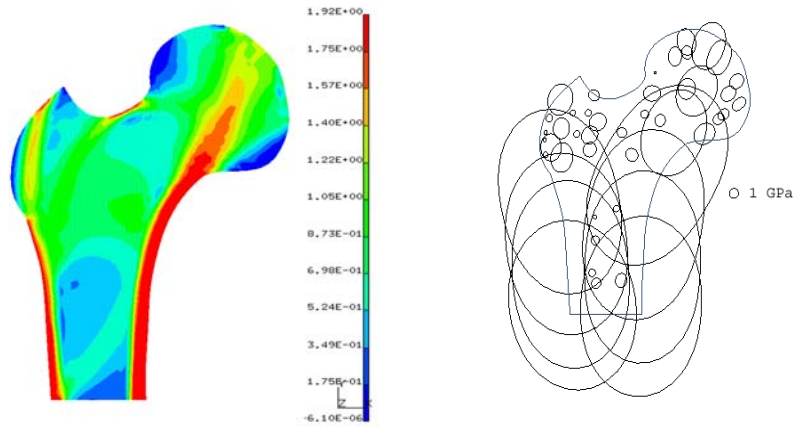
■ Prediction of apparent density distribution



(Doblaré & García, 2001, 2002)

BONE REMODELLING EXAMPLES

■ Prediction of anisotropy



300 days

(Doblaré & García, 2001, 2002)

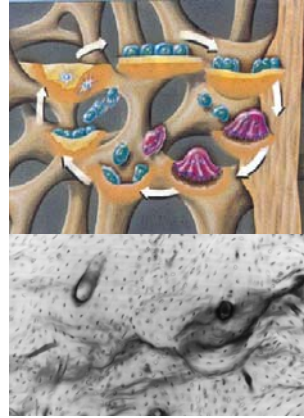
BONE REMODELLING

SOME MECHANOBIOLOGICAL MODELS

- Mechanobiological models: Huiskes et al, 2000; Hernandez et al., 2000; Hazelwood et al, 2001; **García-Aznar et al., 2005**

MECHANOBIOLOGICAL MODEL: main purpose

- Development and validation of a computational model able to:
 - simulate bone remodelling process
 - predict bone stress fractures
- considering:
 - biological effects are controlled by BMU activity
 - actual damage evolution

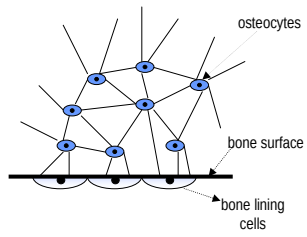


(García-Aznar et al., 2005)

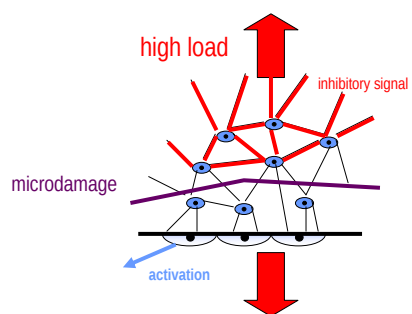
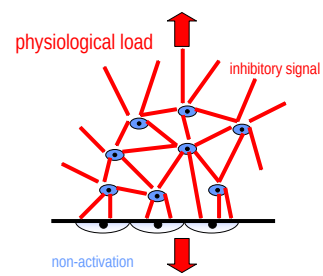
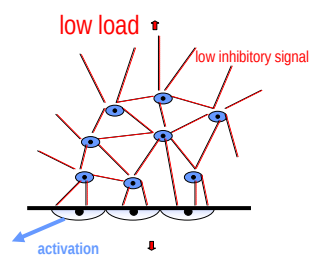
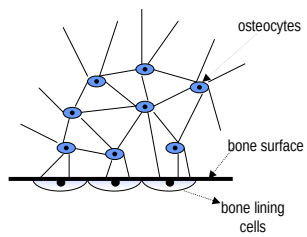
MECHANOBIOLOGICAL MODEL: main assumptions

- The occurrence of BMUs is mainly activated by disuse (Martin, 2000):
 - a low mechanical stimulus
 - microcraks (microdamage) break the communication channels between cells reducing the level of mechanical signal and causing an effect similar to disuse.

Unifying Theory of Bone Remodelling (Martin, 2000)



Unifying Theory of Bone Remodelling (Martin, 2000)



MECHANOBIOLOGICAL MODEL: main assumptions

- The occurrence of BMUs is mainly activated by disuse (Martin, 2000):
 - a low mechanical stimulus
 - microcraks (microdamage) break the communication channels between cells reducing the level of mechanical signal and causing an effect similar to disuse.
- The born rate of BMUs depends on this inhibitory signal, the specific surface and patient and organ-dependent biological factors.
- The inhibitory signal depends on the damage level and a strain measure that characterizes the daily loading history.
- Damage growth and elastic modulus degradation is different for tension and compression.
- Damage is uniformly distributed.
- The mechanical behaviour is linear elastic, heterogeneous and isotropic with the local stiffness depending on the porosity, mineralisation degree and damage level.

MECHANOBIOLOGICAL MODEL: definition

- basic variables
- mechanical stimulus
- BMU birth-rate
- BMU progression
- evolution of bone volume fraction
- focal bone balance
- mineralization
- damage
- material behaviour

MECHANOBIOLOGICAL MODEL: definition

- Basic variables

- subdivision of reference volume:

$$V_T = V_B + V_V$$

$$V_B = V_M + V_O + V_D$$

- bone volume fraction

$$v_b = \frac{V_B}{V_T} = 1 - p, \quad p = \frac{V_V}{V_T}$$

- ash fraction

$$\alpha = \frac{M_M}{M_d} = \frac{\rho_M V_M}{\rho_M V_M + \rho_O V_O}$$

- continuum damage variable

$$d = \frac{V_D}{V_B} = 1 - \frac{E}{E_0}$$

MECHANOBIOLOGICAL MODEL: definition

- Inhibitory signal

- Transmitted by the osteocyte network

- Level of the received signal

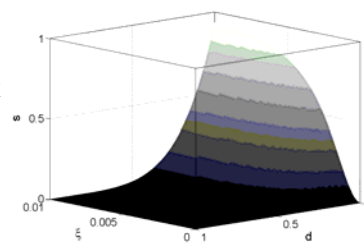
$$s = \frac{\xi}{\xi + c} (1 - d)^k$$

↙ damage

- Daily loading history $\xi = \left(\sum_i N_i \bar{\varepsilon}_i^m \right)^{1/m}$

with $\bar{\varepsilon} = \sqrt{\frac{2U}{E}}$ and U the strain energy density

N_i the daily number of cycles of each load case i

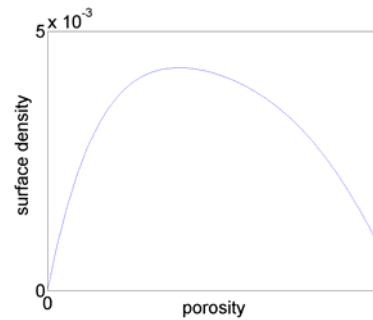


MECHANOBIOLOGICAL MODEL: definition

- BMU birth-rate $\dot{N}_{BMU} = f_{Or} S_V$
 - S_V is specific surface density and function of porosity (Martin, 1984)
 - Origination frequency

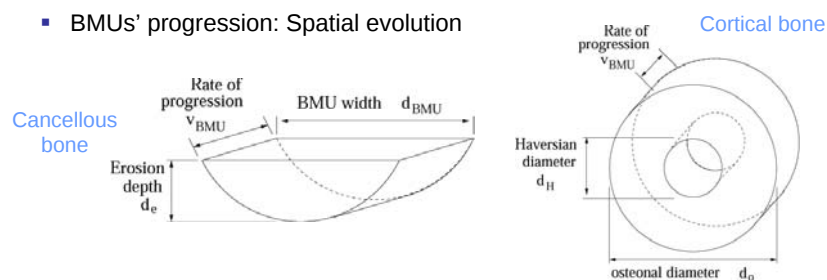
$$f_{Or} = f_{bio} (1 - s)$$

with f_{bio} a specific biological factor
and s the inhibitory signal



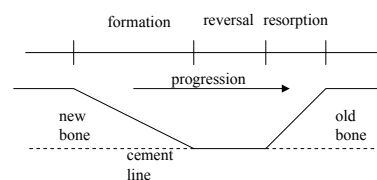
MECHANOBIOLOGICAL MODEL: definition

- BMUs' progression: Spatial evolution



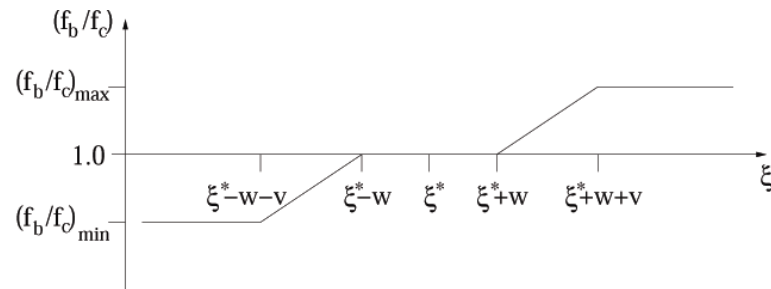
- BMUs' progression: Time evolution

- Resorption interval T_r
- Reversion interval T_l
- Formation interval T_F



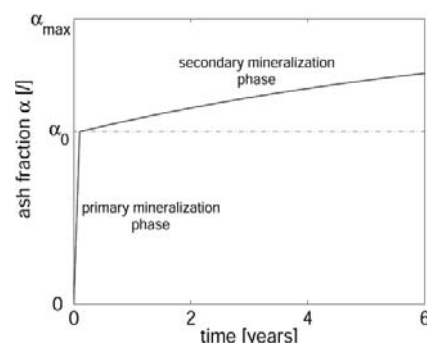
MECHANOBIOLOGICAL MODEL: definition

- Focal bone balance
 - rate of the osteoblasts and osteoclasts activity
 - reference stimulus adaptation $\xi^* = \xi - (\xi - \xi_0^*)e^{-\eta\xi}$



MECHANOBIOLOGICAL MODEL: definition

- Mineralisation
 - time-dependent process
 - two phases
 - the secondary phase takes into account the influence of the remodelling



$$\bar{\alpha}(t) = \frac{v_{b,0} \alpha(t) + \int_0^t \dot{v}_F(\tau) \alpha(t-\tau) - \dot{v}_R(\tau) \bar{\alpha}(\tau) d\tau}{v_b(t)}$$

MECHANOBIOLOGICAL MODEL: definition

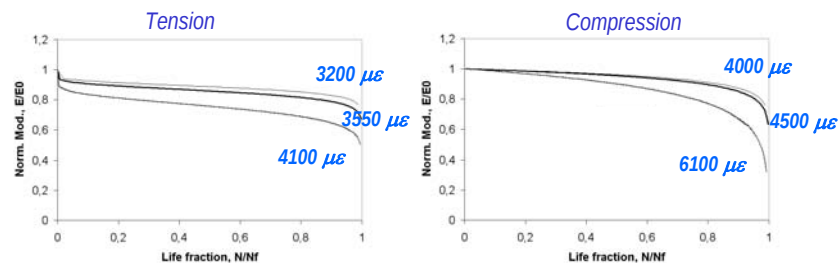
- Damage Growth
 - Continuum Damage Mechanics (CDM)
 - non-linear damage with different effect for tension and compression

$$\frac{\partial d_c}{\partial N} = G_c(d, \tilde{\varepsilon}) = \frac{C_1}{\gamma_1} e^{\gamma_1 d} \tilde{\varepsilon}^{\delta_1}$$

$$\frac{\partial d_t}{\partial N} = G_t(d, \tilde{\varepsilon}) = \frac{C_2}{C_3 \gamma_2} (1 - d)^{1 - \gamma_2} e^{-C_3(1-d)^{\gamma_2}} \tilde{\varepsilon}^{\delta_2}$$

MECHANOBIOLOGICAL MODEL: definition

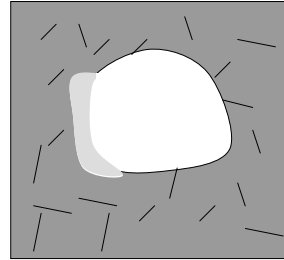
- Damage Growth
 - constants were determined from experimental tests for human femoral bone loaded in different levels for tension and compression (Pattin et al., 1996)



MECHANOBIOLOGICAL MODEL: definition

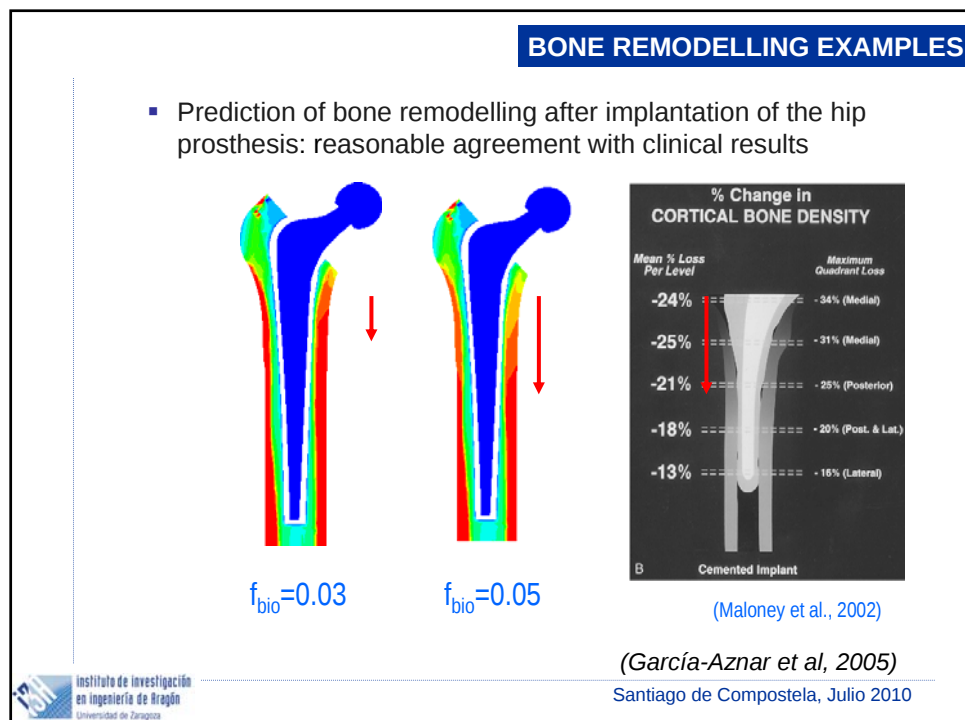
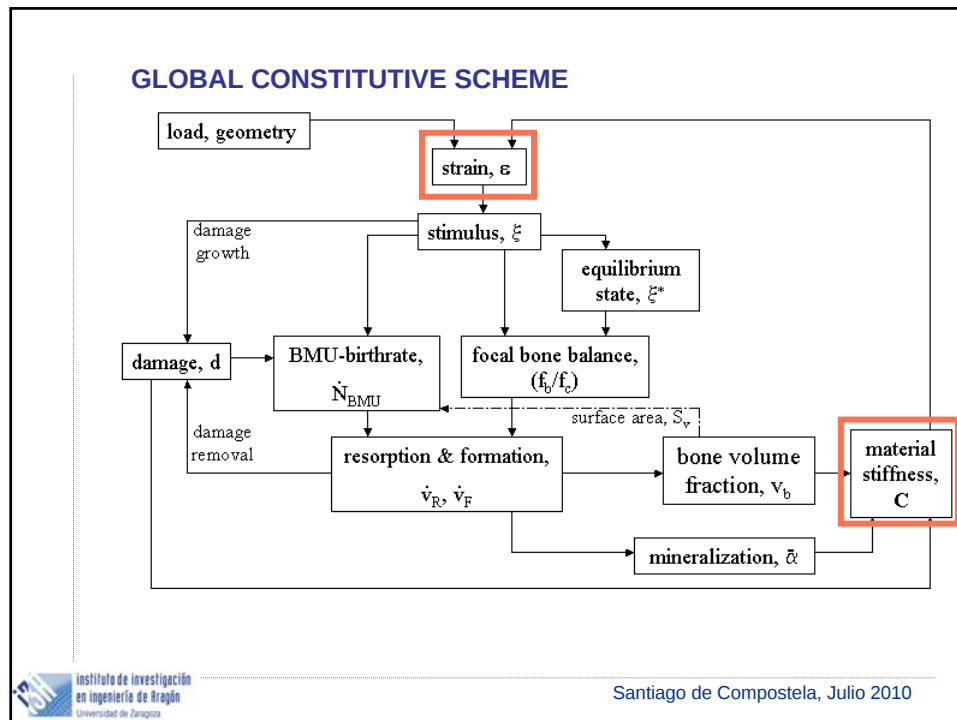
- Damage Repair
 - damage is uniformly distributed
 - damage is repaired as remodelling progresses

$$d = \frac{V_d}{V_B} \Rightarrow \begin{cases} \dot{d} = 0 & \text{(resorption)} \\ \dot{d} = -\dot{v}_F \frac{d^2}{D} & \text{(formation)} \end{cases}$$



MECHANOBIOLOGICAL MODEL: definition

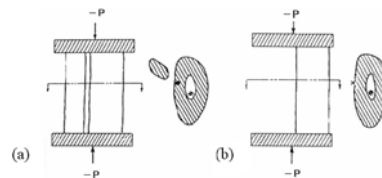
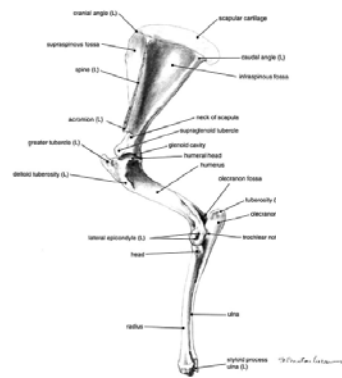
- The **mechanical behaviour** is linear elastic, heterogeneous and isotropic with the local stiffness depending on the porosity, mineralisation degree and damage level.
 - elastic modulus $E = 84,370 \, v_b^{2.58} \alpha^{2.74} (1-d) \, \text{MPa}$
 - Poisson's ratio $\nu = 0.3$
 - constitutive law $\sigma = \frac{E\nu}{(1+\nu)(1-2\nu)} \text{tr} \, \varepsilon + \frac{E}{1+\nu} \varepsilon$



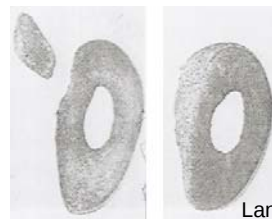
BONE REMODELLING EXAMPLES

SIMULATION OF LANYON EXPERIMENTS (1982) ON ADULT SHEEP (Martínez et al., 2006)

- Ulna osteotomy to change the mechanical environment

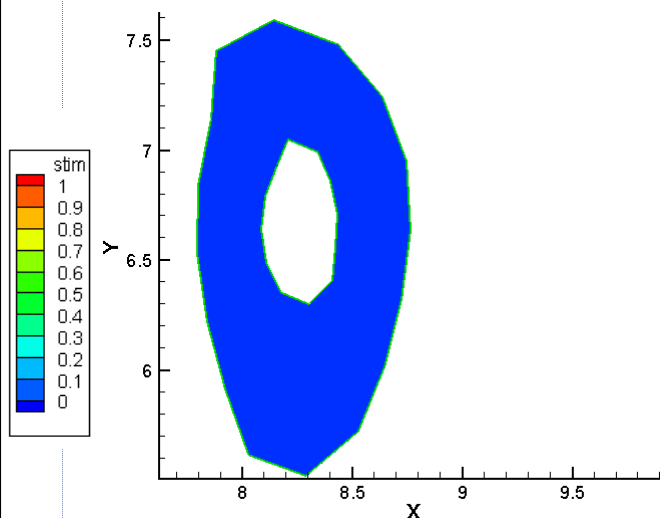


Cowin et al (1985)



Lanyon et al (1982)

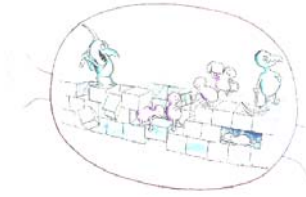
BONE REMODELLING EXAMPLES



After 9 months
(Lanyon et al,1982)

OVERVIEW

- **Bone properties**
- **Role of mechanical factors: bone mechanobiology**
- **Methods of Science: computer simulation**
- **Modelling Bone Mechanobiology:**
 - **Bone Remodelling**
 - **Bone Healing**
 - Bone Distraction
 - Bone Tissue Engineering
- **Final conclusions**



BONE HEALING

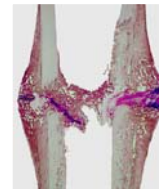
Primary healing

- callus free
- absolute stability
- no interfragmentary movement
 - ✓ perfect anatomic reduction
 - ✓ interfragmentary compression



Secondary healing

- healing via callus or granulation tissue
- similar to bone growth
- involving tissue differentiation in the fracture gaps
- relative stability: less rigid or flexible stabilisation



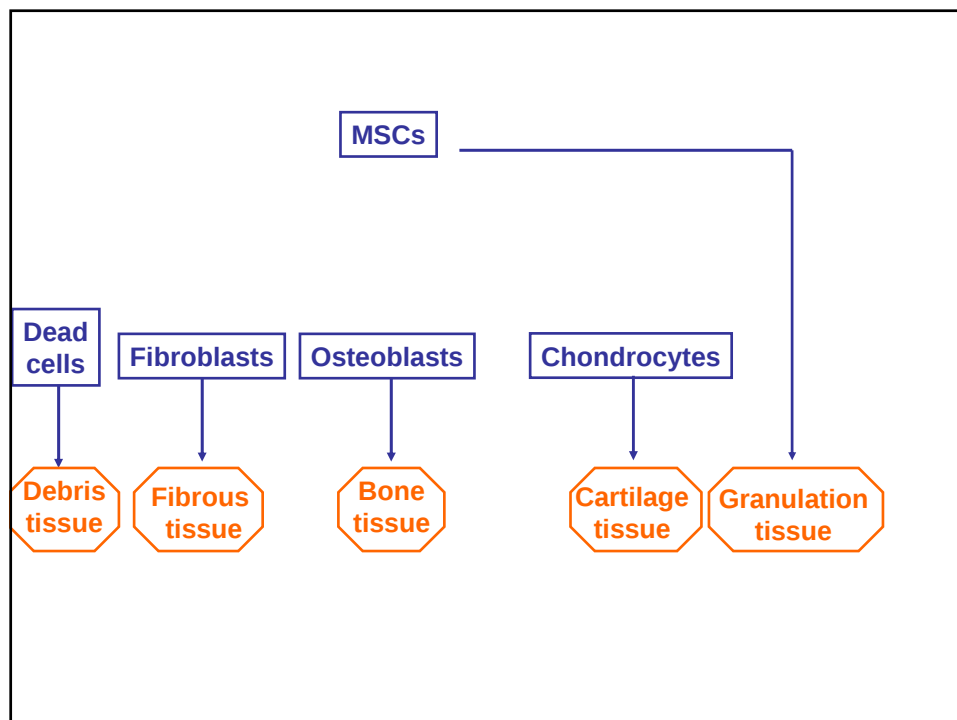
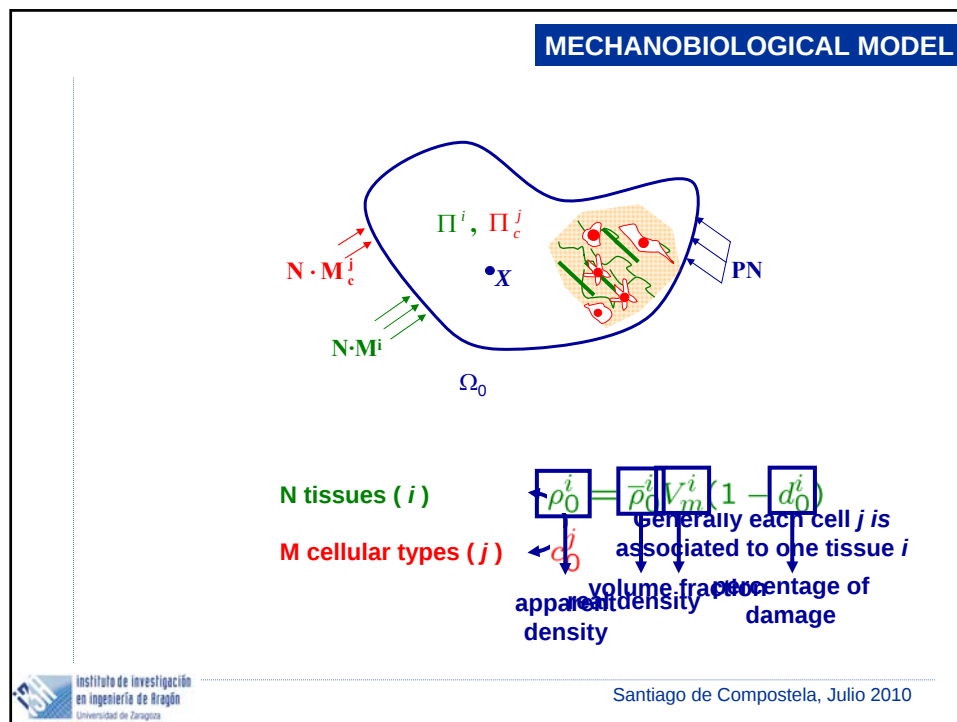
(Claes et al, 2003)

SOME MODELS

- Computational estimation of differentiation rules -> level of strain/stress in a time point of the healing process: Davy and Connolly, 1982; DiGioia et al., 1986; Claes et al. , 1996, 1998, 1999; Carter et al., 1998.
- Evolutionary differentiation models: Kuiper et al., 2000; Ament and Hofer, 2000; Lacroix and Prendergast, 2002; Bailón-Plaza and Van der Meulen, 2001, 2003; Simons et al., 2004; Isaksson et al, 2006.
- Callus growth and tissue differentiation: **Gómez-Benito et al., 2005, 2006; García-Aznar et al., 2007.**

MECHANOBIOLOGICAL MODEL

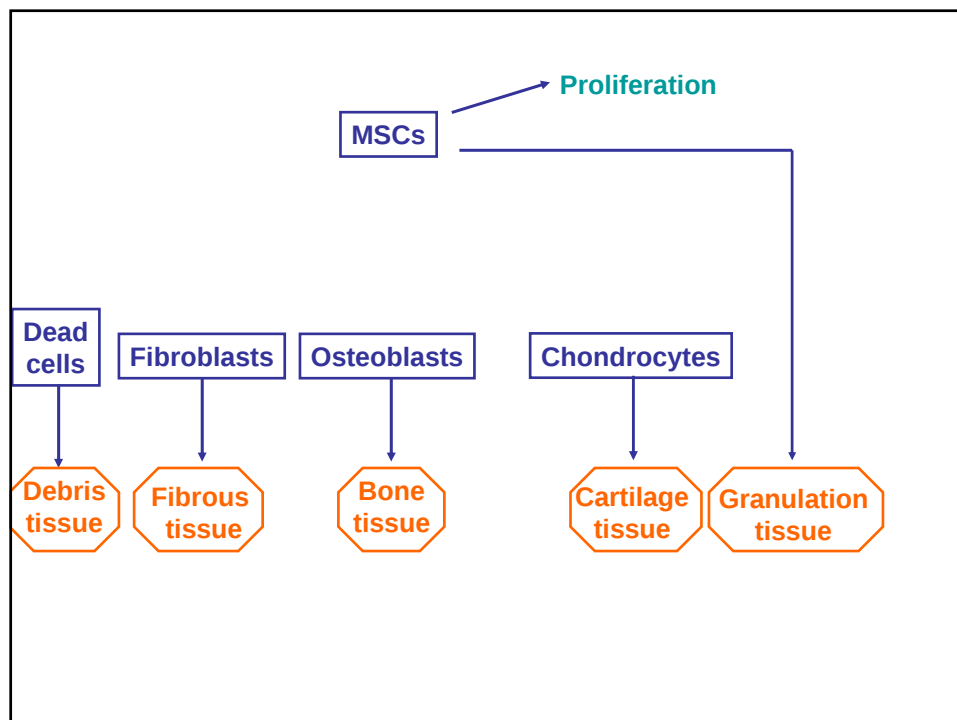
- Balance equations of continuum multiphasic and multicellular tissue mixtures including possible growth have been developed in the continuum setting (generalization of previous formulations by Lubarda & Hoger, 2002; Garikipati et al., 2004, etc.).
- The treatment is macroscopic considering:
 - Tissue growth, differentiation, remodelling and damage
 - Multiple tissues and different types of cells.
 - Sources (sinks) and diffusion of both mass and cells.
 - Energy transfer between the different tissues and cells.

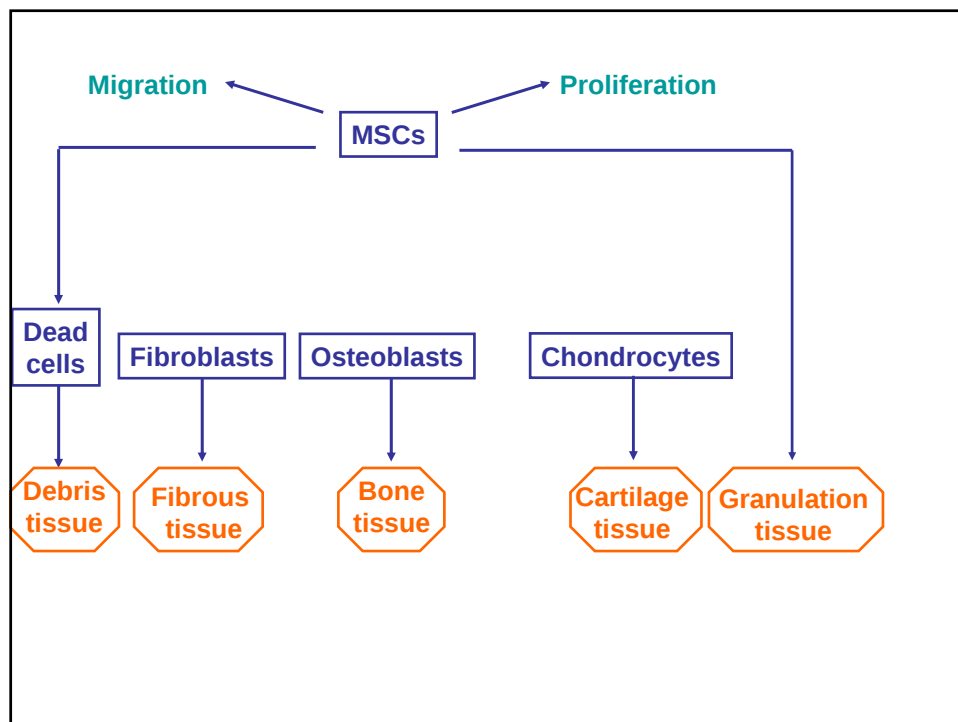
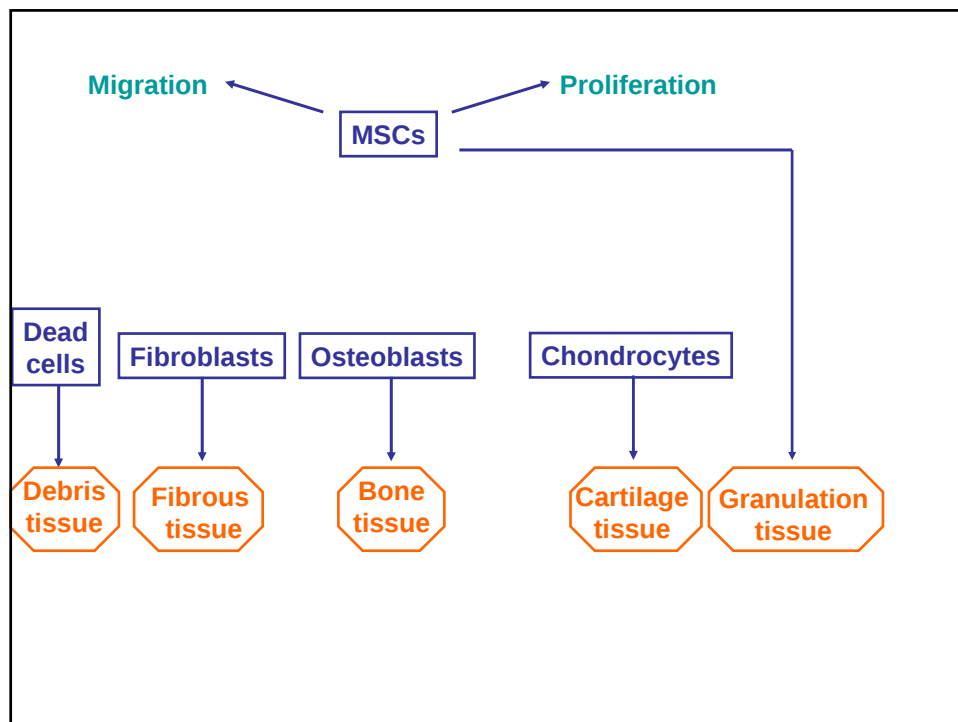


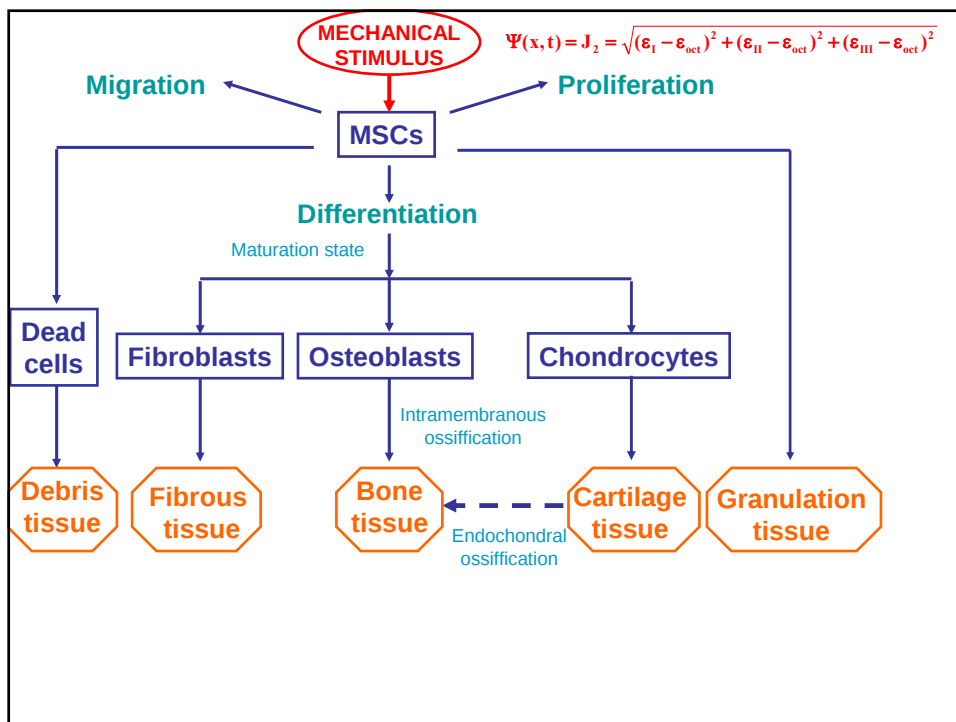
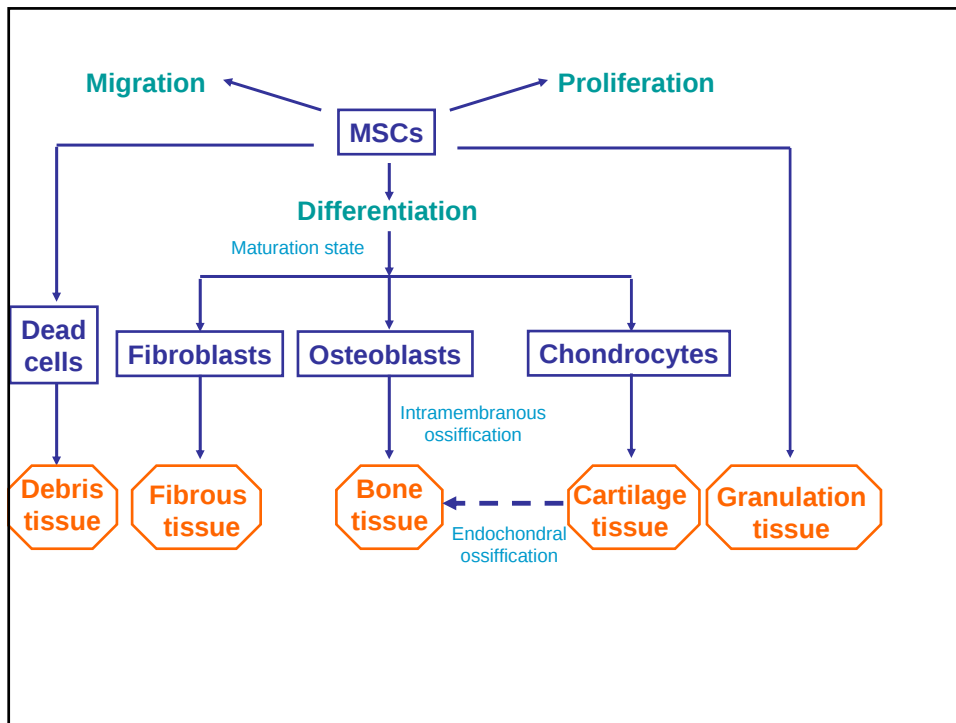
MECHANOBIOLOGICAL MODEL

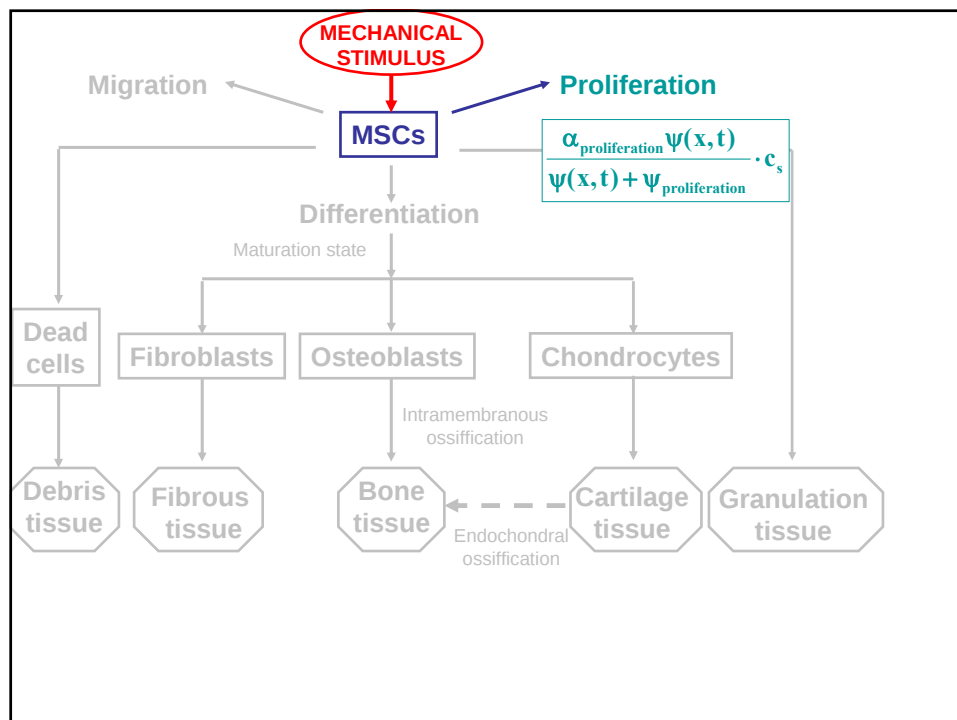
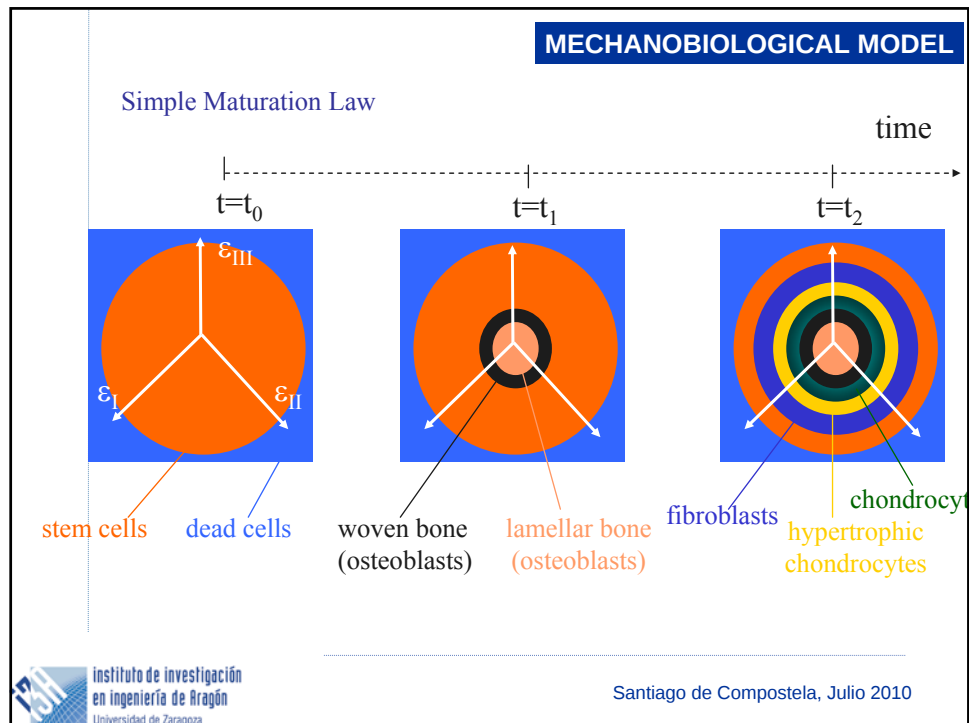
Equation of cell conservation

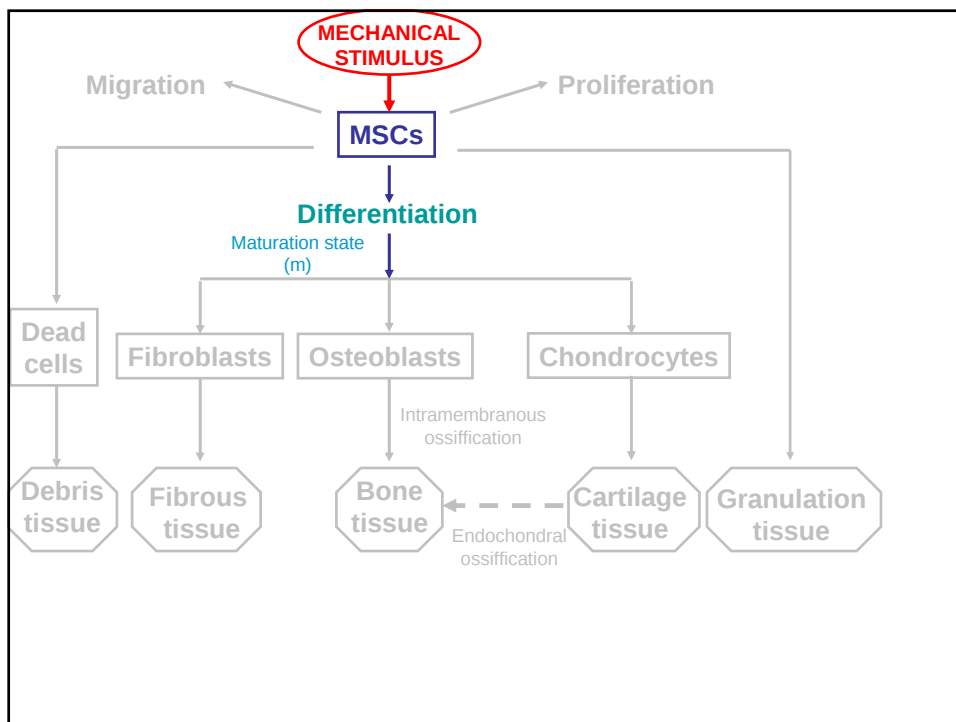
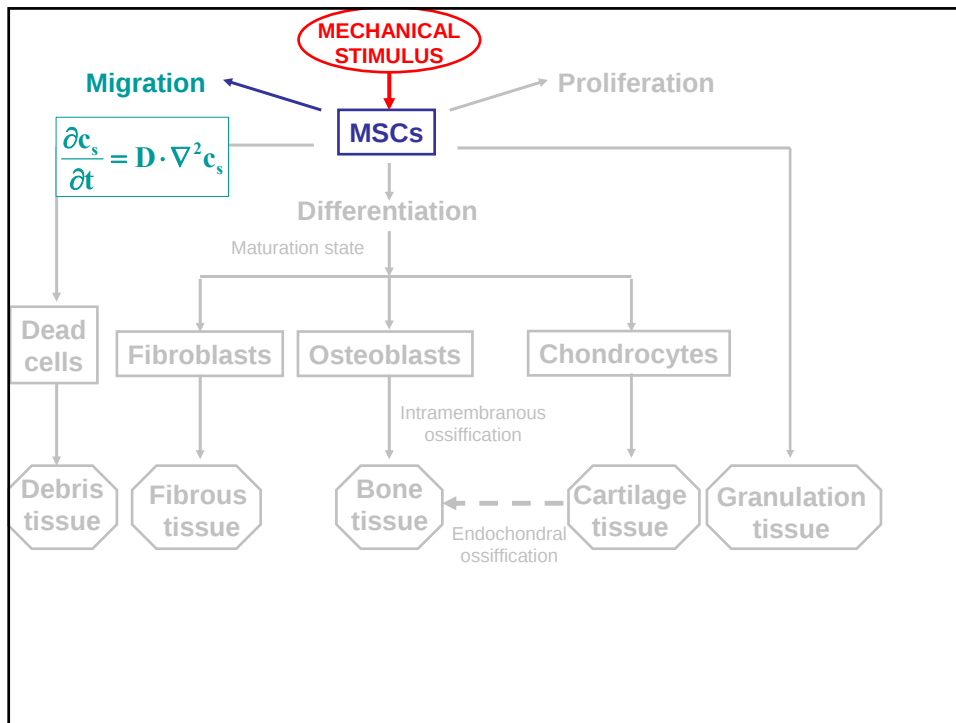
$$\frac{Dc(x,t)}{Dt} = \underbrace{f_{\text{proliferation}}(c, \Psi)}_{\text{Proliferation (mitosis)}} + \underbrace{f_{\text{migration}}(c, V_{\text{disrupted}})}_{\text{Migration}} - \underbrace{f_{\text{differentiation}}(\Psi, m)}_{\text{Differentiation}}$$

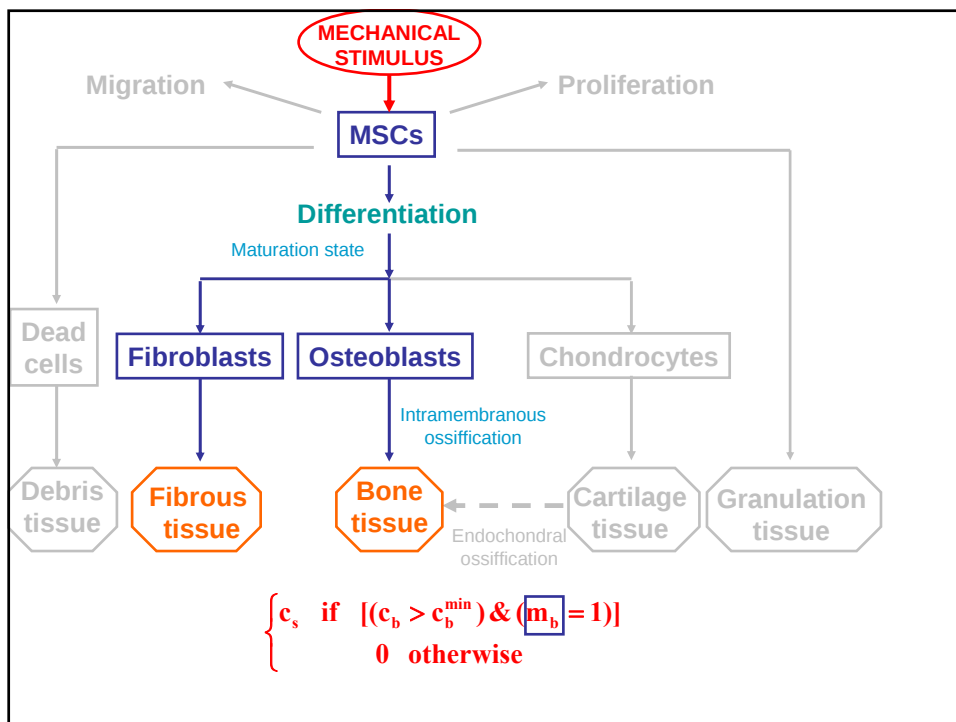
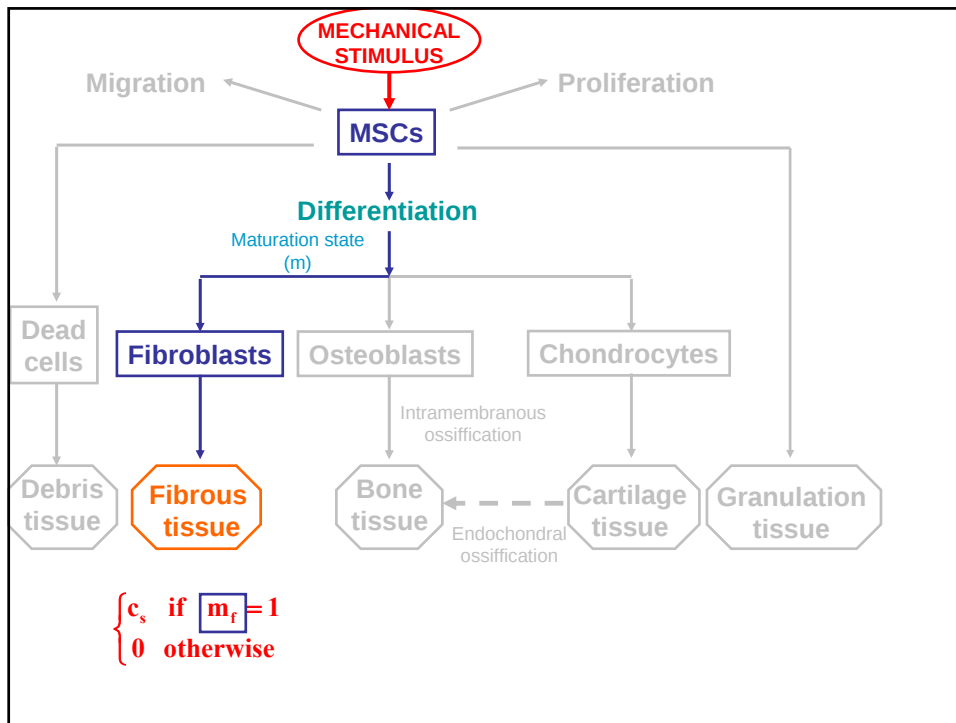


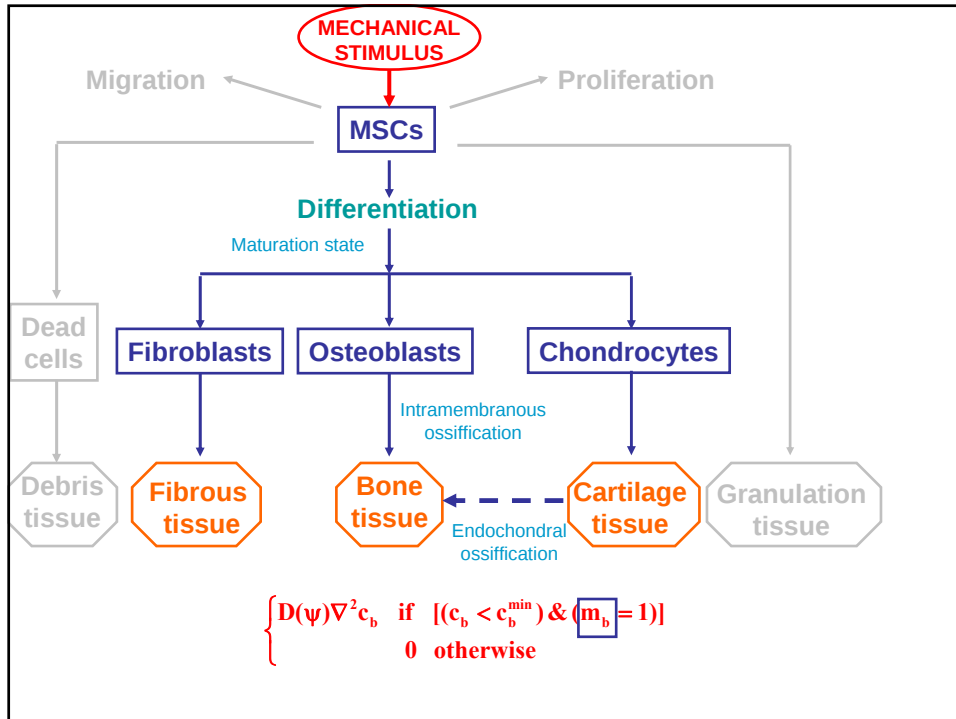
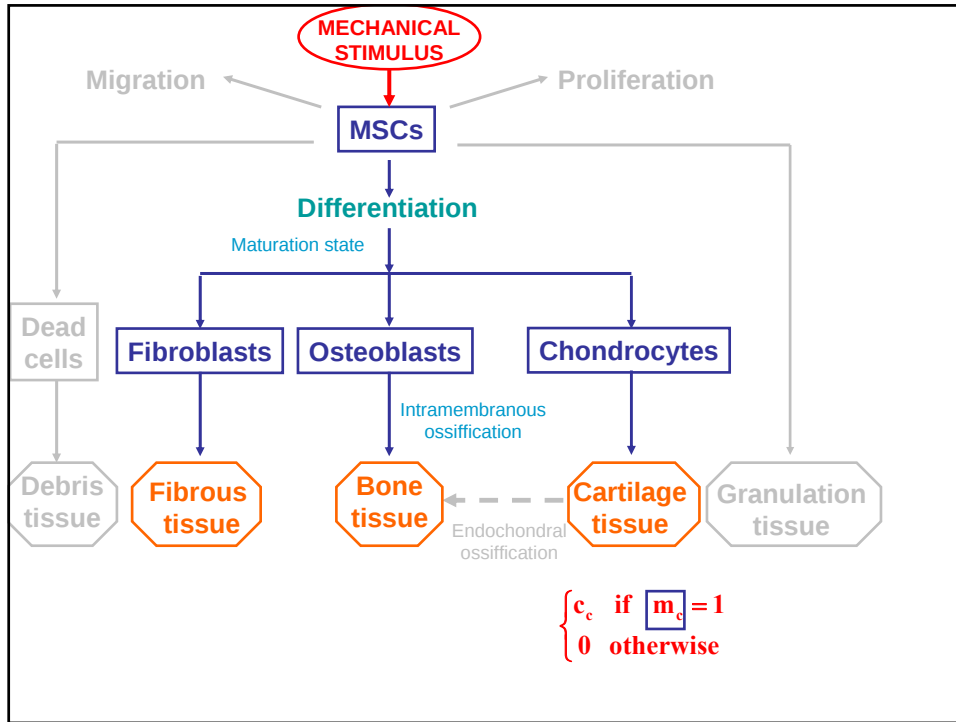


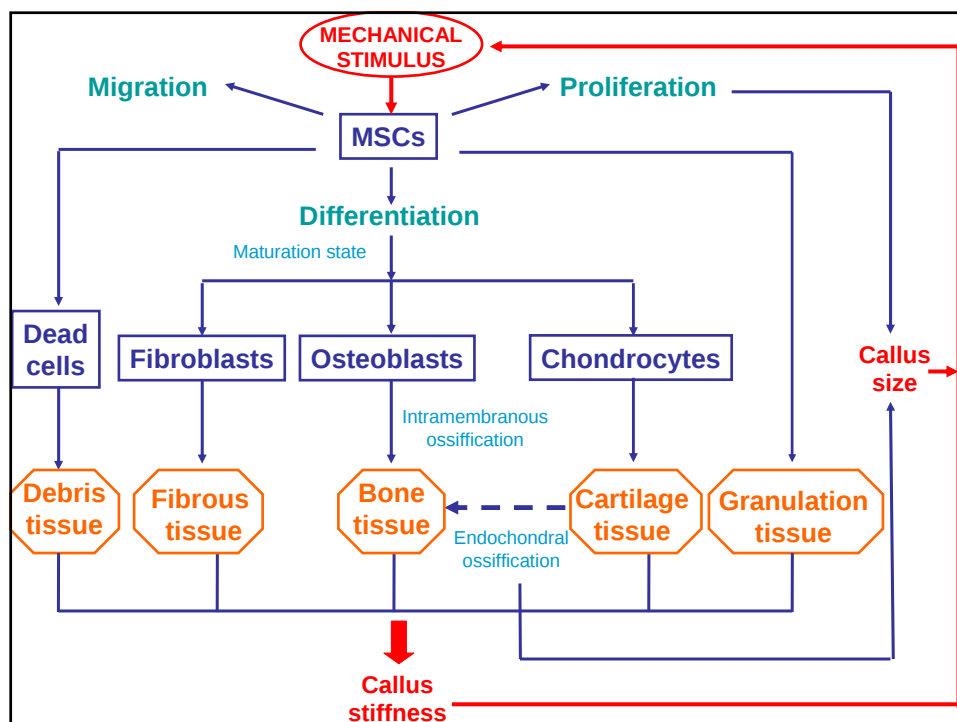
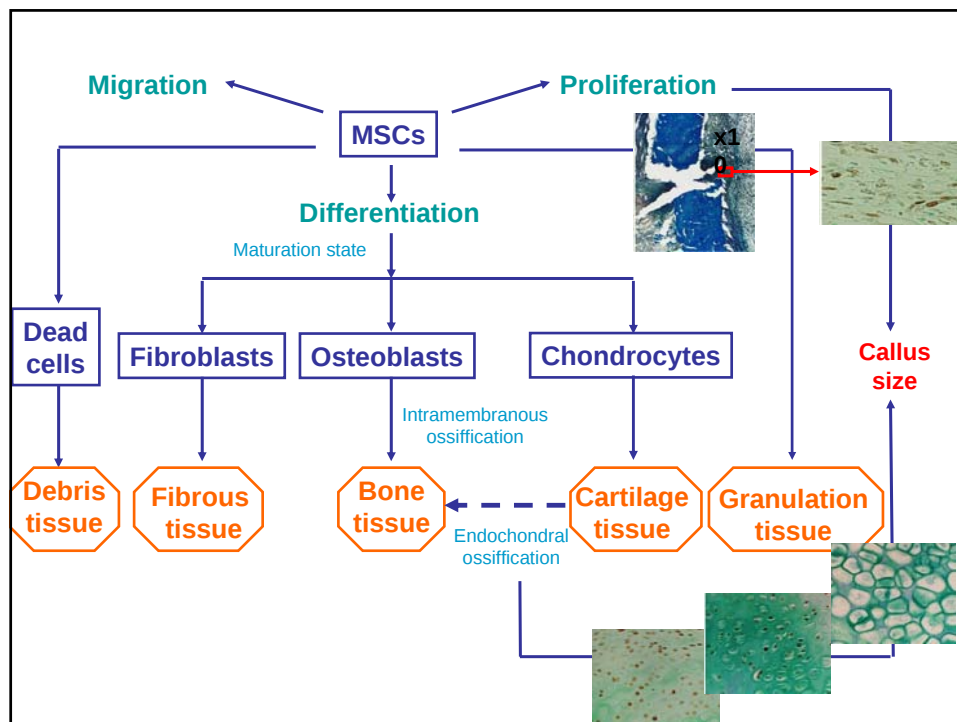












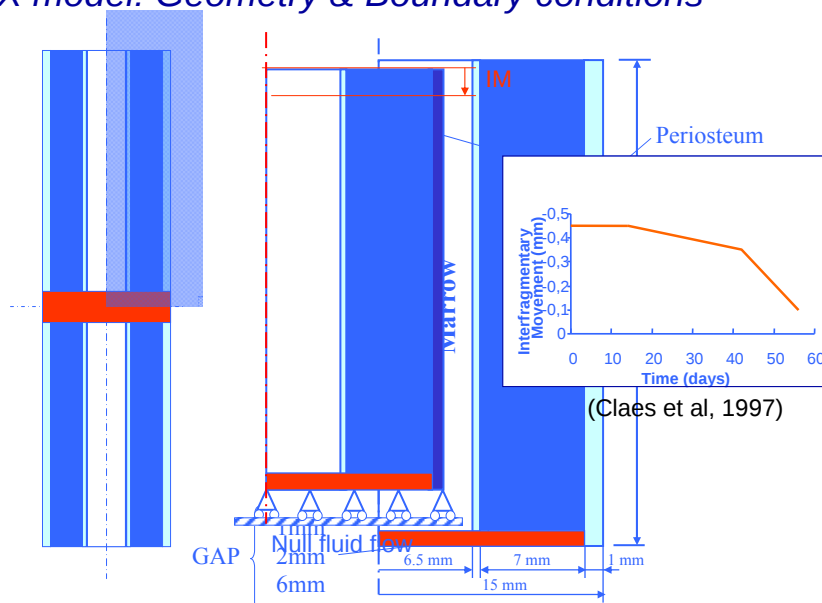
BONE HEALING EXAMPLES

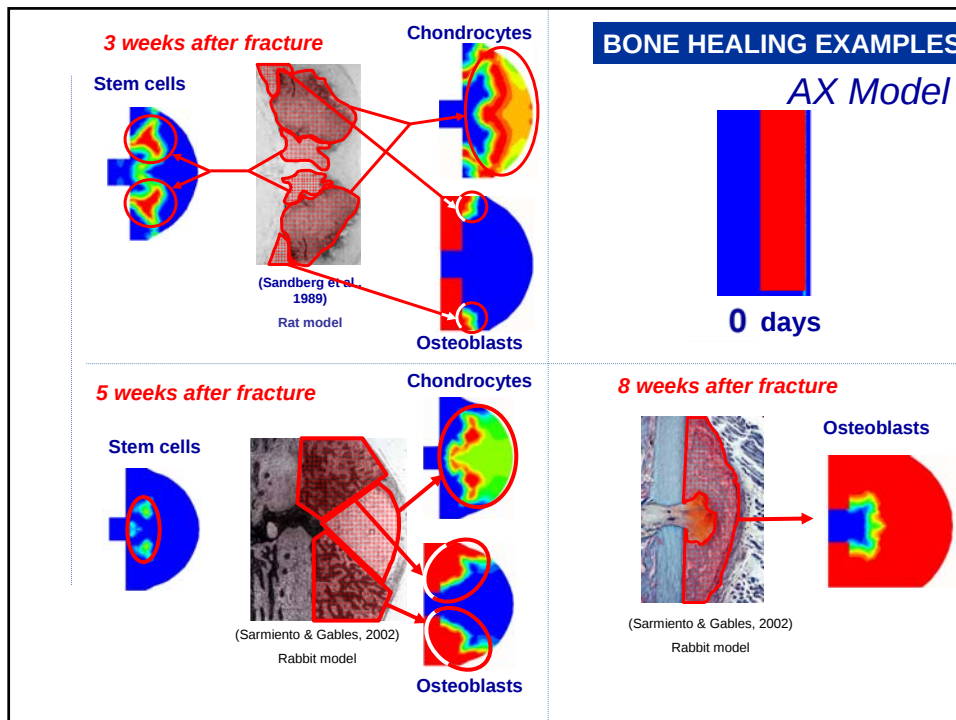
- Axysimmetric human model under compression
- Sheep's model:
 - Interfragmentary movement
 - Bilateral fixator



BONE HEALING EXAMPLES

AX model: Geometry & Boundary conditions





BONE HEALING EXAMPLES

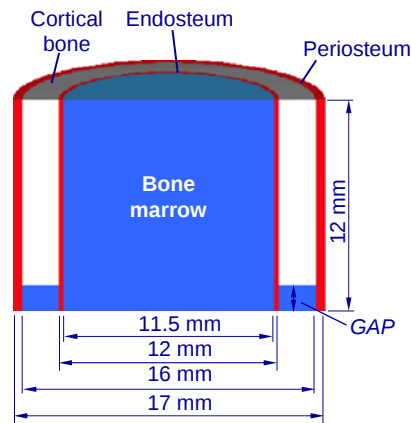
- Axysimmetric human model under compression
- *Sheep's model:*
 - *Interfragmentary movement*
 - Bilateral fixator



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BONE HEALING EXAMPLES

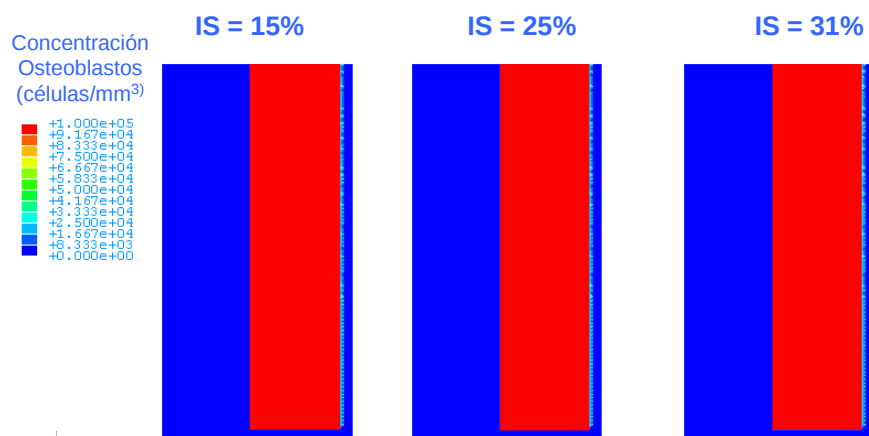
Sheep's geometry



(Augat et al., 2003)

BONE HEALING EXAMPLES

Influence of interfragmentary strain

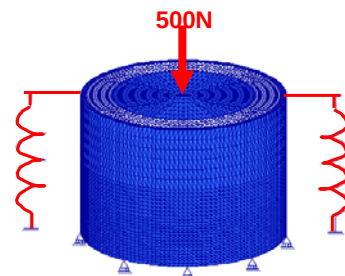


BONE HEALING EXAMPLES

- Axysimmetric human model under compression

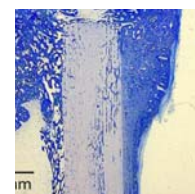
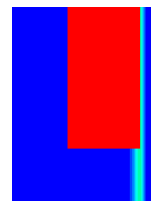
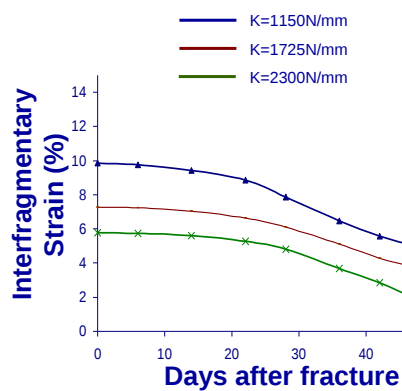
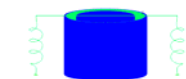
■ *Sheep's model:*

- *Interfragmentary movement*
- *Bilateral fixator*



BONE HEALING EXAMPLES

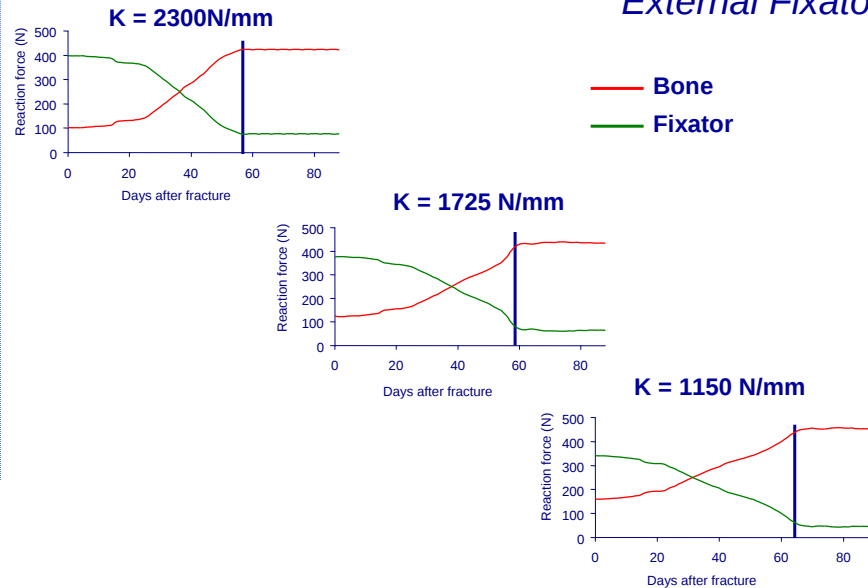
External Fixator



(Augat et al., 2003)

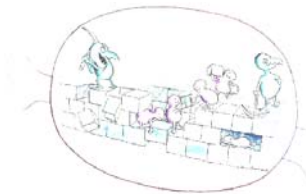
BONE HEALING EXAMPLES

External Fixator



OVERVIEW

- **Bone properties**
- **Role of mechanical factors: bone mechanobiology**
- **Methods of Science: computer simulation**
- **Modelling Bone Mechanobiology:**
 - Bone Remodelling
 - Bone Healing
 - Bone Distraction
 - Bone Tissue Engineering
- **Final conclusions**



BONE DISTRACTION

What is distraction osteogenesis?

A useful technique aimed to induce bone formation in widespread clinical applications. Main fields of application:

Orthopaedics

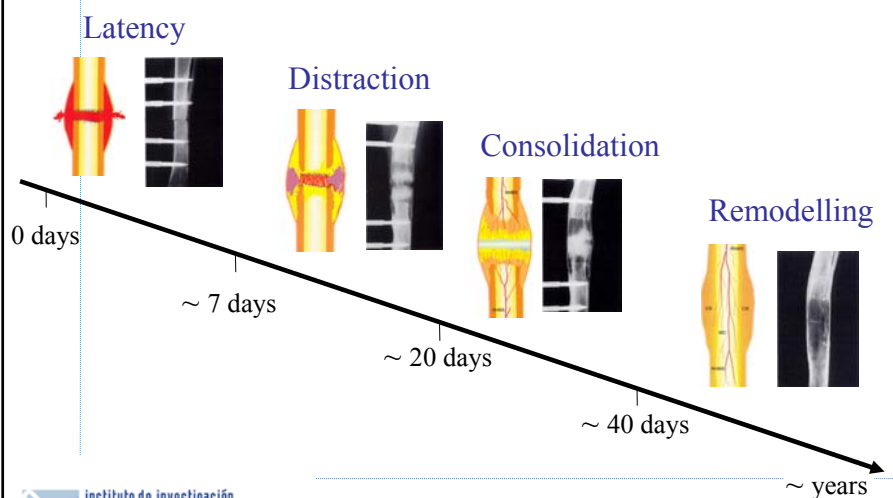


Craneofacial surgery



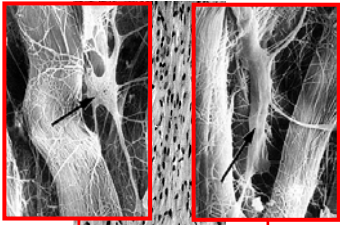
BONE DISTRACTION

Which are the main stages in distraction osteogenesis?



BONE DISTRACTION

Mechanical factors



Non stimulated

Under tension

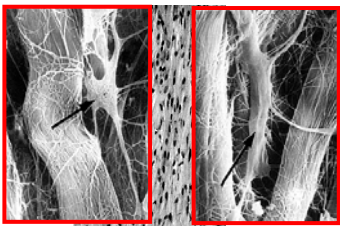
Biological factors


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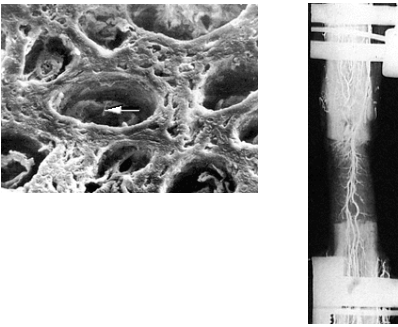
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BONE DISTRACTION

Mechanical factors



Biological factors



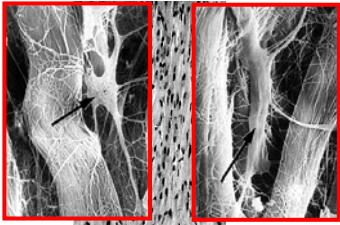
<http://www.globalmednet.com/do-cdrom/Biol/Histomor/rh07.htm>


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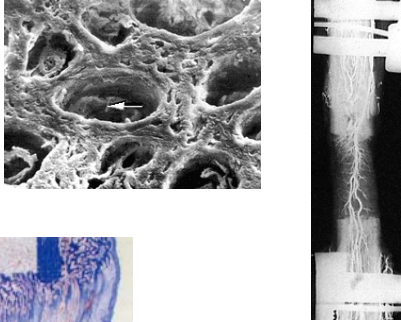
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BONE DISTRACTION

Mechanical factors



Biological factors





Bone tissue formation is 6 times faster
than in a child growth plate

(Anderson *et al.*, J. Bone Joint Surg. 45A.10, 1963)



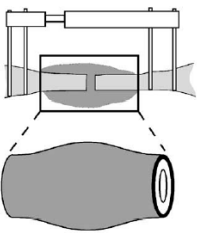
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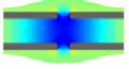
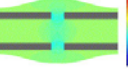
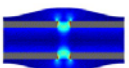
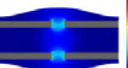
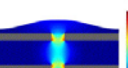
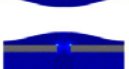
BONE DISTRACTION

Previous models

- Non-evolutive models (Loboa *et al.*, 2004, Boccaccio *et al.*, 2005, Cattaneo *et al.*, 2005, Morgan *et al.*, 2005)



(Morgan *et al.*, 2005)

	Immediately after Distraction	12 Hours after Distraction	
Total Pressure			
Tensile Strain			
Tissue Distraction			
Fluid Velocity			

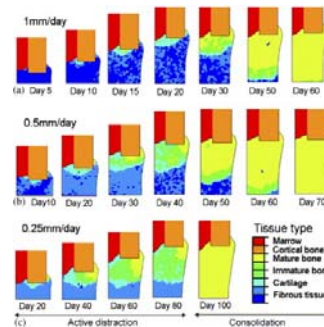
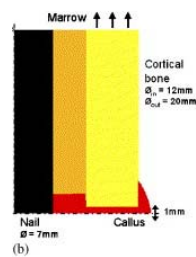


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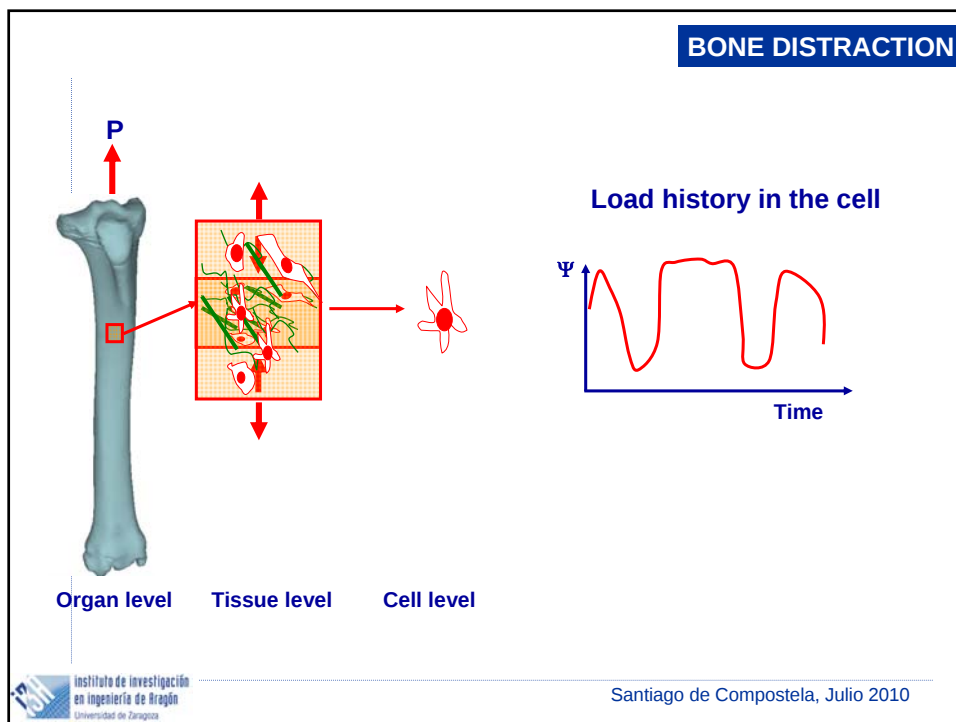
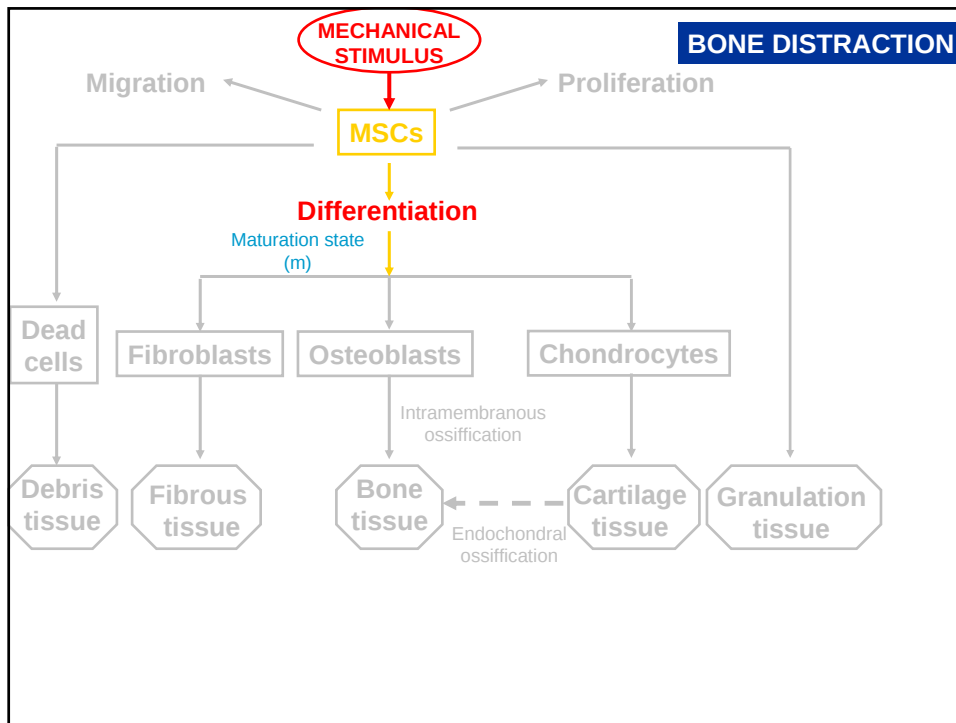
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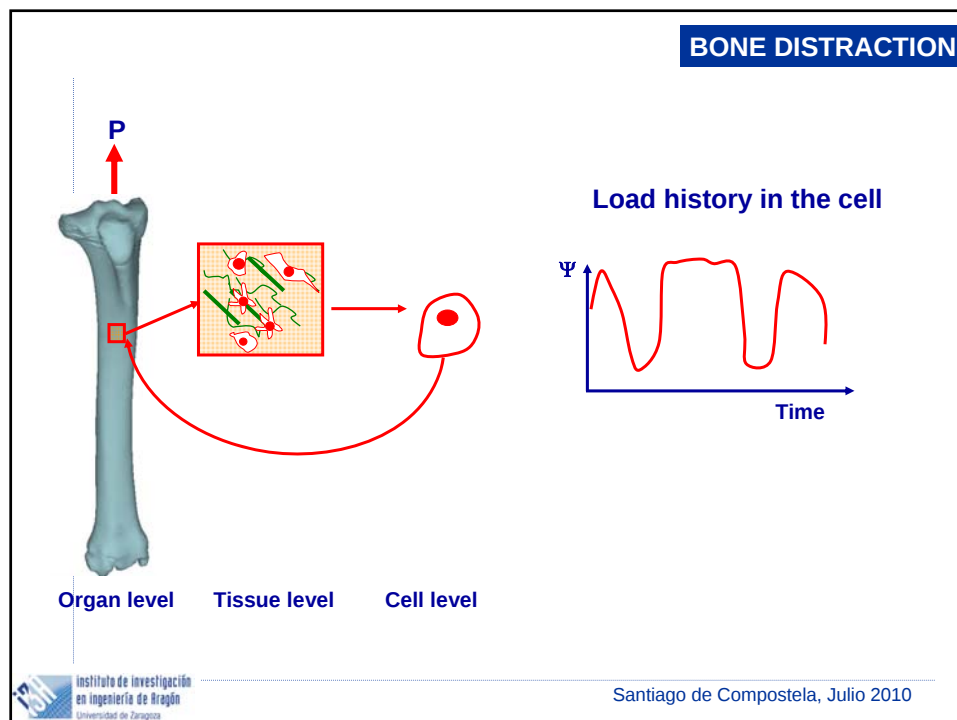
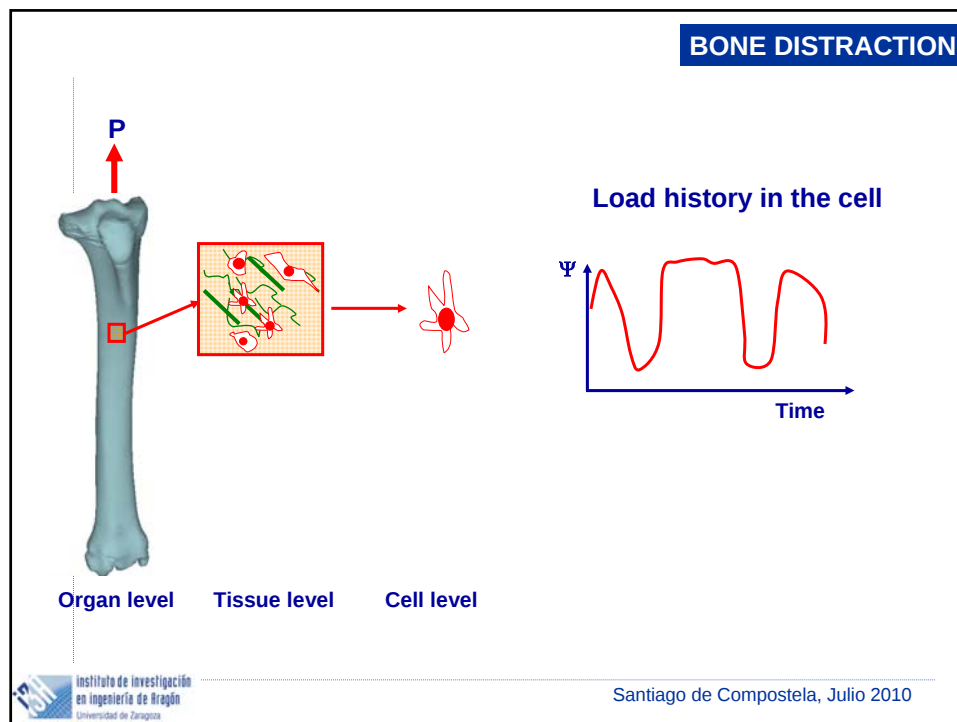
Previous models

- **Non-evolutive models** (Loboa *et al.*, 2004, Boccaccio *et al.*, 2005, Cattaneo *et al.*, 2005, Morgan *et al.*, 2005)
- **Evolutive models** (Isaksson *et al.*, 2007, Boccaccio *et al.*, 2008)

(Isaksson *et al.*, 2006)

- The application of the mechanobiological model of bone healing to distraction osteogenesis presents **limitations**:
 - It is **not able to predict the effect of different distraction rates** => include the **influence of the load history** on cell differentiation
 - **Neglect the effect of pre-traction stresses**





Maturation state of the cell (m_i) – Miner's Rule:

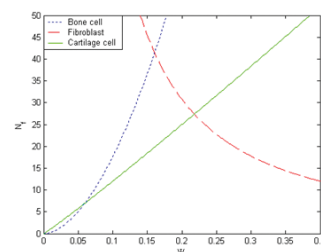
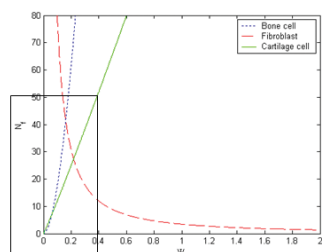
$$m_i = \sum \frac{n}{N_f(\Psi)}$$

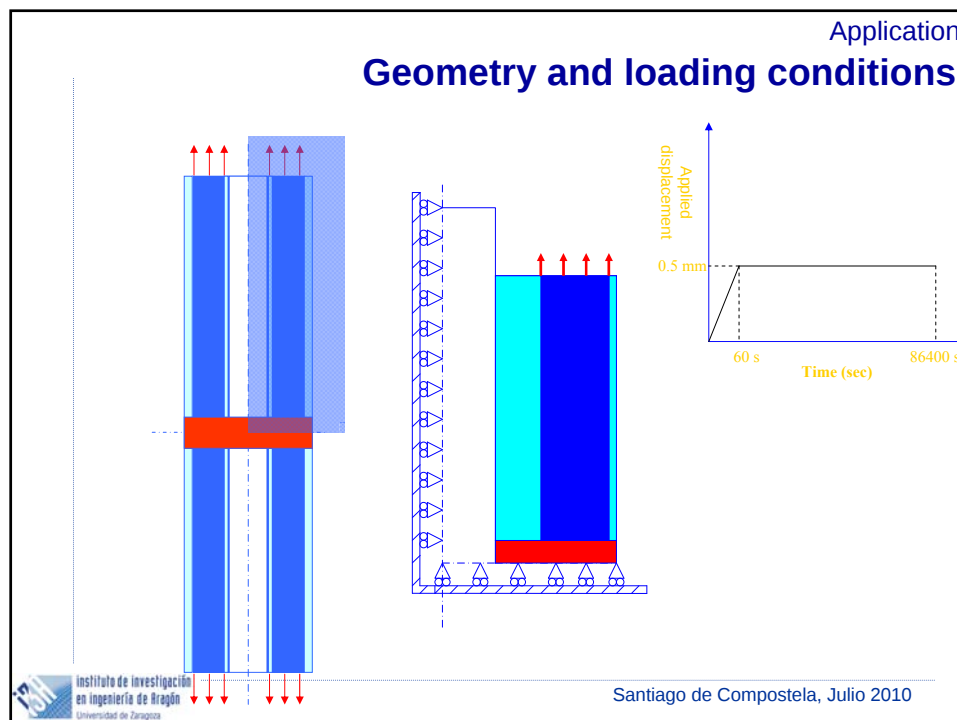
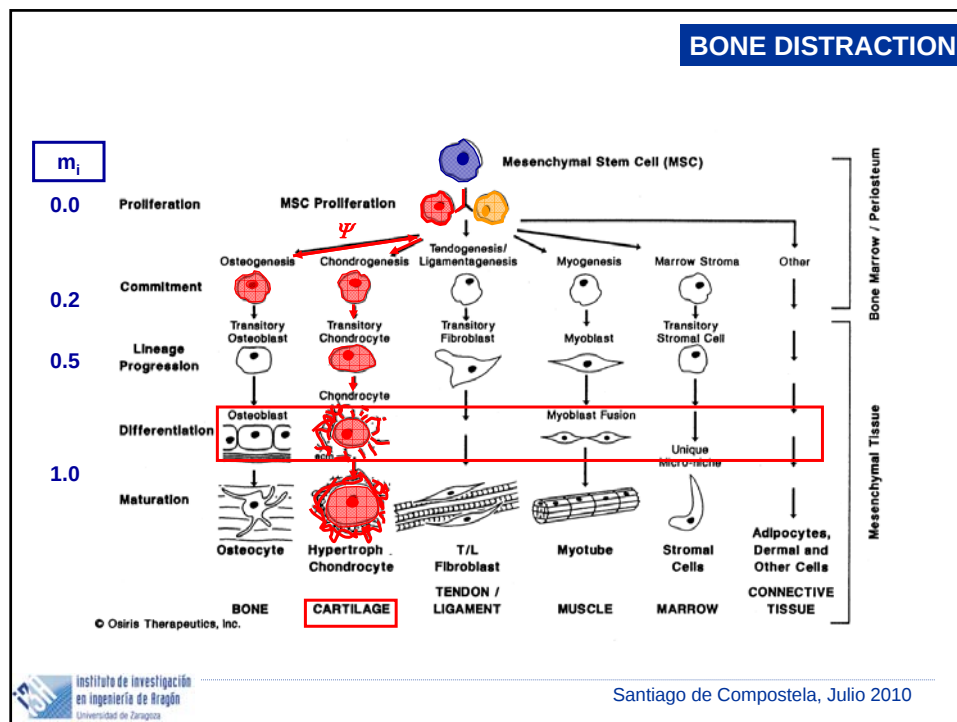
$$N_f = A\Psi^b$$

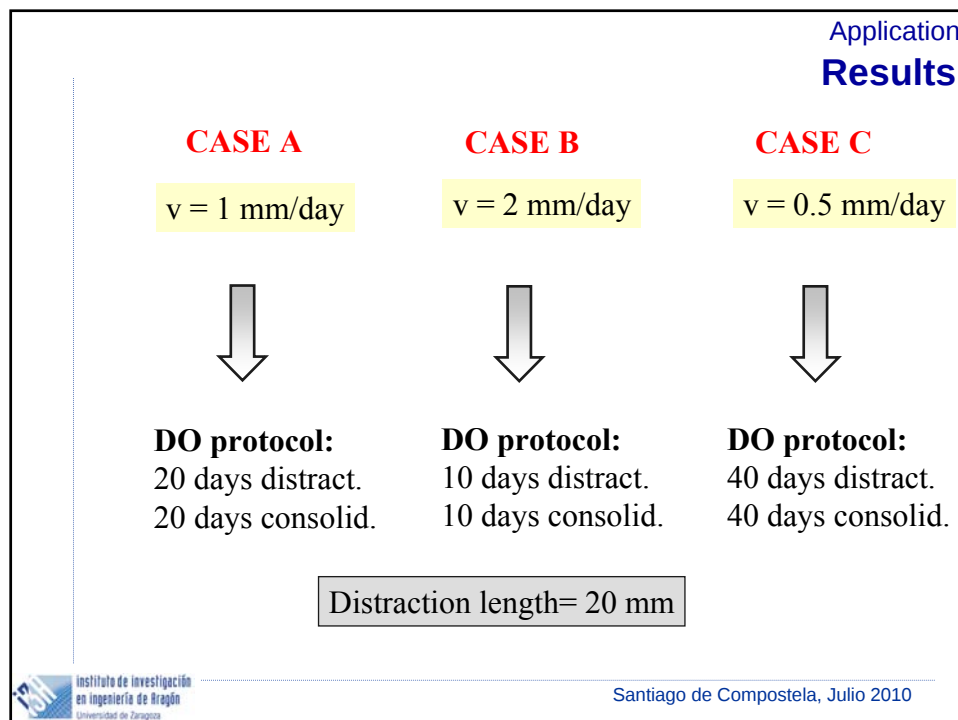
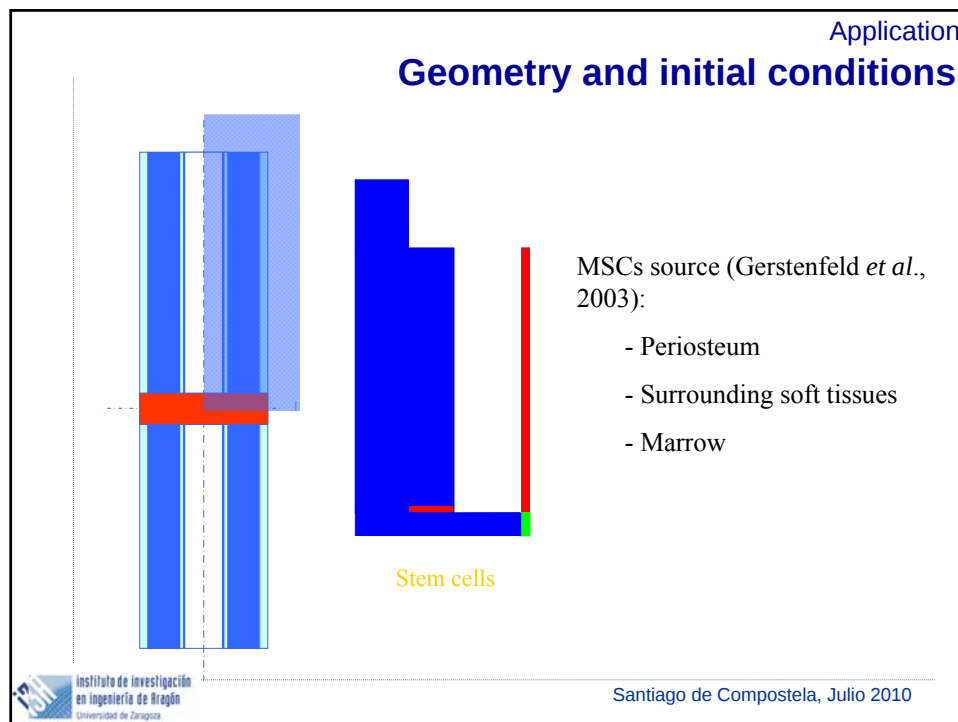
number of days a cell needs to completely
mature at a stimulus level

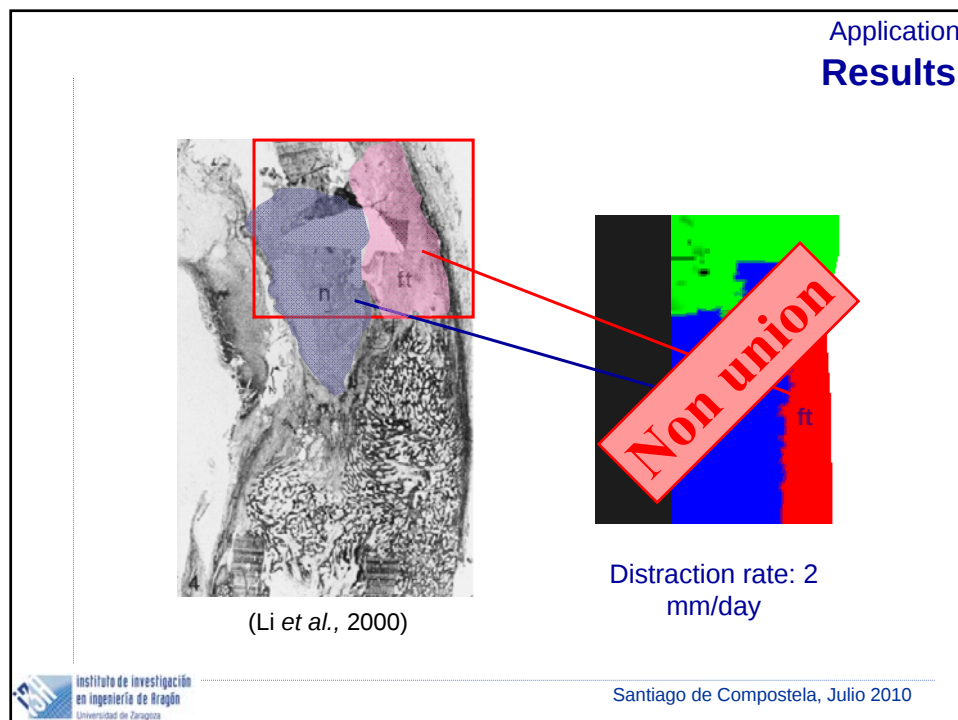
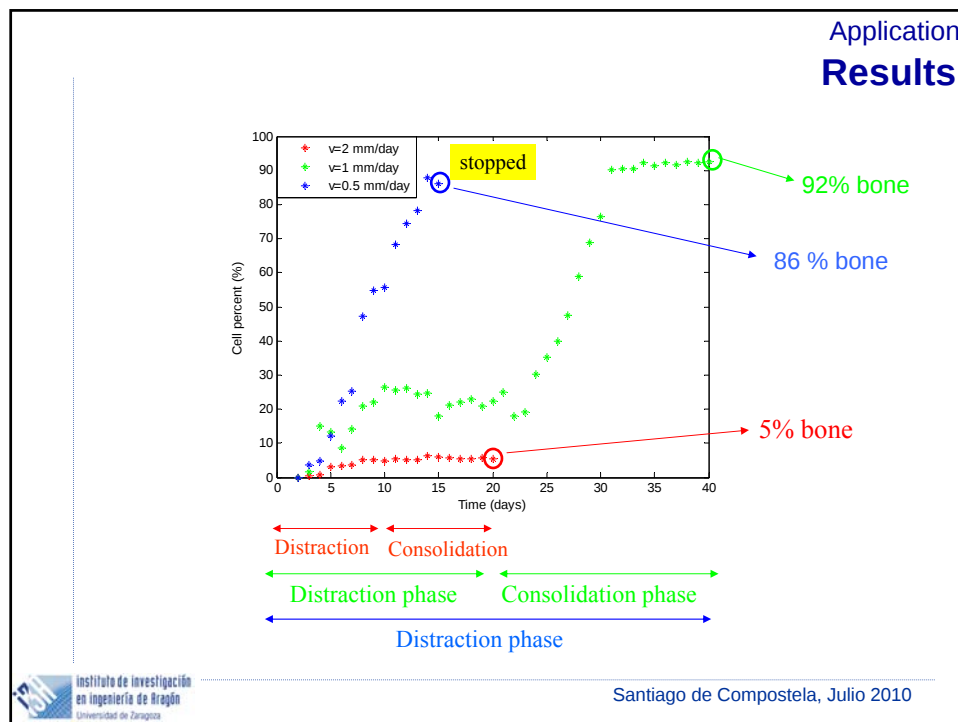
Maturation state of the cell (m_i) – Miner's Rule:

	Bone cells	Fibroblast	cartilage cells
A_i	$1.13 \cdot 10^3$	3.47	137.08
b_i	1.81	-1.36	1.05









Conclusions

- The model of fracture healing (Gómez-Benito et al., 2005) has been extended with the inclusion of the load history in order to predict the main features of distraction osteogenesis under different mechanical environments.
- The inclusion of the load history has allowed the simulation of tissue distributions during limb lengthening at different distraction rates:
 - At low rates, it predicts premature bony union
 - At moderate rates, it predicts successfully bony unions
 - At high rates, it predict fibrous unions or non unions

Conclusions

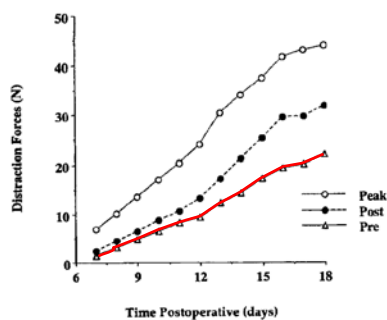
- The main limitations of this model are:
 - the mathematical formulation is developed under the small deformations assumption.
 - we considered a free stress state as initial condition for each day analyzed numerically.
 - the axisymmetric finite element model represents an idealized cylindrical bone under axial loading.

BONE DISTRACTION

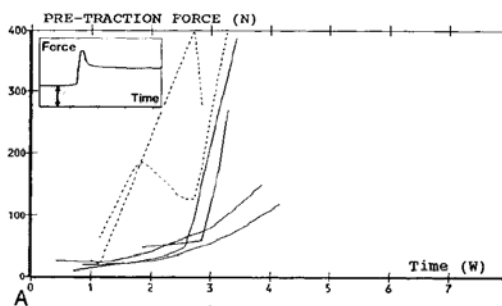
- The application of the mechanobiological model of bone healing to distraction osteogenesis presents **limitations**:
 - It is **not able to predict the effect of different distraction rates** => include the **influence of the load history** on cell differentiation
 - **INCLUDE** the effect of **pre-traction stresses**

BONE DISTRACTION

Experimental Results



(Waanders et al Clin Orthop Relat Res. 1998)



(Brunner et al Clin Orthop Relat Res. 1994)

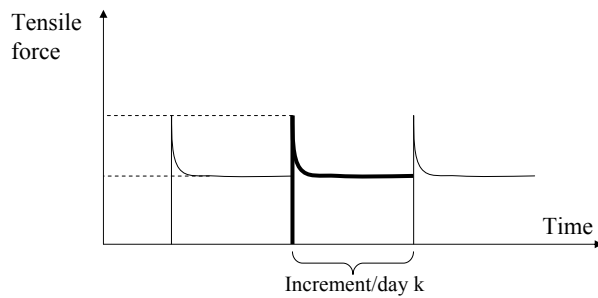


Pre-traction stresses are significant in distraction osteogenesis and need to be considered

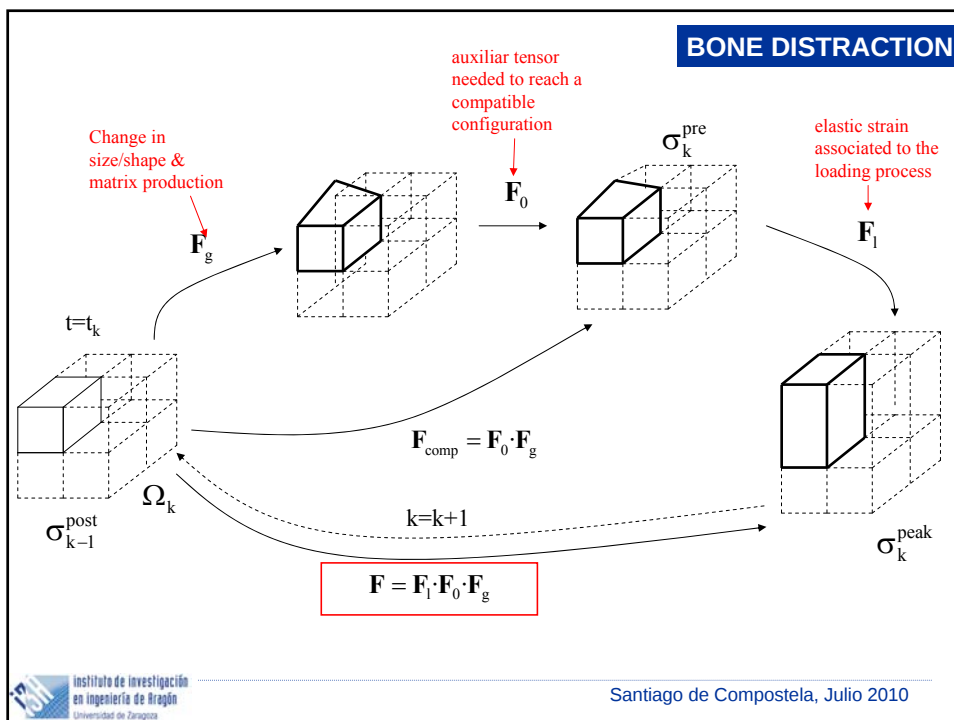
BONE DISTRACTION

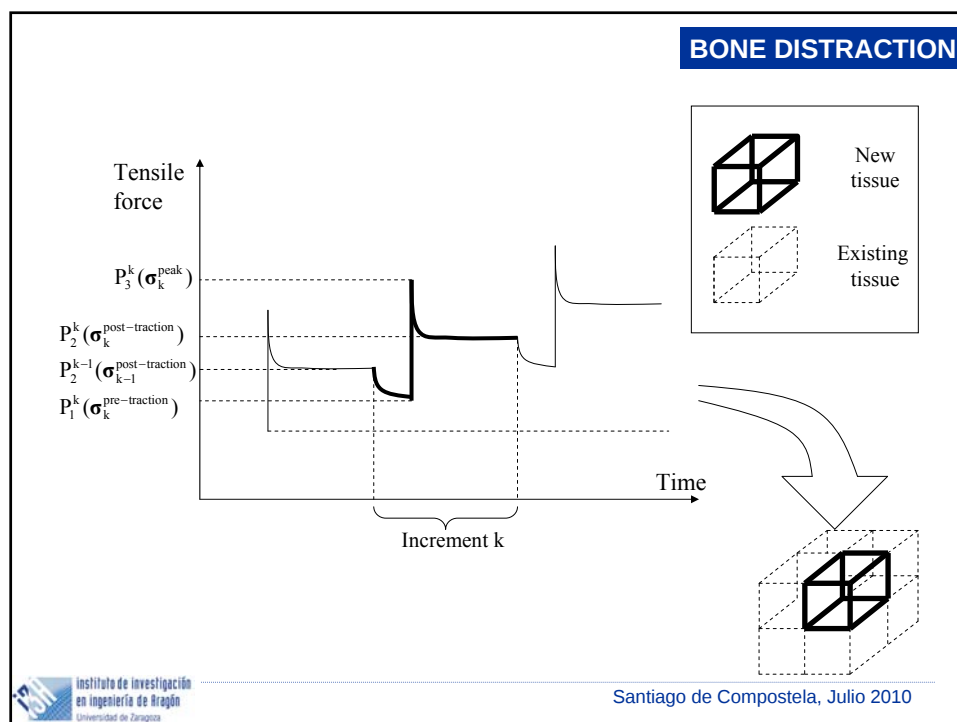
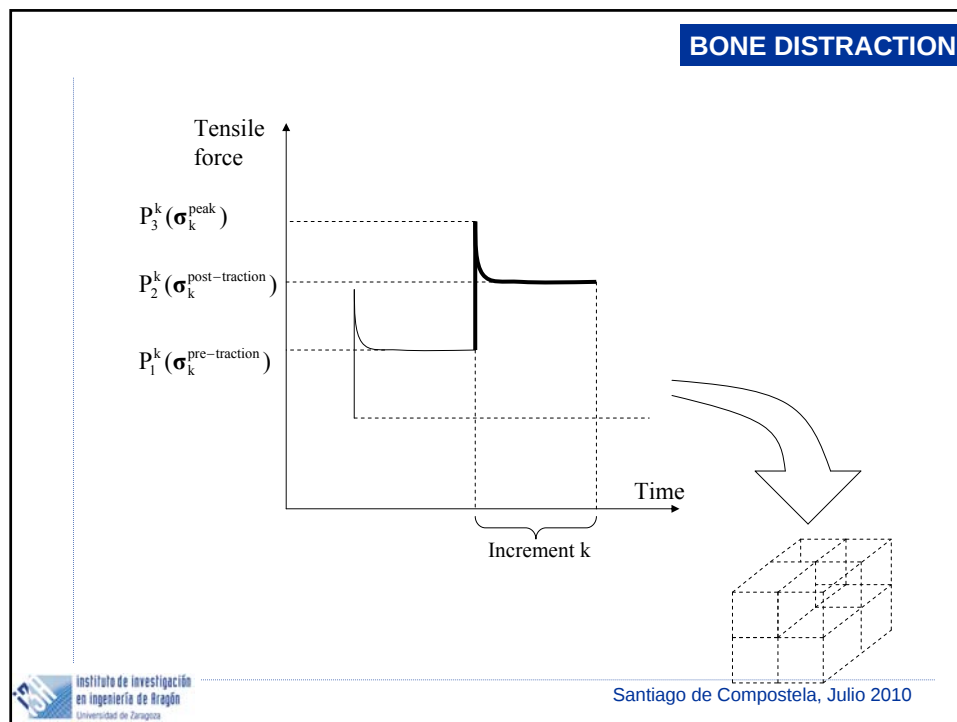
Numerically

Existing mechanobiological models assume that pre-traction stresses are null assuming therefore no stress accumulation after each load step (Isaksson *et al.*, 2007; Boccaccio *et al.*, 2007; Reina-Romo *et al.*, 2009).



BONE DISTRACTION





BONE DISTRACTION

Constitutive equation

$$\boldsymbol{\sigma} = \boldsymbol{\sigma}_{fl} + \boldsymbol{\sigma}_{eff}$$

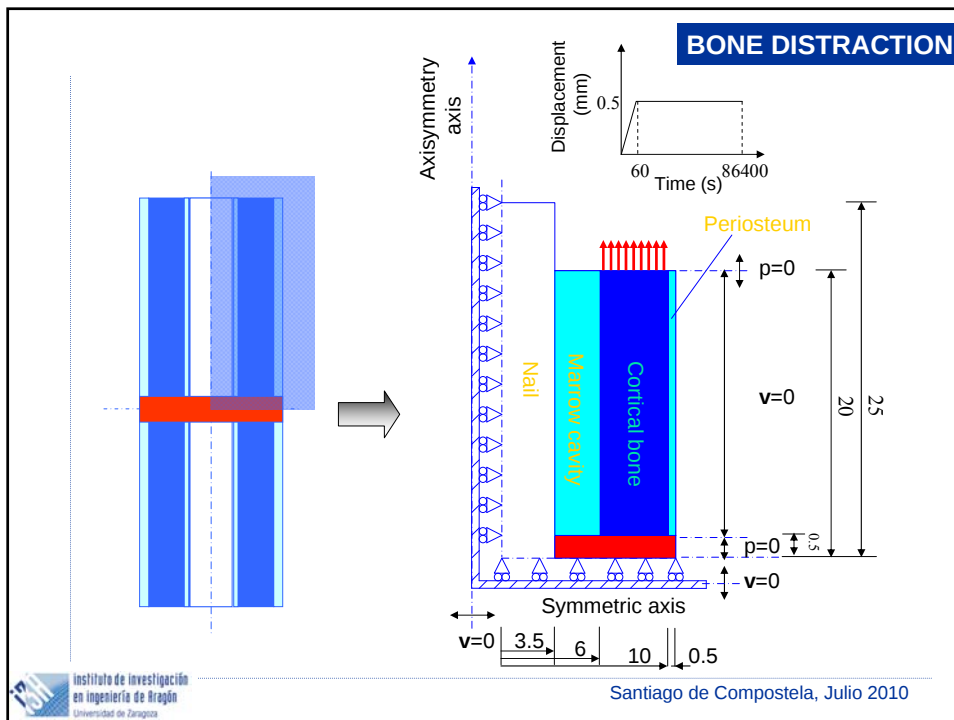
$$\boldsymbol{\sigma}_{fl} = -p\mathbf{1}$$

$$\boldsymbol{\sigma}_{eff} = \sum_{i=1}^N v_{mo}^i \boldsymbol{\sigma}_k^i + \sum_{i=1}^N \frac{1}{\rho^i} \frac{1}{\det \mathbf{F}_k} \int_0^{t_k} \pi^i(\tau) \boldsymbol{\sigma}^i(\mathbf{F}_k, \tau) \det \mathbf{F}_k d\tau$$

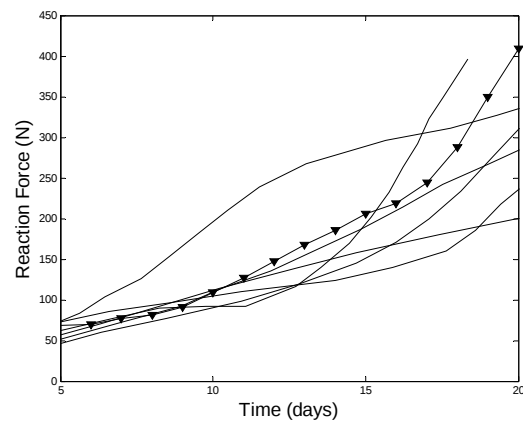
new matrix produced of tissue i per unit volume and time at time τ

stress response for tissue i evaluated at time t_k with respect to the time $\tau \in [0, t_k]$ in which the tissue was created

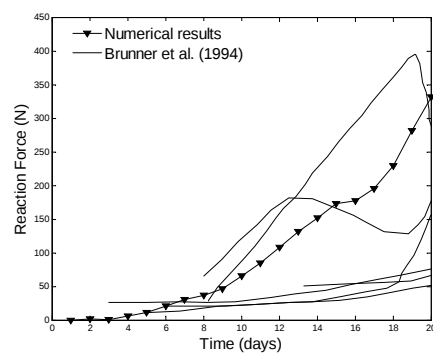
BONE DISTRACTION



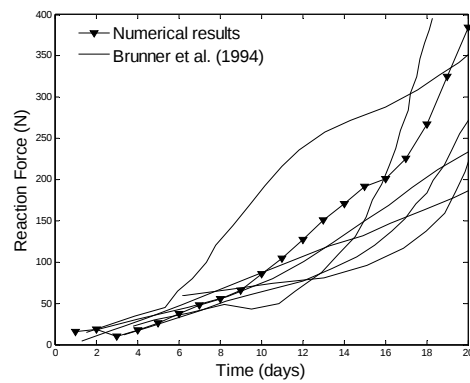
Peak force



Pre-distraction force

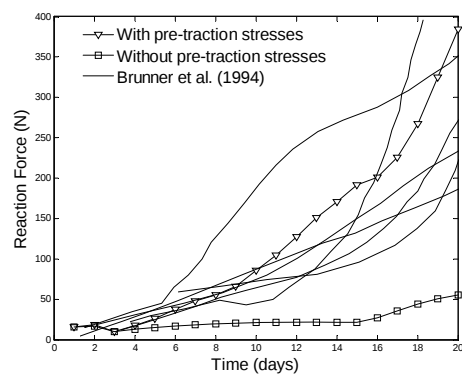


Post-distraction force



BONE DISTRACTION

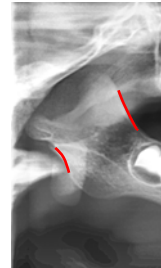
Comparison: with and without pre-traction stresses



BONE DISTRACTION: A CLINICAL CASE



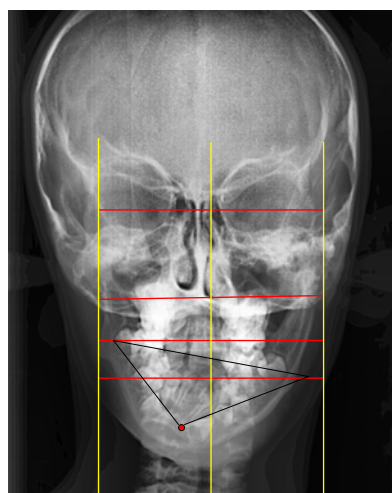
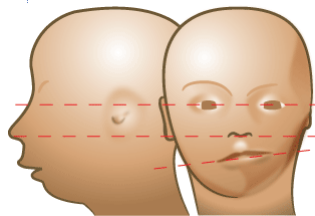
START



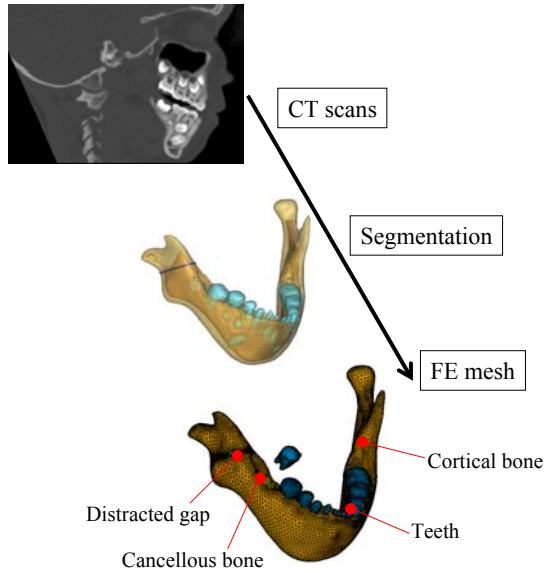
END



BONE DISTRACTION: A CLINICAL CASE



BONE DISTRACTION: A CLINICAL CASE

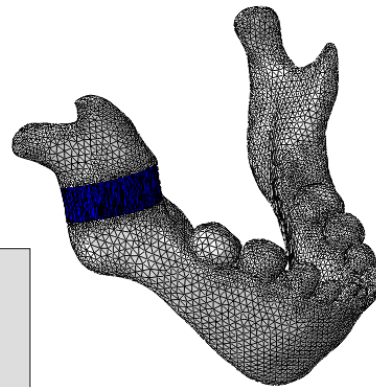


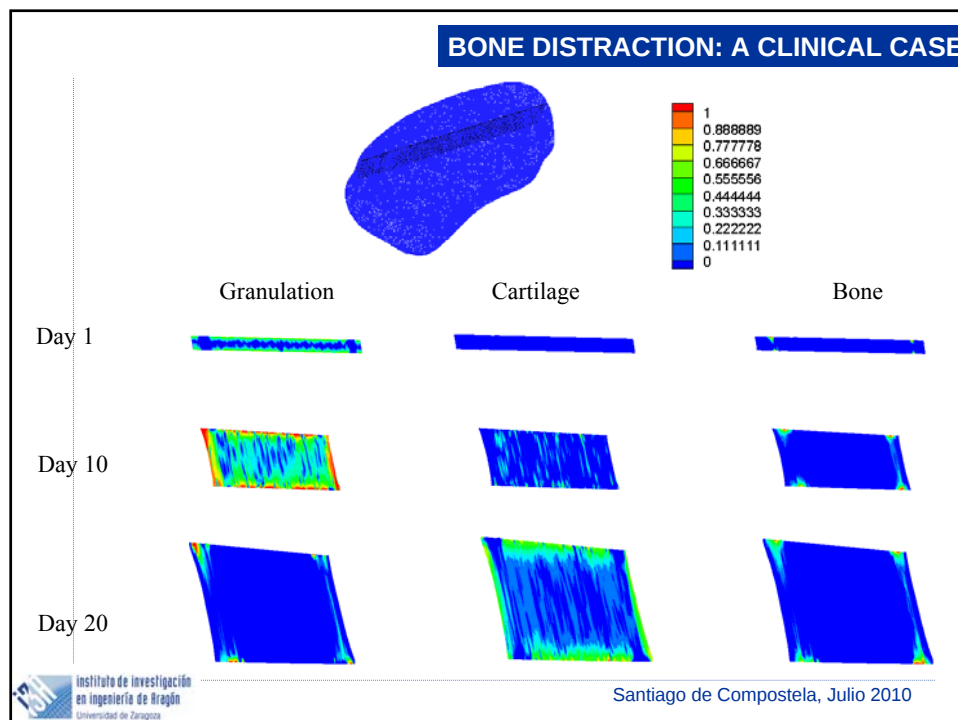
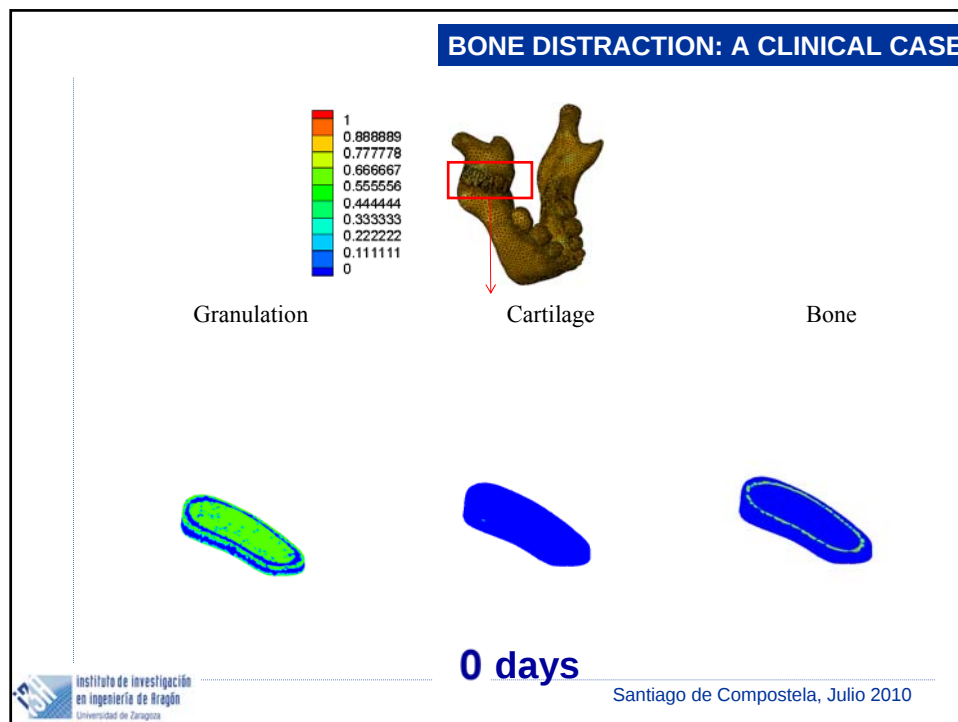
BONE DISTRACTION: A CLINICAL CASE

Initial state
 Geometry, mesh, load, initial conditions

FEA

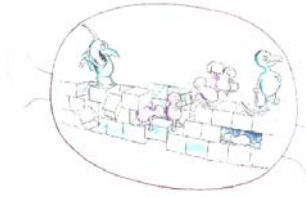
1. **Poroelectric analysis** $\rightarrow \Psi_{\max}, \Psi_{\text{mean}}$
 $\rightarrow$ differentiation, proliferation.
2. **Thermoelastic analysis** $\rightarrow$ gap growth.
3. **Diffusion analysis (MSC)** $\rightarrow$ MSC migration.
4. **Diffusion analysis (osteoblasts)** $\rightarrow$ vascularization



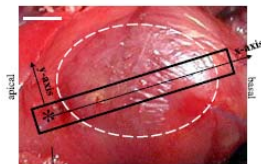


OVERVIEW

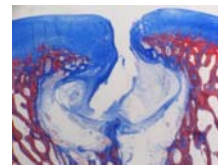
- *Bone properties*
- *Role of mechanical factors: bone mechanobiology*
- *Methods of Science: computer simulation*
- *Modelling Bone Mechanobiology:*
 - *Bone Remodelling*
 - *Bone Healing*
 - *Bone Distraction*
 - *Bone Tissue Engineering*
- *Final conclusions*



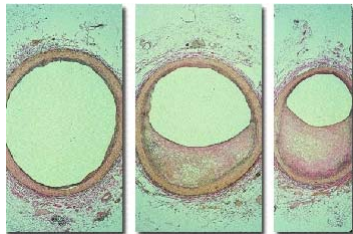
BONE TISSUE ENGINEERING

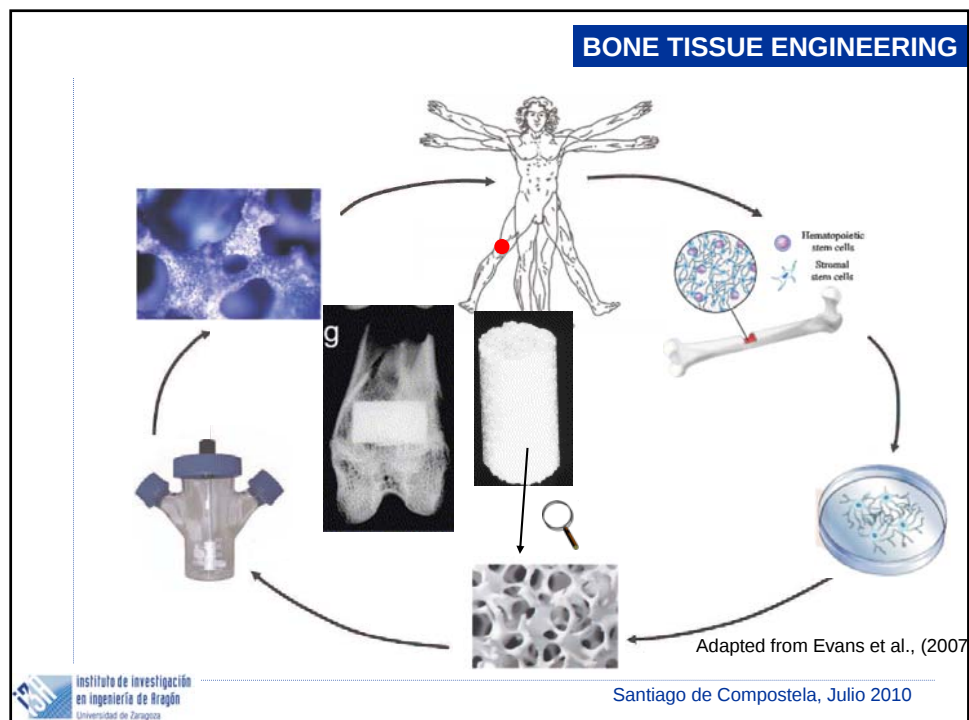
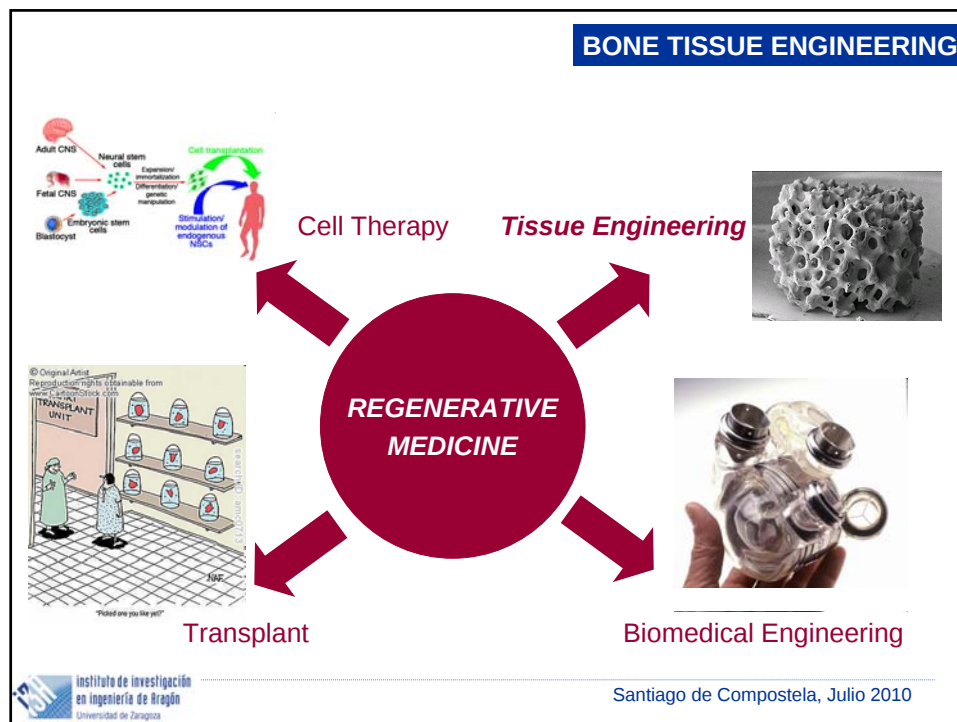


REGENERATIVE MEDICINE:



TO RESTORE THE FUNCTIONALITY OF
DAMAGED TISSUES AND ORGANS





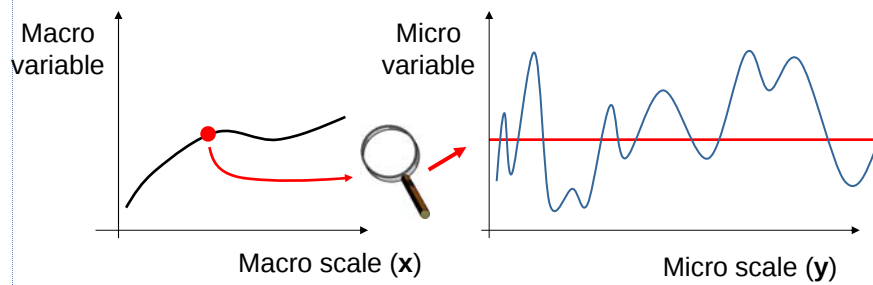
In-silico BTE: literature review

- Modelling from a macroscopic perspective:
 - Simulate osteochondral defects using scaffolds: Kelly and Prendergast (2006).
- Modelling from a microscopic perspective:
 - Macroscopic properties characterization of scaffolds and optimization: Hollister and co-workers (1998-).
 - Simulate bone tissue regeneration and scaffold degradation within a scaffold unit cell. Adachi et al., (2006).
 - Influence of scaffold properties on tissue regeneration: Byrne et al., (2007).

- Objectives:
 - To **evaluate** the potential of **numerical methods** against **experimental measurements**
 - To establish a **multiscale** numerical **model** of BTE taking into account the most relevant biophysical phenomena.
 - Try to elucidate the role of **scaffold parameters** on **bone tissue regeneration** from a numerical point of view.
 - To evaluate the effect of scaffold microstructural **anisotropy** on bone regeneration.
 - Add some light in the task of being BTE a clinical viable reality.

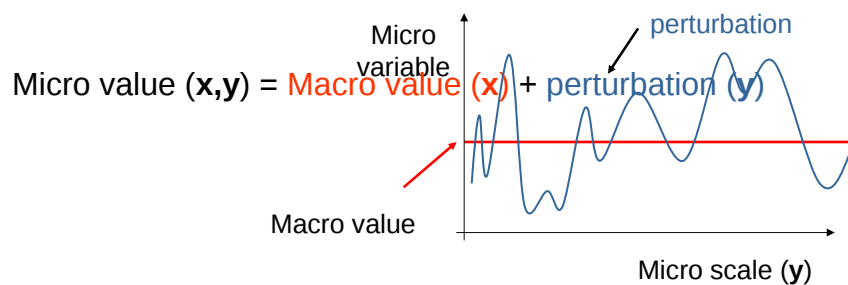
Asymptotic homogenization theory (AHT)

Babuska (1978), Sánchez-Palencia (1980), Suquet (1983), Bakhvalov (1984), Nemat-Nasser (1993), Terada and Kikuchi (1998)...



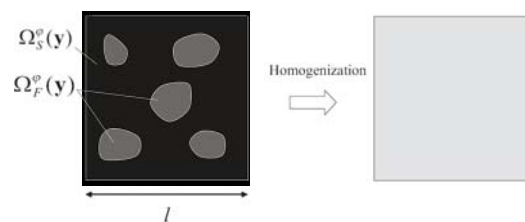
Asymptotic homogenization theory (AHT)

Babuska (1978), Sánchez-Palencia (1980), Suquet (1983), Bakhvalov (1984), Nemat-Nasser (1993), Terada and Kikuchi (1998)...

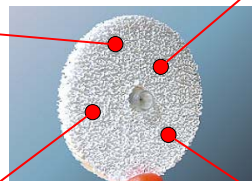
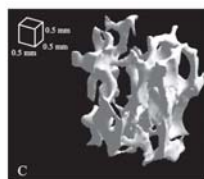
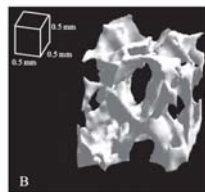


Asymptotic homogenization theory (AHT)

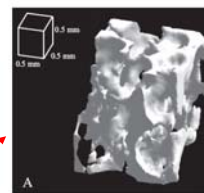
- Biphasic microstructure:
 - Solid domain: mechanics $\Omega_S^\varphi(y) \Rightarrow \mathbf{C}(\mathbf{x})$
 - Fluid domain: fluids $\Omega_F^\varphi(y) \Rightarrow \mathbf{K}(\mathbf{x})$

Application of AHT: properties
characterization of scaffolds

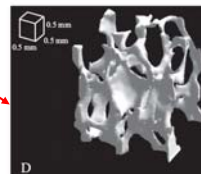
- Numerics. Solid domain:
 - Geometry,



Sponceram® carriers

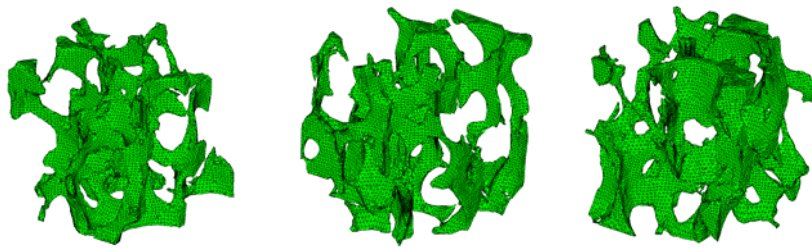


In collaboration with Dr. C. Kasper



Application of AHT: properties characterization of scaffolds

- Numerics. Solid domain:
 - Finite element meshes (RVEs),



In collaboration with Dr. C. Kasper

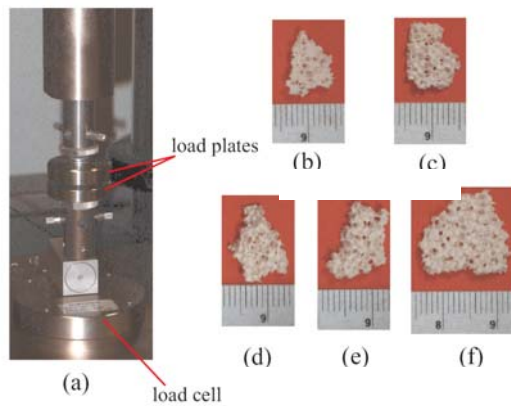
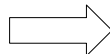
Application of AHT: properties characterization of scaffolds

- Experiments. Solid domain:
 - Setup,

In collaboration with Dr. C. Kasper



Sponceram® carriers



Application of AHT: properties characterization of scaffolds

- Numerics vs Experiments. Solid domain:

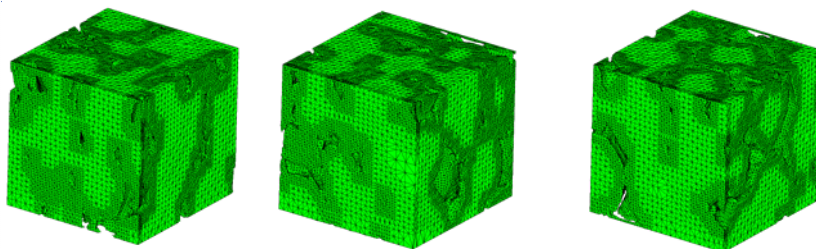
	Porosity %	Experimental E (MPa)	Numerical E (MPa)
S20-90	90.16±0.95	8.73±4.64	8.21±3.06
S30-90	88.75±0.36	11.03±2.24	9.07±1.43
S30-90HA	79.81±2.18	29.40±1.21	30.63±2.61

Sanz-Herrera et al., (JBMRB,2008)

In collaboration with Dr. C. Kasper

Application of AHT: properties characterization of scaffolds

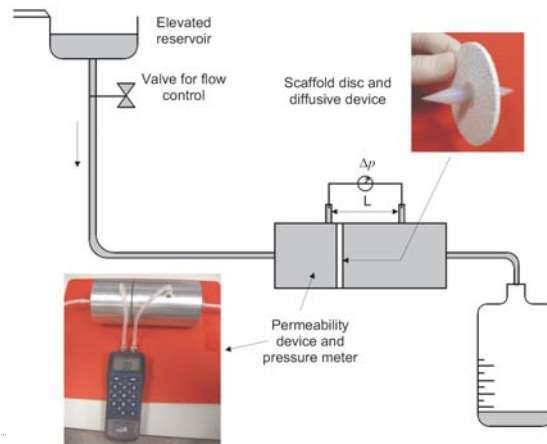
- Numerics. Fluid domain:
 - Finite element meshes (RVEs),



Application of AHT: properties characterization of scaffolds

- Experiments. Fluid domain:

- Setup,



In collaboration with Dr. C. Kasper

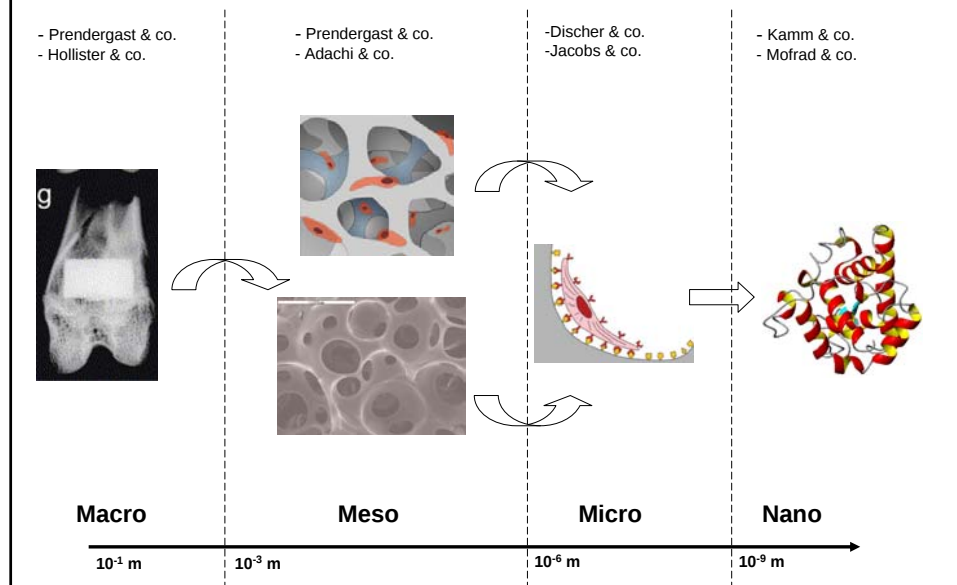
Application of AHT: properties characterization of scaffolds

- Numerics vs Experiments. Solid domain:

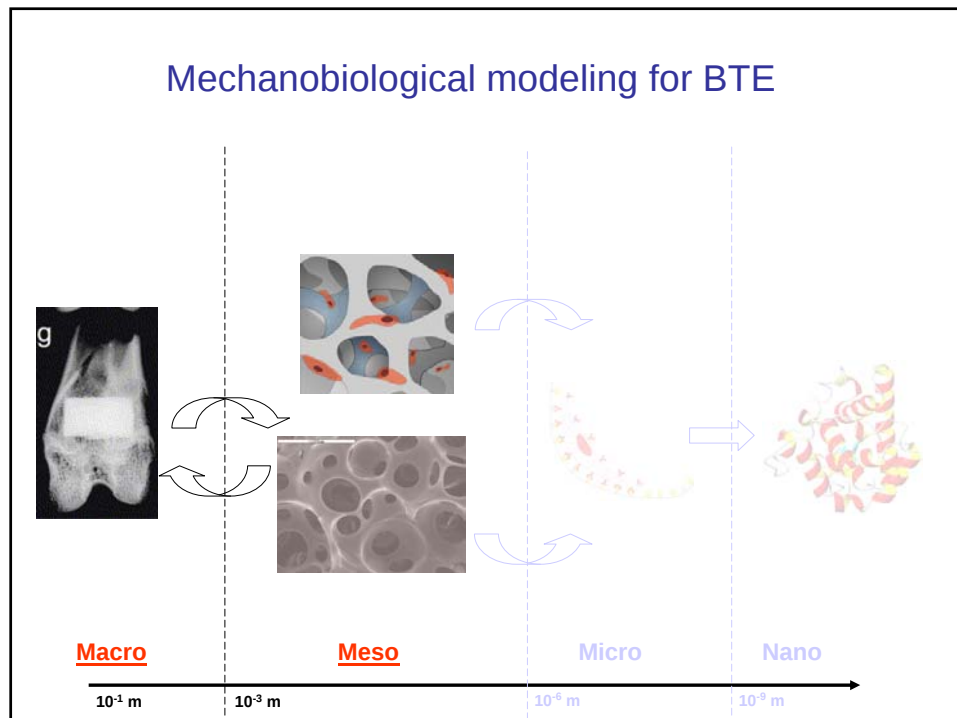
	Porosity (%)	Experimental $k \text{ (m}^2\text{)}$	Numerical $k \text{ (m}^2\text{)}$
S20-90	90.16 ± 0.95	$3.76 \cdot 10^{-8} \pm 4.38 \cdot 10^{-9}$	$3.49 \cdot 10^{-8} \pm 2.56 \cdot 10^{-9}$
S30-90	88.75 ± 0.36	$3.17 \cdot 10^{-8} \pm 3.62 \cdot 10^{-9}$	$2.31 \cdot 10^{-8} \pm 6.25 \cdot 10^{-10}$
S30-90HA	79.81 ± 2.18	$1.79 \cdot 10^{-8} \pm 4.09 \cdot 10^{-9}$	$1.88 \cdot 10^{-8} \pm 1.25 \cdot 10^{-9}$

Sanz-Herrera et al., (JBMRB, 2008)

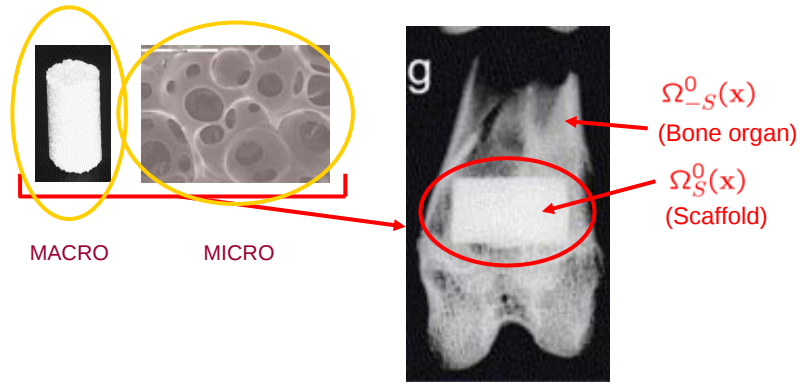
Mechanobiological modeling for BTE



Mechanobiological modeling for BTE



Mechanobiological modeling for BTE



Mechanobiological modeling for BTE



Bone organ:

- Remodelling and adaptation

↑ MACRO

Mechanobiological modeling for BTE

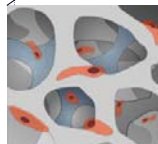


Scaffold:

- Cell migration
- Mechanics

↑
MACRO

Mechanobiological modeling for BTE



Scaffold microstructure:

- Homogenization (fluid and solid phases)
- Localization problem (solid phase)
- Microscopic model of bone growth
- Scaffold resorption

↑
MACRO

↓
MICRO

Analysis of the macroscale (bone organ)



Bone organ:

- Remodelling and adaptation

↑
MACRO

Analysis of the macroscale (bone organ)

- Macroscopic mathematical models of bone remodelling:
 - **Homeostatic value of a mechanical stimulus:** Pauwels (1960); Carter et al., (1987); Huiskes et al., (1987, 2000); **Beaupré et al., (1990)**; Cowin et al., (1992).
 - **Global optimality criteria:** Terrier et al., (1997); Fernandes et al., (1999); Adachi et al., (2001).
 - **Damage repair models:** Prendergast and Taylor (1994); Zidi and Ramtani (2000); Doblaré and García (2001); Hazelwood et al. (2001); García-Aznar et al. (2005)
- As a first approach, we use the isotropic model proposed by Beaupré et al. (1990).

Analysis of the macroscale (scaffold)



Scaffold:

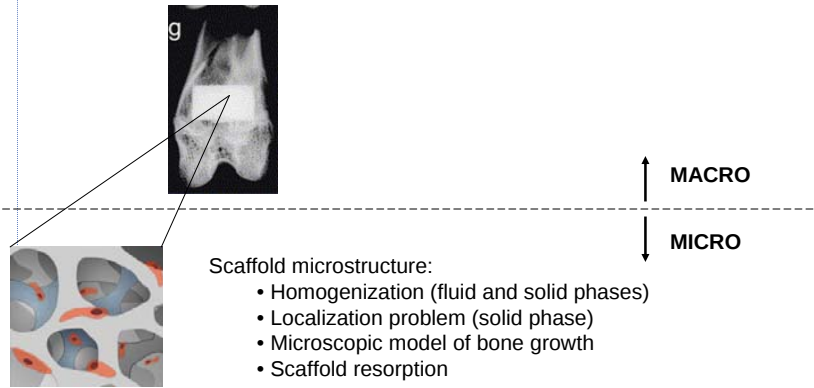
- Cell migration
- Mechanics

↑ **MACRO**

Analysis of the macroscale (scaffold)

- Model of scaffold bone ingrowth (Ohgushi and Caplan, 1999):
 - Insertion of the scaffold: immunological response and hematoma. Osteoprogenitor and MSC's migration.
 - Cell attachment onto the scaffold microsurface and biolayer formation.
 - Cell differentiation to osteoblasts.
 - Osteoblasts synthesize immature (woven or primary) bone.
 - Primary bone becomes mature and fully mineralized bone.

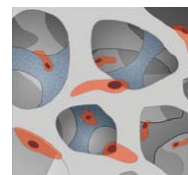
Analysis of the microscale (scaffold)



Analysis of the microscale (scaffold)

- Bone growth,

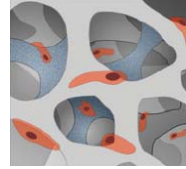
$$\dot{M} = \dot{M}(\bar{c}, \Psi) \quad \text{in} \quad \partial\Omega_S^{\mathcal{Q}}(\mathbf{y})$$



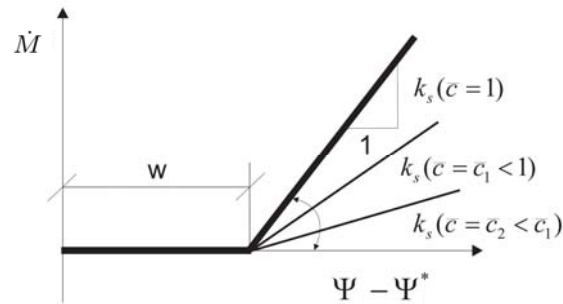
Analysis of the microscale (scaffold)

- Bone growth,

$$\dot{M} = \dot{M}(\bar{c}, \Psi) \quad \text{in} \quad \partial\Omega_S^\varphi(\mathbf{y})$$



$$\dot{M} = \begin{cases} k_s(\bar{c}) \cdot (\Psi - \Psi^* - w) & \text{if } \Psi - \Psi^* > w \\ 0 & \text{otherwise} \end{cases}$$

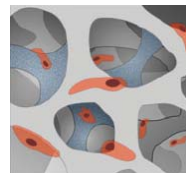


Hypothesis: no bone resorption during healing (Biskobing, 2000).

Analysis of the microscale (scaffold)

- Bone growth,

$$\dot{M} = \dot{M}(\bar{c}, \Psi) \quad \text{in} \quad \partial\Omega_S^\varphi(\mathbf{y})$$



$$\dot{M} = \begin{cases} k_s(\bar{c}) \cdot \boxed{\Psi} - \Psi^* - w & \text{if } \Psi - \Psi^* > w \\ 0 & \text{otherwise} \end{cases}$$

Evaluated microscopically through,

$$\epsilon^\varphi(\mathbf{u}^\varphi) = \epsilon_{kh}^0(\mathbf{I}_{kh} + \epsilon^\varphi(\chi_{kh}))$$

$$\Psi = \sum_{i=1}^N \left[n_i^{1/m} \left(\frac{\hat{\rho}}{\rho} \right)^2 \bar{\sigma}_i \right] \quad \bar{\sigma}_i = \sqrt{2EW}$$

Analysis of the microscale (scaffold)

- Scaffold degradation, hydrolysis, (Adachi et al., 2006),

Analysis of the microscale (scaffold)

- Scaffold degradation, hydrolysis, (Adachi et al., 2006),
 - Water content,

$$\dot{d} = \alpha \Delta d \quad \text{in} \quad \Omega_S^{\varphi}(y) \\ + \text{boundary and initial conditions}$$

Analysis of the microscale (scaffold)

- Scaffold degradation, hydrolysis, (Adachi et al., 2006),
 - Water content,

$$\dot{d} = \alpha \Delta d \quad \text{in} \quad \Omega_S^\varphi(\mathbf{y})$$

+ boundary and initial conditions

- Molecular weight rate,

$$\dot{W}(d) = -\beta d$$

Analysis of the microscale (scaffold)

- Scaffold degradation, hydrolysis, (Adachi et al., 2006),
 - Water content,

$$\dot{d} = \alpha \Delta d \quad \text{in} \quad \Omega_S^\varphi(\mathbf{y})$$

+ boundary and initial conditions

- Molecular weight rate,

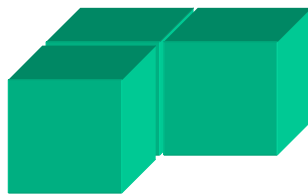
$$\dot{W}(d) = -\beta d$$

- Mechanical properties,

$$\mathbb{C}^\varphi(W(t)) = \mathbb{C}^\varphi(t=0) \frac{W(t)}{W_0} \quad \text{in} \quad \Omega_S^\varphi(\mathbf{y})$$

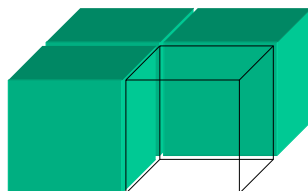
Computational implementation: voxel-FEM

- Bone growth



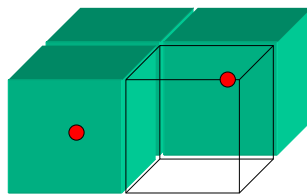
Computational implementation: voxel-FEM

- Bone growth
 - Identify voxel candidates.



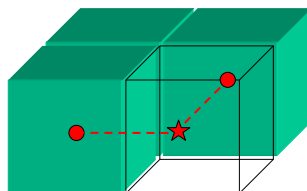
Computational implementation: voxel-FEM

- Bone growth
 - Identify voxel candidates.
 - Compute M at adjacent voxel candidate centroids.



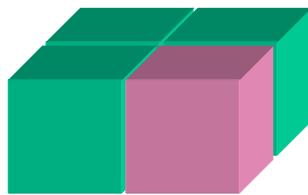
Computational implementation: voxel-FEM

- Bone growth
 - Identify voxel candidates.
 - Compute M at adjacent voxel candidate centroids.
 - Lagrangian interpolation of M to voxel candidate centroid.



Computational implementation: voxel-FEM

- Bone growth
 - Identify voxel candidates.
 - Compute M at adjacent voxel candidate centroids.
 - Lagrangian interpolation of M to voxel candidate centroid.
 - If $M > M_c$ then shift voxel candidate into bone voxel.



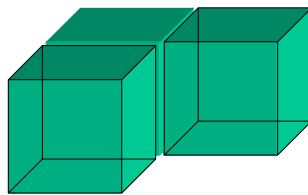
Computational implementation: voxel-FEM

- Scaffold degradation



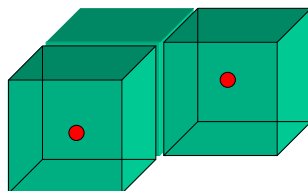
Computational implementation: voxel-FEM

- Scaffold degradation
 - Identify boundary voxels.



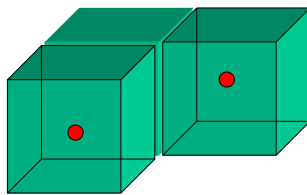
Computational implementation: voxel-FEM

- Scaffold degradation
 - Identify boundary voxels.
 - Compute W at boundary voxel centroids.



Computational implementation: voxel-FEM

- Scaffold degradation
 - Identify boundary voxels.
 - Compute W at boundary voxel centroids.
 - If $W < W_c$,



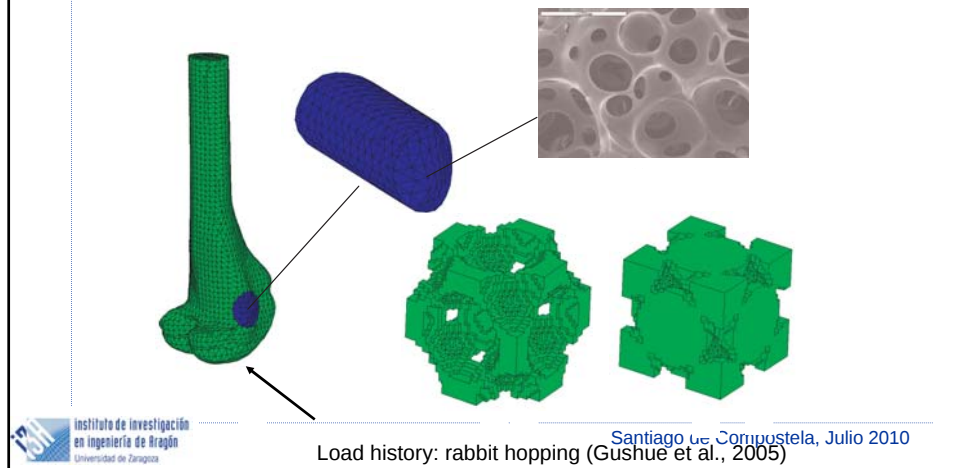
Computational implementation

- Scaffold degradation
 - Identify boundary voxels.
 - Compute W at boundary voxel centroids.
 - If $W < W_c$,
 - Then remove boundary voxel.



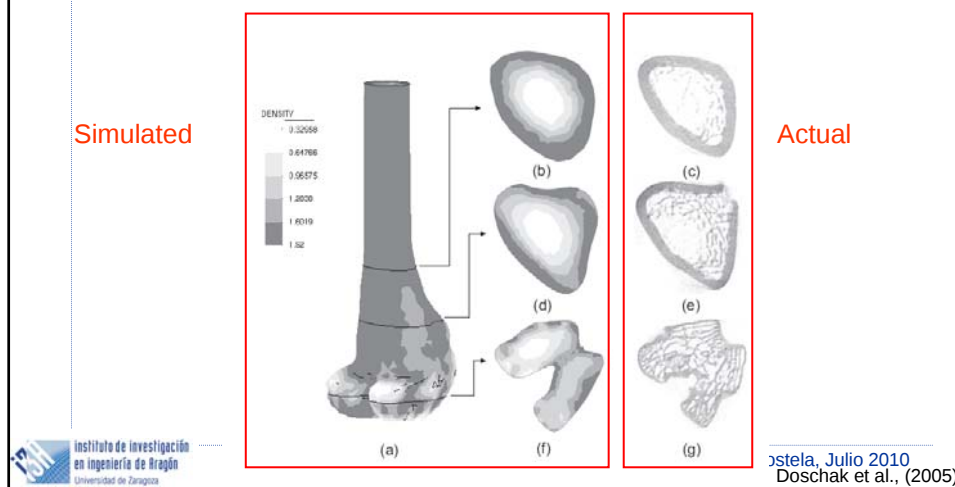
Example application: Distal femoral condyle

Problem definition (Sanz-Herrera et al, CMAME 2008)

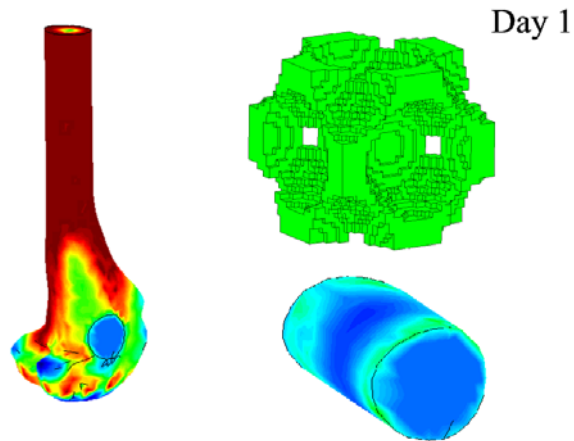


Example application: Distal femoral condyle

- Initial density distribution (g/cc),

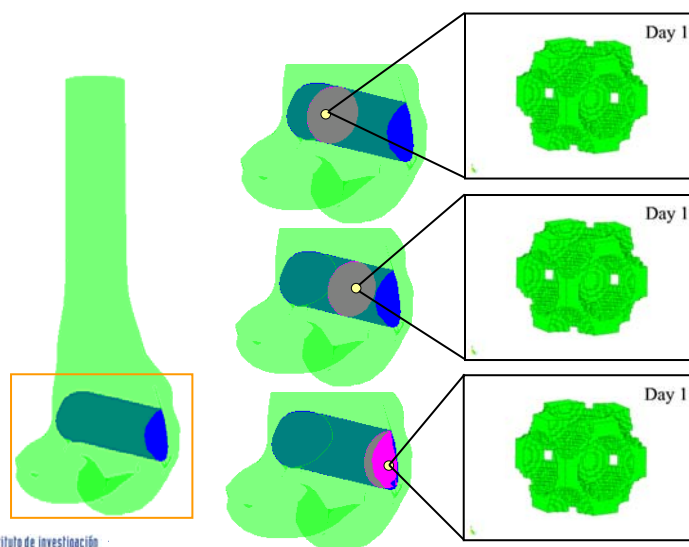


Example application: Distal femoral condyle



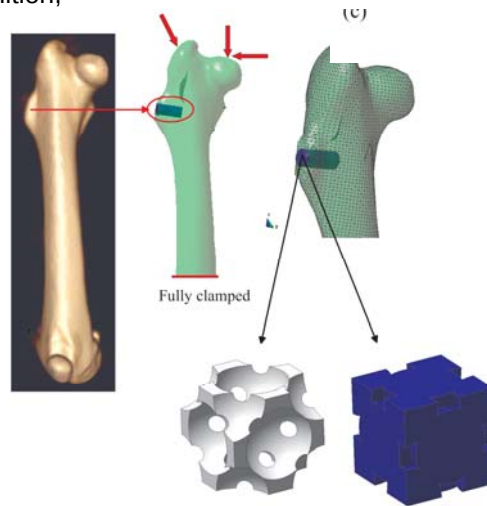
Example application: Distal femoral condyle

- Multiscale results,



Proximal femur: parametric analysis

- Problem definition,



Proximal femur: parametric analysis

- Parametric analysis, effects of:
 - Porosity.
 - Pore size.
 - Biomaterial stiffness.
 - Resorption kinetics.
 - Scaffold pre-seeding.

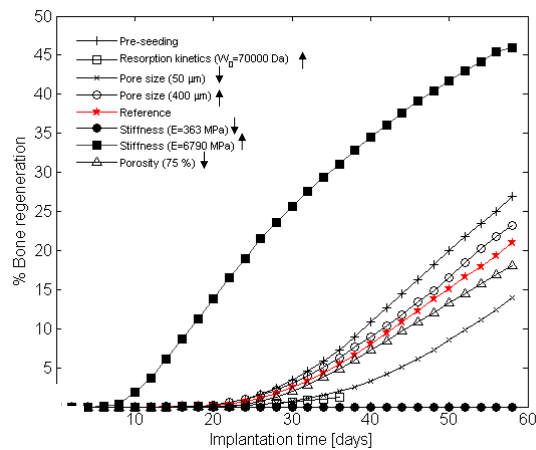
Reference scaffold parameters

Parameter	Description	Value
Φ	Scaffold porosity	0.9
E	Bulk biomaterial Young's modulus (MPa)	1570
ν	Bulk biomaterial Poisson's ratio	0.39
α	Hydrolysis model parameter ($\text{mm}^2 \text{day}^{-1}$)	4×10^{-4}
β	Hydrolysis model parameter (day^{-1})	4000
W_0	Bulk biomaterial molecular weight (Da)	114,000
W_c	Molecular weight value under which the biomaterial degrades (Da)	10,000
R	Pore size (μm)	200

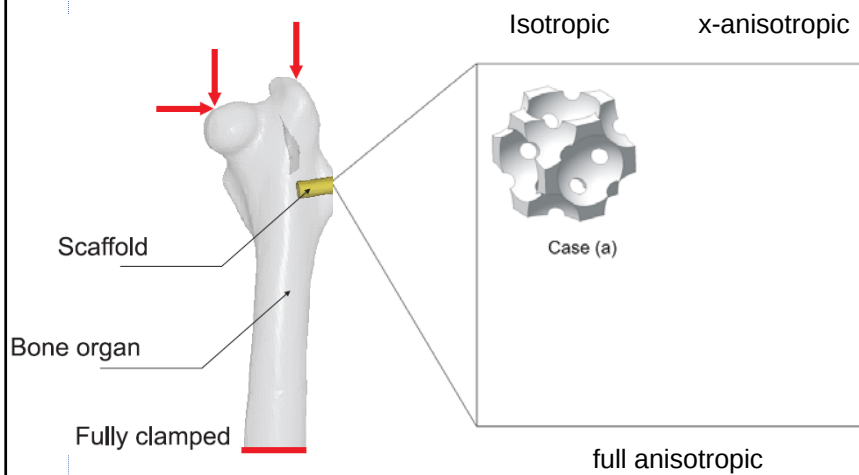
Proximal femur: parametric analysis

- Macroscopic quantities: bone regeneration,

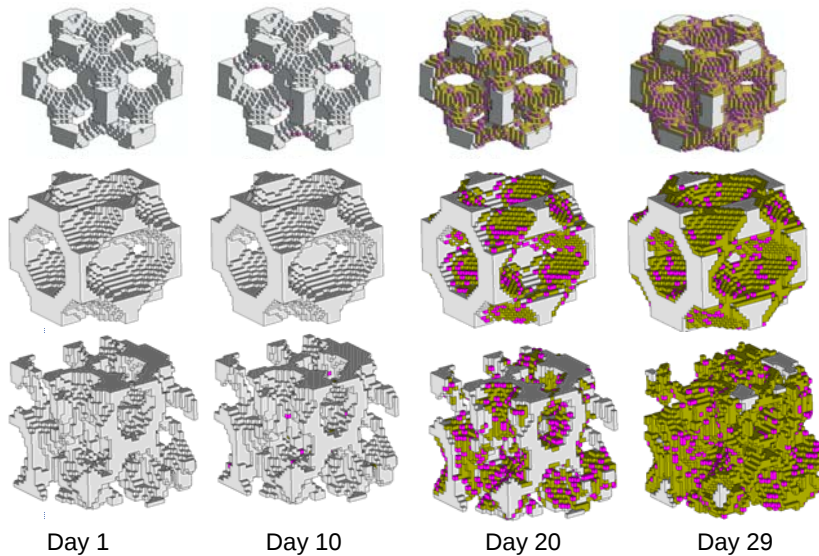
Acta Biomaterialia (2008)



Analysis of scaffold microstructural anisotropy



Analysis of scaffold microstructural anisotropy

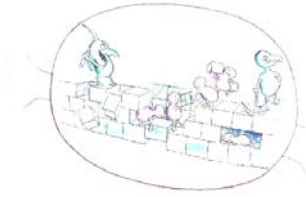


5.1. Some Conclusions

- The evolutionary microstructural geometry results into a non-usual non-linear problem in mechanics.
- Bone remodelling due to the implantation of a scaffold is negligible except in a localized zone surrounding the scaffold.
- Bone growth at scaffold microsurfaces occurs isotropically for symmetric patterns.
- Scaffold microstructure determines the subsequent architecture of the formed bone.
- When anisotropy is present in the scaffold microarchitecture, bone regeneration tends to follow the direction of maximum stiffness.

OVERVIEW

- *Bone properties*
- *Role of mechanical factors: bone mechanobiology*
- *Methods of Science: computer simulation*
- *Modelling Bone Regeneration:*
 - *Bone Healing*
 - *Bone Distraction*
 - *Bone Tissue Engineering*
- *Final conclusions*



FINAL CONCLUSIONS

- ❖ These models require further experimental validation.
- ❖ Check the influence of different mechanical stimulus and other mechanoregulation rules
- ❖ Incorporate additional coupled biophysical stimuli: **multiphysics** analysis
- ❖ Going down through scales trying to explore mechanotransduction at cellular level in connection with organ level: **multiscale** analysis
- ❖ But multiscale and multiphysics approach require a **high computational cost**

**Development of new numerical techniques
that allow to reduce this cost**

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and Material Modelling

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