

A Deterministic Model for Computing the Radiation Dose in Cancer Treatment

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ABSTRACT

When a X-ray beam (i.e. a beam of photons) passes into an absorbing medium such as body tissues, some of the energy carried by the beam is transferred to the medium where it may produce biological damage. The energy deposited per unit mass of the medium is known as the absorbed dose and is a very useful quantity for the prediction of biological effects. The deterministic model for dose calculation consists of two integro-differential transport equations and the expression for the absorbed dose. In order to evaluate the absorbed dose we should know the amount of the photons in the tissue and the amount of the electrons (primary and secondary electrons). The transport equation for the photons was solved in [2]. In this work we give the Fokker-Planck approximation of the Boltzmann transport equation for electrons in a slab like geometry.

RESULTS

Approximation of the Boltzmann transport equation for the electrons is written as the Fokker-Planck asymptotic equation, which is

$$\Omega_e \cdot \nabla \psi_e(r, \Omega_e, \epsilon_e) - [T_{Mott}(r, \epsilon_e) + T_M(r, \epsilon_e)] L \psi_e - \frac{\partial}{\partial \epsilon_e} [S_M(r, \epsilon_e) \psi_e] = Q(r, \Omega_e, \epsilon_e) \quad (1)$$

In order to obtain the discretization of (1), we divide the boundary (∂Q) of the domain (Q) which we are working into two parts: irradiated part Γ and the rest surface Λ ($\partial Q = \Gamma \cup \Lambda$).

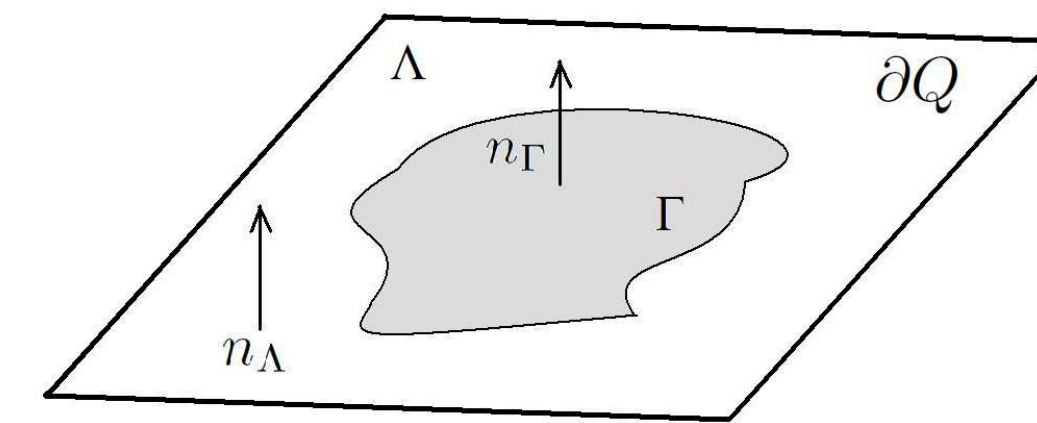


Fig. 1: For the sake of simplicity, the surface depicted is a rectangle.

The boundary conditions are chosen as

$$\begin{aligned} \psi_e(r, \Omega_e, \epsilon_e)|_{\Gamma} &= \psi_e^0(s, \Omega_e, \epsilon_e), \quad \text{for } n_{\Gamma} \cdot \Omega_e < 0, \\ \psi_e(r, \Omega_e, \epsilon_e)|_{\Lambda} &= 0, \quad \text{for } n_{\Lambda} \cdot \Omega_e < 0, \end{aligned} \quad (2)$$

n_{Γ} and n_{Λ} being the outer normal of Γ and Λ .

As we are considering slab like geometry, we consider that absorbed dose depends only on depth z , which is leading to a mathematically one-dimensional problem in space.

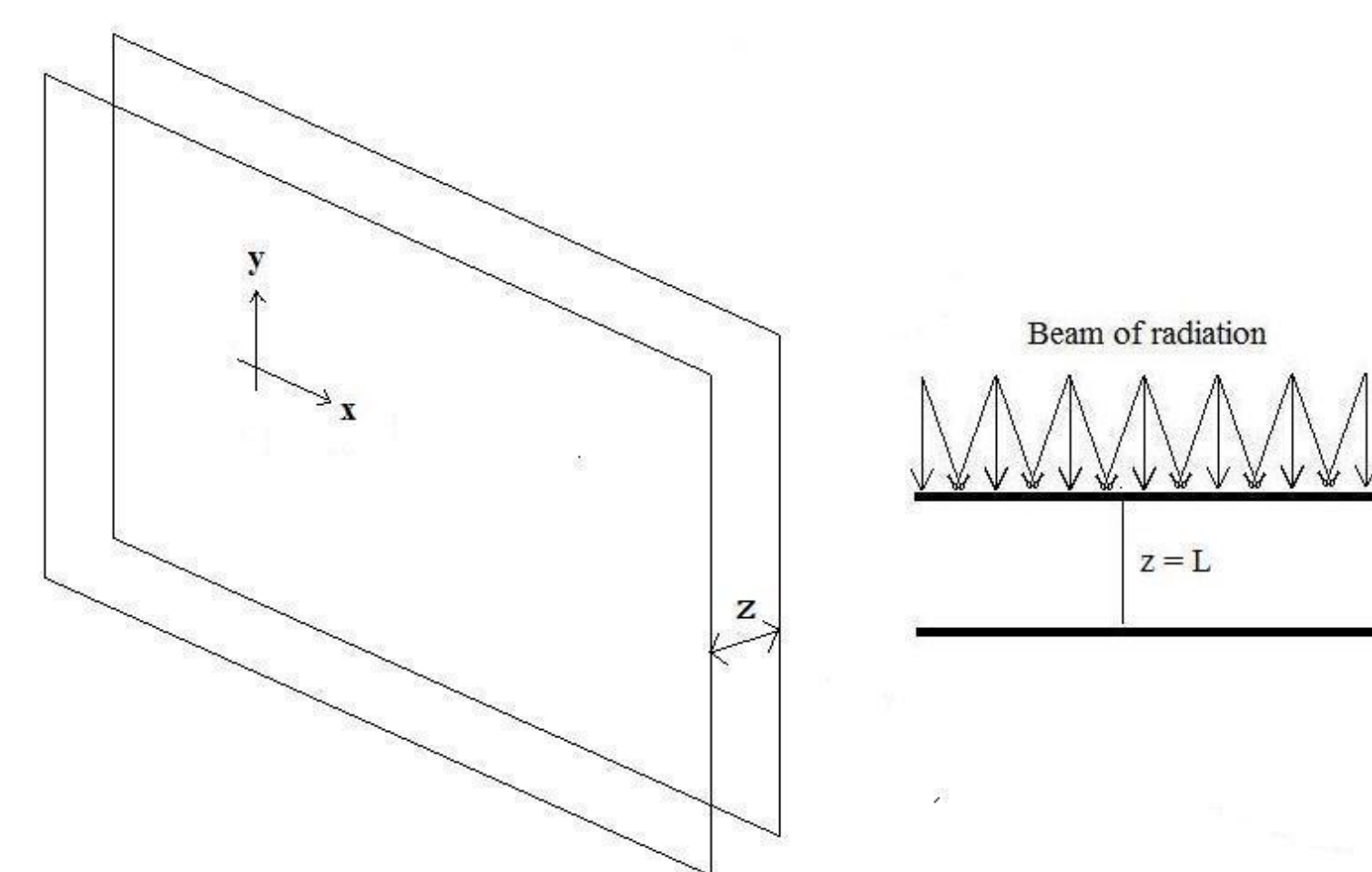


Fig. 2: The slab geometry

$$\begin{aligned} \mu_e \frac{\partial}{\partial z} \psi_e(z, \mu_e, \epsilon_e) - [T_{Mott}(z, \epsilon_e) + T_M(z, \epsilon_e)] L \psi_e - \frac{\partial}{\partial \epsilon_e} [S_M(z, \epsilon_e) \psi_e] &= Q(z, \mu_e, \epsilon_e) \end{aligned} \quad (3)$$

The $(z-\mu_e-\epsilon_e)$ -domain is subdivided into a grid. Substituting the approximations of the derivatives to the Fokker-Planck equation we will have

$$\begin{aligned} & -\frac{\mu_j}{\Delta z_{i-1}} \psi_{i-1,j,k} - A_{i,k} \frac{1 - \mu_{j-\frac{1}{2}}^2}{(\Delta \mu_{j-1}) \bar{\mu}} \psi_{i,j-1,k} - B_{i,k} \frac{\psi_{i,j,k+1}}{\Delta \epsilon_k} + \\ & + \left(\frac{\mu_j}{\Delta z_{i-1}} + A_{i,k} \left[\frac{1 - \mu_{j+\frac{1}{2}}^2}{(\Delta \mu_j) \bar{\mu}} + \frac{1 - \mu_{j-\frac{1}{2}}^2}{(\Delta \mu_{j-1}) \bar{\mu}} \right] + \frac{B_{i,k}}{\Delta \epsilon_k} \right) \psi_{i,j,k} - \\ & C_{i,k} \psi_{i,j,k} - A_{i,k} \frac{1 - \mu_{j+\frac{1}{2}}^2}{(\Delta \mu_j) \bar{\mu}} \psi_{i,j+1,k} = Q_{i,j,k} \end{aligned} \quad (4)$$

for $\mu_j > 0$

$$\begin{aligned} & -A_{i,k} \frac{1 - \mu_{j-\frac{1}{2}}^2}{(\Delta \mu_{j-1}) \bar{\mu}} \psi_{i,j-1,k} - B_{i,k} \frac{\psi_{i,j,k+1}}{\Delta \epsilon_k} + \\ & + \left(-\frac{\mu_j}{\Delta z_i} + A_{i,k} \left[\frac{1 - \mu_{j+\frac{1}{2}}^2}{(\Delta \mu_j) \bar{\mu}} + \frac{1 - \mu_{j-\frac{1}{2}}^2}{(\Delta \mu_{j-1}) \bar{\mu}} \right] - C_{i,k} \right) \psi_{i,j,k} + \\ & + \frac{B_{i,k}}{\Delta \epsilon_k} \psi_{i,j,k} - A_{i,k} \frac{1 - \mu_{j+\frac{1}{2}}^2}{(\Delta \mu_j) \bar{\mu}} \psi_{i,j+1,k} + \frac{\mu_j}{\Delta z_i} \psi_{i+1,j,k} = Q_{i,j,k} \end{aligned} \quad (5)$$

for $\mu_j < 0$.

We obtain similar equations for the boundary points.

REFERENCES

- [1] Hensel, H., Iza-Teran, R. and Siedow, N. *Deterministic model for dose calculation in photon radiotherapy*. Physics in Medicine and Biology, **51**, 675–693.
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