CAN THE NORMALITY OF THE SEMI VARIANCE BE IMPROVED? EVIDENCE FROM FINANCIAL STOCK INDEXES WITH HOURLY, DAILY, QUARTERLY AND ANNUAL DATA OF DJIA AND SP500

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Abstract

This study examines the financial and statistical properties of the variance and semi variance (SV). Since the mean-variance approach and its extended mean-semi variance approach assume normality of returns, it has been observed that practical and computational problems emerged in the cases of portfolio optimization and estimation risk. The reliability of the semi variance has to be re-examined. This paper shows that the variance and its partial domain (semi variance) produce non normal estimates when the mean returns are normally distributed. Accordingly, a new proposed measure of risk, Mean Semi Deviations (MSD), is introduced which focuses on the measurement of the percentage returns lost from the average. The financial and statistical properties of the three measures of risk are tested and examined taking into account the risk-return theoretical relationship using data from index returns (DJIA and S&P500). The data patterns used are hourly, daily, quarterly and annual data. The financial results of the paper show that the MSD outperforms the variance and the SV in terms of its association to mean returns. The statistical properties show that the MSD produces estimates that are normally distributed and less volatile for all patterns of data (except for daily data) which outperforms the variance and the SV. The contribution of the paper is that it shows a prerequisite approach to be followed for testing the normality and volatility of any downside risk measure before using it for portfolio optimization, selection and estimation risk.

JEL classification: B23, O16

Keywords: Risk measures, Variance, Semi Variance, Mean Semi Deviations, DJIA, S&P500

1. Introduction

The semi variance (*hereinafter* SV) was introduces to the literature by Markowitz (1959, 1987) to measure the downside risk which concerns the risk-averse investors.[1] Basically, these investors focus on the below-average returns (Mao, 1970). Since then, the SV has been used as a risk measure for the portfolio construction. In this regard, the latter has been formulated based on certain statistical assumptions about the distribution of returns. The fundamental assumption is the normality of the returns distribution. Therefore, the efficiency of the portfolio depends to a large extent on the properties of the returns distribution. The problem arises when the returns are not normally distributed. In this case, the first and second moments (i.e., mean-variance) of the distribution do not guarantee an efficient portfolio construction. For example, Markowitz (1959, pp. 286-288) stated the limitations of the mean-variance analysis. Grootveld and Hallerbach (1999) state that the asymmetrical return distributions render the variance a deficient measure of investment risk. In addition, the variance as a measure of risk may miss its

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1 Chronologically, Fisher (1906) was the first to consider “the chance of earnings falling below the interest-paying line.”

link with the distributions of security returns. Accordingly, since studies in the literature have considered the use of mean-semi variance (i.e., Hogan and Warren, 1974; Marmer and Louis Ng, 1993; Kaplan and Alldredge, 1997; Ballestero, 2005) instead of mean-variance, the issue of data normality is still valid. This means that if the data are not normally distributed, the use of the mean-semi variance can not guarantee an efficient portfolio in addition to a low degree of estimation accuracy. This requires a re-examination of the semi variance itself as a measure of downside risk when the data are not normally distributed. Accordingly, the objective of this paper is to test the normality and volatility of the SV estimates in comparison with its full domain (variance) counterpart. A new proposed partial domain Mean Semi Deviation (*hereinafter* MSD) is also compared with its counterpart (SV).

The contribution of the paper is as follows. First, the paper introduces and examines the MSD as a new measure of downside risk which has financial and statistical properties that outperform the variance and the semi variance. The MSD produces normal estimates even when the data are not normally distributed. This characteristic outperforms the variance and semi variance which produces non normal estimates when the data are normally distributed. The statistical and other financial advantages of the MSD help improve the results of the mean-downside risk framework of portfolio selection and optimization. Second, since it has been realized that each study uses a data (returns) pattern that varies from one study to another, this paper examines the four common patterns of data which are hourly, daily, quarterly and annual data.

The paper is organized as follows. Section 2 discusses the relevant studies that use the mean-variance and mean-semi variance approach and the highlight the problems raised because of the normality assumption. Section 3 introduces the MSD as a new measure of downside risk. Section 4 examines the operational (financial and statistical) properties of the three measures of risk; variance, SV and MSD. Section 5 concludes.

2. Variance, Semi Variance and Normality Assumption

Financial modeling and hypothesis testing are based, in most cases, on theoretical assumptions about how the financial variables and parameters are distributed. In particular, the mean-variance efficiency of portfolio selection and CAPM has been tested under the two assumptions; normality and non normality. Kandel and Stambaugh (1987) used the mean-variance framework to test the efficiency of an unobservable portfolio based on its correlation with a proxy portfolio (NYSE-AMEX).

Gibbons et al., (1989) provide mean-variance test of portfolio efficiency that are valid only under the normality assumption.[2] They provide several intuitive interpretation of the test including a simple mean-standard deviation which improves the portfolio efficiency. Affleck-Graves and McDonald (1989) tested the Gibbons's et al. test under non normality and concluded that the test is robust with respect to typical levels of non normality. These results show that the mean-standard deviation worked better off the mean-variance when testing the portfolio efficiency. Zhou (1993) tested the mean-variance efficiency of the

[2] This is true since the CAPM theory asserts that the market portfolio is mean-variance efficient.

CRSP index under the normality assumption and concluded that the efficiency of the index is rejected in half (three of the six consecutive ten-year subperiods from 1926-1986) of the periods at the 5 percent level.

3. Mean Semi Deviations: a New Proposed Measure of Risk

It is well known in the literature of statistics that the variance measure is concerned basically with the deviations of the observations from the mean. Accordingly, the semi variance is calculated the same way as the variance but taking into account only the negative deviations (downside curve) from the mean. The quadratic form of the variance and the semi variance has resulted in many computational restrictions when working out problems of portfolio optimization. As a result, Kono (1988), Kono and Yamazaki (1991) Simaan (1997) and Kono and Koshizuka (2005) introduced and tested the absolute deviation as a piecewise linear risk function which improved the portfolio optimization problem substantially.[3] Still assumed that the returns are normally distributed.

In this paper, a new measure of the downside returns is developed by the author to measure the percentage return lost from the average, which reflects a true meaning of returns' risk. The Mean Semi-Deviations (*hereinafter* MSD) takes the form:

$$\text{Mean Semi-Deviations} = \frac{\Sigma\left(\bar{x} - x_i\right)}{n}\begin{cases} x_i < \bar{x} \\ 0 \end{cases}$$

The MSD takes into account the downside observations as well, i.e., the negative deviations from the distribution's mean. The conventional variance form calculates the deviations as $\left(x_i - \bar{x}\right)^2$, which results in compounding values in cases of loss. The proposed MSD calculates the deviation as $\left(\bar{x} - x_i\right)$. When taking into account only the negative deviations from the mean, the total deviations measure the return lost from the average, which reflects a true meaning of downside risk. This is to be considered a simple way for measuring risk in downside observations. Other models of downside risk in the literature measure the deviations the same way $\left(\bar{x} - x_i\right)$ using conditional forms such as the well known models presented by Stone (1973), Damant and Satchell (1996), Sortino and Price (1994).

4. Testing the Operational Aspects of MSD using Indices Data

Data and Analysis. This section shows the extent to which the MSD is operational using real return data from stock market indices.[4] The operationality is addressed by using different data patterns which include hourly, daily, quarterly and annual data. The subsections that follow describe the data and the results of using MSD as a measure of risk.[5] The statistical properties are examined using the Skewness[6], Kurtosis[7] and the

[3] The mean-absolute deviation model was originally proposed by Hazell (1971) who is an agricultural economist.

[4] Return calculation ignores the dividends, thus is computed as $(P_1 - P_o)/P_o$.

[5] The descriptive statistics of the data are reported in tables A-D in the appendix.

Anderson-Darling (*hereinafter* AD) test for normality (Anderson and Darling, 1952, 1954)[8]. The AD test is run under the hypothesis that: H_0: The data are drawn from normal distribution. H_a: The data are drawn from none normal distribution.

Hourly Data. The hourly trading data are obtained from the DJIA for three consecutive days 1st, 2nd and 3rd September 2004. Each day trading is classified into hours from 9:30 AM until around 16:00 PM, thus producing a total of 21 periods. Table 1 in the Annex presents the risk measures and mean returns for the DJIA-Hourly Data. Table (1) shows the estimates of the three risk measures and the geometric mean returns for the DJIA hourly trading. The volatility (AMD)[9] of the variance and semi variance estimates (0.000012 and 0.000002 respectively) are lower than that of the MSD (0.233942).

Nevertheless, the distribution characteristics vary from one measure to another. That is, the AD normality test shows that the variance and semi variance produce not normally distributed estimates, while the MSD produces normal estimates. This is obvious since the Skewness of the variance and semi variance (0.8758 and 1.235 respectively) are higher than the Skewness of the MSD (0.7804). In addition, the kurtosis of the variance and semi variance (-0.3211 and 0.7644 respectively) show contradicting results since the former is negative (flat) and the latter is positive (peak). In this case, the MSD has much smaller kurtosis (0.0511) than the variance and the SV.

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[6] Skewness $= \frac{n}{(n-1)(n-2)}\sum\left([x_i - \overline{x}] \div s\right)^3$

[7] Kurtosis $= \left\{\frac{n(n+1)}{(n-1)(n-2)(n-3)}\sum\left([x_i - \overline{x}] \div s\right)^4\right\} - \frac{3(n-1)^2}{(n-2)(n-3)}$

[8] The Anderson-Darling test is an alternative to the chi-square and Kolmogorov-Smirnov goodness-of-fit tests. The Anderson-Darling test is used to test if a sample of data came from a population with a specific distribution. It is a modification of the Kolmogorov-Smirnov (K-S) test and gives more weight to the tails than does the K-S test. The K-S test is distribution free in the sense that the critical values do not depend on the specific distribution being tested. The Anderson-Darling test makes use of the specific distribution in calculating critical values. This has the advantage of allowing a more sensitive test and the disadvantage that critical values must be calculated for each distribution. (Stephens 1974, 1976, 1977, 1979). The AD test computes as follows. $W_n^2 = -n - \frac{1}{n}\sum_{j=1}^{n}(2j-1)\left[\ln\upsilon_j + \ln(1-\upsilon_{n-j+1})\right]$. The critical values for the Anderson-Darling test are dependent on the specific distribution that is being tested. Tabulated values and formulas have been published (Stephens, 1974, 1976, 1977, 1979) for a few specific distributions (normal, lognormal, exponential, Weibull, logistic, extreme value type 1).

[9] In accordance with the MSD, the volatility is measured by the Absolute Mean Deviations (AMD) $= \left|\frac{\sum_{i=1}^{n}(\overline{x} - x_i)}{n}\right|$

Table 1. Risk Measures and Mean Returns for the DJIA-Hourly Data

	Variance[a]	G mean[b]	SV[a]	MSD[b]
Period 1	0.0017	0.0226	0.0009	0.1433
Period 2	0.0007	-0.0123	0.0004	0.0977
Period 3	0.0006	-0.0056	0.0003	0.0865
Period 4	0.0019	-0.0112	0.0009	0.1482
Period 5	0.0006	0.0031	0.0003	0.0926
Period 5	0.0007	-0.0104	0.0004	0.0985
Period 7	0.0007	0.0227	0.0003	0.1014
Period 8	0.0015	0.0096	0.0007	0.1306
Period 9	0.0006	-0.0073	0.0003	0.1762
Period 10	0.0003	0.0037	0.0002	0.0697
Period 11	0.0002	-0.0015	0.0001	0.0640
Period 12	0.0006	0.0334	0.0003	0.0902
Period 13	0.0011	0.0315	0.0005	0.1206
Period 14	0.0010	0.0141	0.0004	0.1200
Period 15	0.0017	0.0028	0.0007	0.1423
Period 16	0.0008	-0.0026	0.0004	0.1071
Period 17	0.0004	0.0090	0.0002	0.0778
Period 18	0.0003	-0.0046	0.0002	0.0685
Period 19	0.0013	0.0039	0.0002	0.0791
Period 20	0.0005	0.0078	0.0003	0.0876
Period 21	0.0006	-0.0375	0.0003	0.0956
Mean	0.0008	0.0034	0.0004	0.1046
AMD	0.000012	0.008067	0.000002	0.233942
Skewness	0.8758	-0.2014	1.2350	0.7804
Kurtosis	-0.3211	1.0374	0.7644	0.0511
AD (P-value)	0.009*	0.438****	0.001*	0.206****

* Not Normally Distributed.** Normally Distributed at the level 10%. *** Normally Distributed at the level 5%. **** Normally Distributed at the level 1%. [a] Figures are multiplied by 1,000,000
[b] Figures are multiplied by 10,000

Table 2. Correlation Matrix of Risk-Return – DJIA Hourly Data-1-3/9/2004

	Variance	G mean	SV	MSD
Variance	1.0000			
G mean	0.1616	1.0000		
SV	0.8975	0.1272	1.0000	
MSD	0.6832	0.0575	0.7377	1.0000

Table 2 shows the correlation matrix of the risk-return characteristics. The matrix shows the extent of the association between the risk measures and their relationship with index returns. Number of observations are drawn from table (2) as follows.

1. Elements of consistency are realized between the correlation coefficients of the variance, the SV and the MSD. The correlation between each two measures are positive.
2. The estimates of the variance-semi variance are very close (0.8975), thus correlated with each other.
3. The estimates of the MSD are closer to those of the SV (0.7377) than to those of the variance (0.6832) which means that the MSD works out the same way as the SV does.
4. The variance-mean and the SV-mean coefficients (0.1616 and 0.1272 respectively) show elements of consistency. Since the SV measures the negative deviations (return losses) only, its coefficient is lower than the variance. In this case, the MSD has a lower coefficient (0.0575) which indicates that it is more informative since it does not exacerbate the amount of losses.

Figure 1 in the Annex shows graphical representation of the estimates of the three risk measures and the mean returns of the DJIA hourly data. The graph shows that the observed MSD-returns relationship is positive and much lower than variance-mean and the SV-mean positive relationship. This shows that, when using hourly data, the estimates of the MSD turn out to be more informative.

Daily Data. The daily data are obtained from the DJIA for the period August 30th until September 10th 2004. The data shows the trading activity every ½ hour covering the whole trading day. Each day was considered separately and treated as one unit that consists of several observations (½ hour trading).

Table (3) shows the estimates of the three risk measures and the geometric mean returns for the DJIA daily trading. The volatility (AMD) of the variance and SV estimates (0.00002 and 0.00004 respectively) are higher than that of the MSD (0.00001). The distribution characteristics of each measure are relatively similar. That is, the AD normality test shows that the estimates of the three risk measures and the mean returns are normally distributed, although the MSD outperforms the variance and the SV since the p-value of the MSD (0.831) is higher than that of the variance (0.592) and the SV (0.824). The kurtosis of the variance, SV and MSD (-0.9119, -0.5866, and -0.507 respectively) are similar in the negative trend (flat). The Skewness adds more insights into the distributional characteristic since the Skewness of the SV (0.0538) is much lower than those of the variance and MSD (0.3964 and 0.1407 respectively).

Table 4 shows the correlation matrix of the risk-return characteristics. The matrix is to show the extent of the reliability of the risk measures and their relationship with index returns. Number of observations are drawn from table (4) as follows.

1. As with the hourly data, elements of consistency are realized between the correlation coefficients of the variance, the SV and the MSD. The correlation between each two measures are positive.

2. The estimates of the variance-MSD (0.9668) are closer than the variance-semi variance (0.8375), which means that the MSD works out the same way as the variance does when using daily data.
3. The variance-mean and the SV-mean coefficients (0.5622 and 0.5435 respectively) show elements of consistency. Since the SV measures the negative deviations (return losses) only, its coefficient is lower than the variance. In this case, the SV has a lower coefficient (0.5435) than the MSD (0.5804), which means that the former is relatively more informative.

Table 3. Risk Measures and Mean Returns for the DJIA-Daily Composite Price Index

	Variance[1]	G mean[2]	SV[3]	MSD[4]
8/30/2004	0.070	-4.720	0.038	0.334
8/31/2004	0.500	3.349	0.149	0.867
9/1/2004	0.360	-0.290	0.107	0.624
9/2/2004	0.230	9.685	0.096	0.553
9/3/2004	0.140	-2.046	0.066	0.443
9/7/2004	0.510	5.517	0.145	0.803
9/8/2004	0.230	-2.268	0.100	0.545
9/9/2004	0.180	-1.763	0.085	0.537
9/10/2004	0.320	1.988	0.173	0.711
Mean	0.2822	1.0502	0.1066	0.6019
AMD	0.00002	0.00002	0.00004	0.00001
Skewness	0.3964	0.7995	0.0538	0.1407
Kurtosis	-0.9119	0.1123	-0.5866	-0.5070
AD (P-value)	0.592****	0.518****	0.824****	0.831****

Notes: Figures are multiplied by 100,000. [2] Figures are multiplied by 10,000. [3] Figures are multiplied by 100,000. [4] Figures are multiplied by 1000 * Not Normally Distributed. ** Normally Distributed at the level 10%. *** Normally Distributed at the level 5%. **** Normally Distributed at the level 1%

Table 4. Correlation Matrix of Risk-Return, DJIA-Daily Composite Price Index-30-8/10-9-2004

	Variance	G mean	SV	MSD
Variance	1.0000			
G mean	0.5622	1.0000		
SV	0.8375	0.5435	1.0000	
MSD	0.9668	0.5804	0.9187	1.0000

Figure 2 in the Annex shows graphical representation of the estimates of the three risk measures and the mean returns of the daily DJIA data. The graph shows that the observed MSD-returns relationship is positive and much higher than variance-mean positive relationship. This shows that, when using daily data, the estimates of the SV turn out to be relatively reliable than the variance and the MSD.

Quarterly Data. The quarterly data are obtained from the S&P500 for the period 1988-2004. Each year was considered as one unit that consists of 4 observations (4 quarters).

Table (5) shows the estimates of the three risk measures and the geometric mean returns for the S&P500 quarterly data. The volatility (AMD) of the MSD estimate (0.00001) is lower than that of the variance and semi variance (0.00005 and 0.00002 respectively)..

Table 5. Risk Measures and Mean Returns for the S&P 500 - Quarterly Stock Returns

	Variance	G mean	SV	MSD
1988	0.0010	-0.0304	0.0012	0.0173
1989	0.0035	-0.0338	0.0013	0.0243
1990	0.0165	-0.0234	0.0060	0.0472
1991	0.0021	-0.0183	0.0030	0.0387
1992	0.0002	-0.0276	0.0011	0.0231
1993	0.0011	0.0021	0.0023	0.0287
1994	0.0017	-0.0282	0.0044	0.0337
1995	0.0002	-0.0626	0.0009	0.0207
1996	0.0006	-0.0377	0.0003	0.0086
1997	0.0029	-0.0887	0.0001	0.0062
1998	0.0131	-0.0376	0.0007	0.0159
1999	0.0070	-0.0377	0.0002	0.0116
2000	0.0035	0.0663	0.0001	0.0062
2001	0.0142	0.0050	0.0008	0.0183
2002	0.0158	0.0794	0.0031	0.0425
2003	0.0035	-0.0664	0.0011	0.0231
2004	0.0025	-0.0110	0.0003	0.0123
Mean	0.0052	-0.0206	0.0016	0.0222
AMD	0.00005	0.00002	0.00002	0.00001
Skewness	1.1574	1.1098	1.5331	0.5771
Kurtosis	-0.2975	1.6858	1.9370	-0.4699
AD (P-value)	0.000*	0.021****	0.003*	0.544****

* Not Normally Distributed.** Normally Distributed at the level 10%. *** Normally Distributed at the level 5%. **** Normally Distributed at the level 1%

The distribution characteristics of each measure are different from those produced by the daily data, but similar to those of the hourly data. That is, the AD normality test shows that the estimates of the mean returns and MSD are normally distributed and the estimates of the variance and the SV are not normally distributed. This is due to the fact that the Skewness of the MSD is much smaller (0.5771) than that of the variance and the SV (1.1574 and 1.5331 respectively). This brings the distribution of the MSD to normal settings than the variance and the semi variance. The kurtosis shows that the variance and the MSD has a flat distribution (negative kurtosis -0.2975 and -0.4699 respectively), while the kurtosis of the SV shows contradicting distribution since it has peaked (positive kurtosis) distribution (1.937). In addition, since the estimates of the MSD and the mean returns are normally distributed, this shows a considerable degree of conformity since the normal MSD estimates are drawn from normal mean returns

Table 6. Correlation Matrix of Risk-Return of S&P 500 Quarterly

	Variance	G mean	SV	MSD
Variance	1.0000			
G mean	0.3747	1.0000		
SV	0.3656	0.1771	1.0000	
MSD	0.3886	0.2596	0.9142	1.0000

Table 6 shows the correlation matrix of the risk-return characteristics. The matrix is to show the extent of the reliability of the risk measures and their relationship with index returns. Number of observations are drawn from table (6) as follows.

1. As with the hourly and daily data, elements of consistency are realized between the correlation coefficients of the variance, the SV and the MSD. The correlation between each two measures are positive.
2. The estimates of the MSD-SV are closer (0.9142) than those of the variance-SV (0.3656) and variance-MSD (0.3886), which means that the MSD works out the same way the SV does when using quarterly data.
3. The variance-mean and the SV-mean coefficients (0.3747 and 0.1771 respectively) show elements of consistency. Since the SV measures the negative deviations (return losses) only, its coefficient is lower than the variance. In this case, the SV outperforms the MSD which has a higher coefficient (0.2596). This means that, when using quarterly, the SV is more informative than the variance and the MSD.

Figure 3 in the Annex shows graphical representation of the estimates of the three risk measures and the mean returns of the quarterly S&P500 data. The graph shows that the observed MSD-returns relationship is positive and higher than SV-mean relationship and lower than the variance-mean relationship.

Annual Data. The annual data are obtained from the S&P500 composite price index for the years 1800 until 2000. The data were divided into periods of 10 years resulting in 20 periods.

Table 7 shows the estimates of the three risk measures and the geometric mean returns for the S&P500 annual data. The volatility (AMD) of the MSD estimate (0.0206) is higher than that of the SV (0.0078) and lower than that of the variance (0.0228). The distribution characteristics of each measure are relatively similar to those shown by the hourly and quarterly data, but different from those produced by the daily. That is, the AD normality test shows that the estimates of the mean returns, SV and MSD are normally distributed and the estimates of the variance are not normally distributed. This is due to the fact that the Skewness of the variance and the SV (2.0295 and 1.3928 respectively) are much higher than that of the MSD (0.1357). The same pattern is realized in the kurtosis. The kurtosis of the variance and the SV (6.4920 and 2.9051 respectively) are much higher than that of the MSD (0.7932). This means that the MSD tends to show normality characteristics than the variance and the SV.

Table 7. Risk Measures and Mean Returns for the S&P 500-Annual Composite Price Index

	Variance	G mean	SV	MSD
Period 1	0.0034	0.0045	0.0015	0.0227
Period 2	0.0064	-0.0178	0.0029	0.0298
Period 3	0.0020	0.0030	0.0008	0.0168
Period 4	0.0074	-0.0087	0.0036	0.0329
Period 5	0.0362	0.0244	0.0100	0.0630
Period 6	0.0316	-0.0498	0.0141	0.0647
Period 7	0.0391	0.0989	0.0068	0.0606
Period 8	0.0307	0.0281	0.0087	0.0554
Period 9	0.0125	-0.0237	0.0045	0.0386
Period 10	0.0140	0.0409	0.0068	0.0429
Period 11	0.0456	0.0279	0.0197	0.0737
Period 12	0.0348	-0.0280	0.0141	0.0668
Period 13	0.0411	0.0846	0.0189	0.0710
Period 14	0.1059	-0.0365	0.0339	0.1057
Period 15	0.0233	0.0679	0.0106	0.0575
Period 16	0.0373	0.1103	0.0123	0.0680
Period 17	0.0193	0.0472	0.0087	0.0550
Period 18	0.0392	0.0395	0.0184	0.0733
Period 19	0.0163	0.0930	0.0073	0.0500
Period 20	0.0223	0.1486	0.0106	0.0596
Mean	0.0284	0.0327	0.0107	0.0554
AMD	0.00002	0.00002	0.00001	0.00001
Skewness	2.0295	0.4260	1.3928	0.1357
Kurtosis	6.4920	-0.5574	2.9051	0.7932
AD (P-value)	0.014*	0.721****	0.136****	0.345****

* Not Normally Distributed.** Normally Distributed at the level 10%. *** Normally Distributed at the level 5%. **** Normally Distributed at the level 1%

Table 8. Correlation Matrix of Risk-Return of S&P 500 -Annual Composite Price Index – 1801-2000

	Variance	G mean	SV	MSD
Variance	1.0000			
G mean	-0.0652	1.0000		
SV	0.9359	-0.0579	1.0000	
MSD	0.9257	0.1185	0.9407	1.0000

Table 8 shows the correlation matrix of the risk-return characteristics. The matrix is to show the extent of the reliability of the risk measures and their relationship with stock returns. Number of observations are drawn from table (8) as follows.

1. As with the hourly, daily and quarterly data, elements of consistency are realized between the correlation coefficients of the variance, the SV and the MSD. The correlation between each two measures is positive.
2. As with quarterly data, the estimates of the MSD-SV are closer (0.9407) than those of the variance-SV (0.9359) and variance-MSD (0.9257), which means that the MSD works out the same way the SV does when using annual data. In addition, These correlation coefficient of the risk measures are much higher than those produced by the hourly, daily and quarterly data.
3. The variance-mean and the SV-mean coefficients (-0.0652 and -0.0579 respectively) show elements of consistency since the latter coefficient is smaller, in absolute value, than the variance. In this case, the SV outperforms the MSD which has a higher coefficient (0.1185). An element of contradiction is also observed since the MSD-mean is positive and those of the variance-mean and the SV-mean are negative. This means that, using annual data, the MSD may not reflect the true trend of the risk-return theoretical relationship.

Figure 4 in the Annex shows graphical representation of the estimates of the three risk measures and the mean returns of the annual S&P500 data. The graph shows that the observed MSD-returns relationship is positive and contradicts both the variance-mean and the SV-mean negative relationships. This shows that, when using annual data, the estimates of the variance and the SV turn out to be relatively reliable. This is the same result obtained when using daily and quarterly data.

5. Conclusion

This paper examines the financial and statistical properties of three risk measures; the variance, the semi variance and mean semi deviations (MSD) as a new proposed measure of risk. Two general observations can be derived. First, the variance and SV produce non-normal estimates when the mean returns are normally distributed. This occurred when using hourly and quarterly data. In addition, the variance and SV produced non normal estimates when the data (returns) are normally distributed. This occurred when using hourly data. In the case of annual data, the variance was inferior producing non normal estimates when the mean returns are normally distributed. On the other hand, the MSD outperforms the variance and the SV that it (MSD) produces normal estimates when the mean returns are either normally or non-normally distributed. This provides evidence that the MSD can highly likely improve the portfolio optimization problem since it is represented as a linear program. In addition, the MSD, being a linear risk function, has the same advantage of the mean deviation. That is, the MSD can help derive an equilibrium relation between the market portfolio and an individual asset which presents the CAPM model. Second, the volatility of the MSD estimates is less than that of the variance and SV for all data patterns. Here, a relevant point of comparison can be made. Since Bond and Satchell's (2002) paper is very related to this paper, their results present a relative comparison to the results of this paper. That is, (1) contrary to Bond and Satchell, under the assumption of symmetric distribution, the variance is not the most efficient to use, rather the MSD outperforms since the latter produces normal estimates. (2) consistent with Bond and Satchell, the variance is more volatile than the semi variance. In addition, the results in this paper show that the SV is less volatile in the case of the hourly data only (3) although Bond and Satchell use monthly data, their results are relatively consistent with the results obtained in this paper that the SV is inefficient when using daily and quarterly returns.

The general conclusion is that the MSD can better represent the downside risk factor in terms of the normality and volatility (except for the hourly data) of the estimates. This has also the advantage of improving the estimation accuracy of risk when using simulation analysis. It is worth to note that since this paper focuses on examining the distributional properties of the proposed measure of risk (MSD), the issues of its advantage(s) to portfolio optimization problems and estimation of risk are held to further independent research. Nevertheless, at this stage we can at least be sure that the normality of the MSD estimates outperforms those of the variance and semi variance.

Table 9 summarizes the financial properties (risk-return) concluded by the results of the correlation matrix for the four data patterns. The table shows the interrelationship between the three measures of risk and the relation to the mean returns.

Table 9. Properties of Risk-Return Relationship

Relationship	Annual	Quarterly	Daily	Hourly
Mean-Variance	Inconsistent	Inconsistent	Consistent	Inconsistent
Mean-Semi Variance	Consistent	Inconsistent	Consistent	Inconsistent
Mean-MSD	High Consistency	High Consistency	High Consistency	High Consistency
Variance-MSD	Inconsistent	Inconsistent	Consistent	Inconsistent
Variance-SV	Inconsistent	Consistent	Consistent	Consistent
MSD-SV	Consistent	Inconsistent	Consistent	Inconsistent

Table (9) shows that:

1- The mean-variance relationship is inconsistent for all data patterns but the daily data.
2- The mean-semi variance relationship is inconsistent for the quarterly and hourly data, but consistent for the annual and daily data.
3- The mean-MSD relationship is highly consistent for all data patterns.
4- The variance-MSD relationship is inconsistent for all data patterns but the daily data.
5- The variance-SV relationship is consistent for all data patterns but for the annual data.
6- The MSD-SV relationship is inconsistent for the quarterly and hourly data, but consistent for the annual and daily data.

The results obtained in this paper warrant further research. That is, since this paper is concerned with normality tests of the SV and its full domain (variance) and partial domain (MSD) counterparts, this approach can be extended to other measures in the downside risk family. The approach of this paper is also to be considered a prerequisite to the use of any downside risk measure for solving portfolio optimization problems and estimation risk. These issues will be considered independently in series of future research.

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Appendix

Figure 1. Risk measures and the mean returns of the DJIA hourly data.

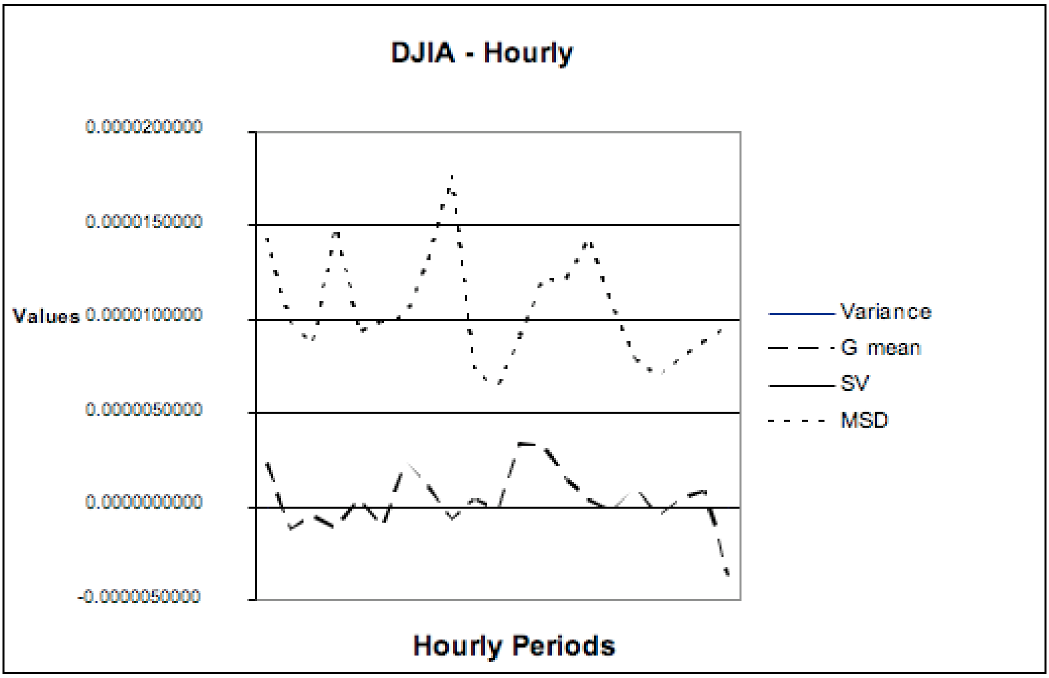


Figure 2. Risk measures and the mean returns of the daily DJIA data.

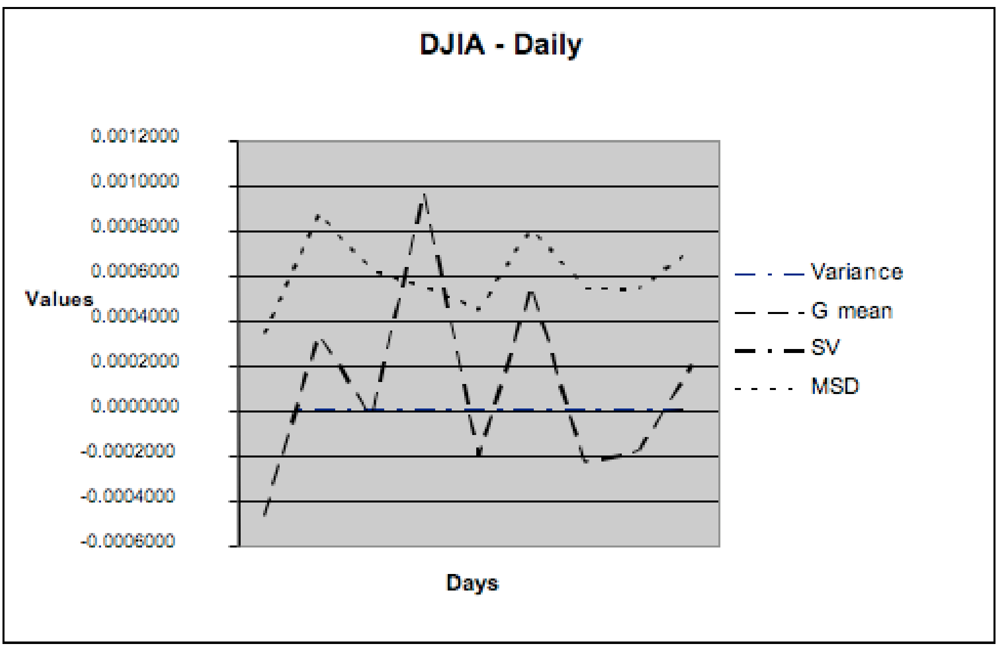


Figure 3. Risk measures and the mean returns of the quarterly S&P500 data.

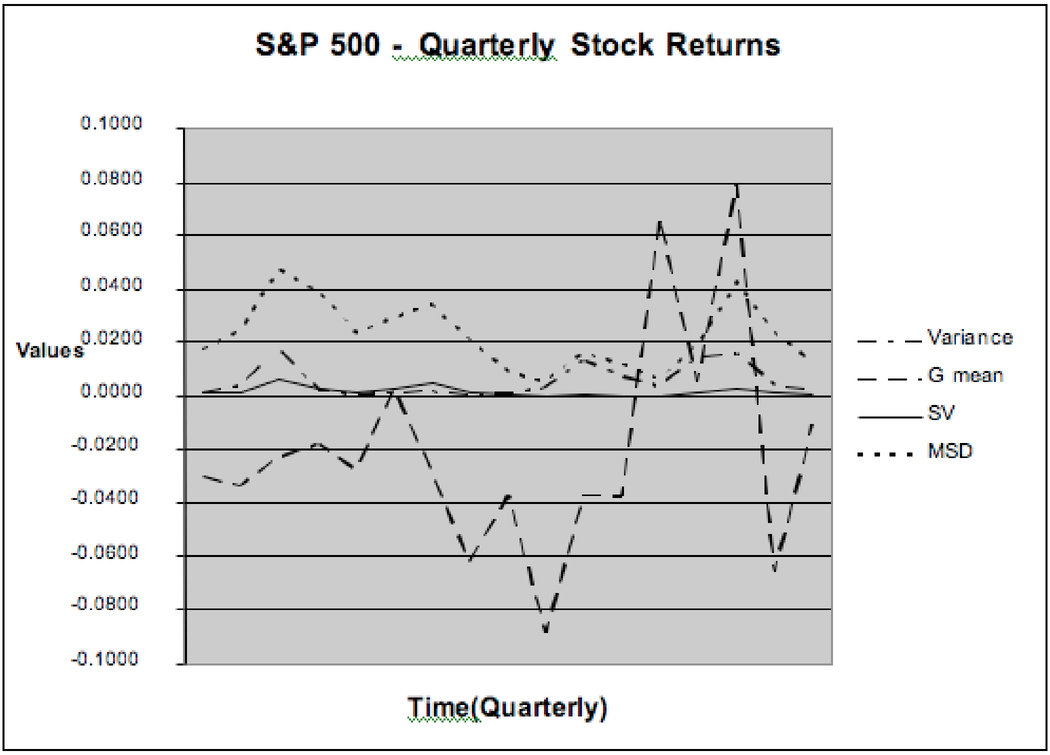
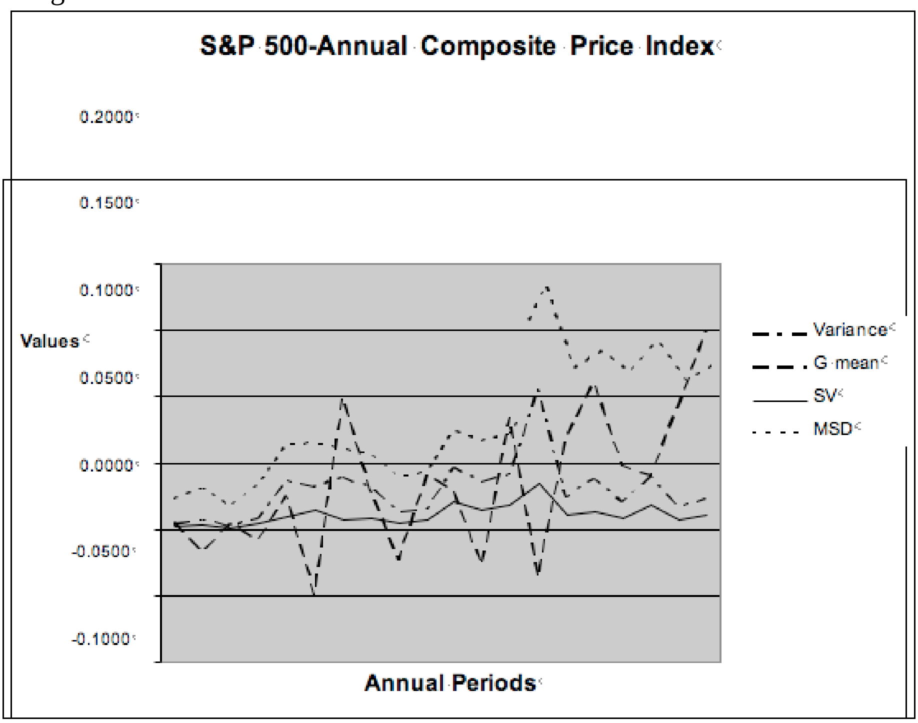
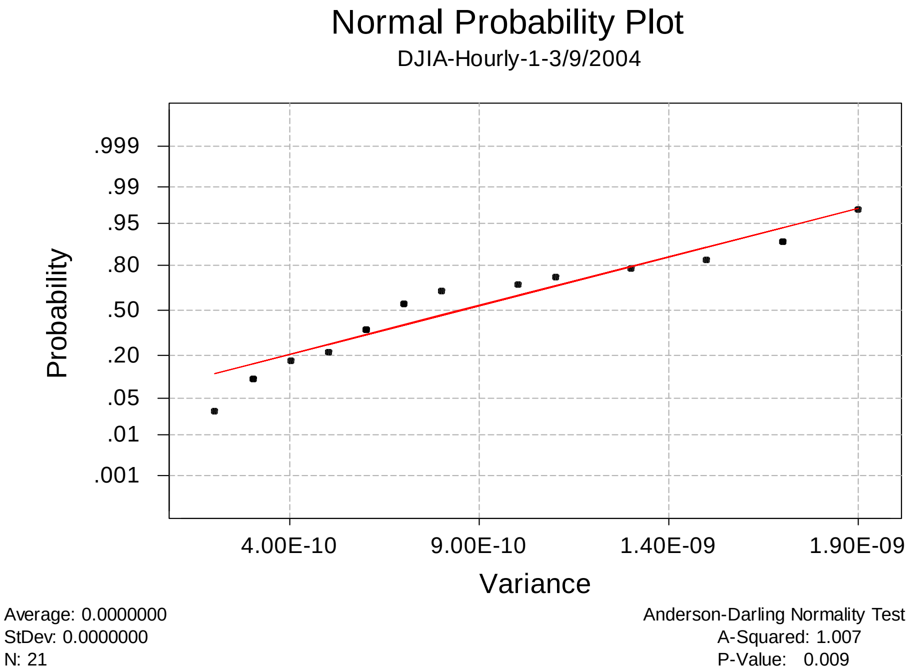
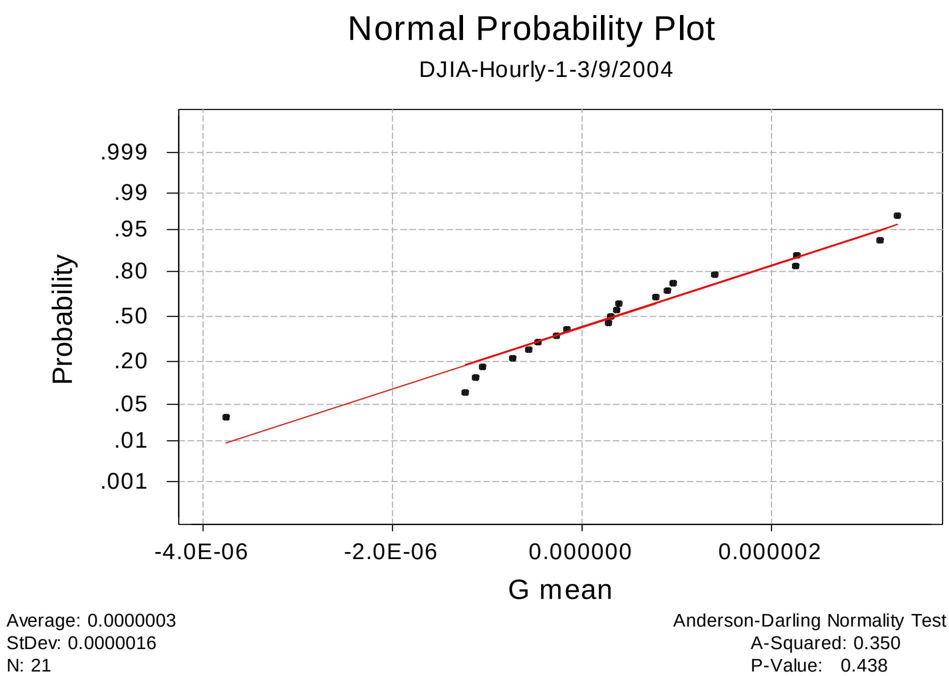
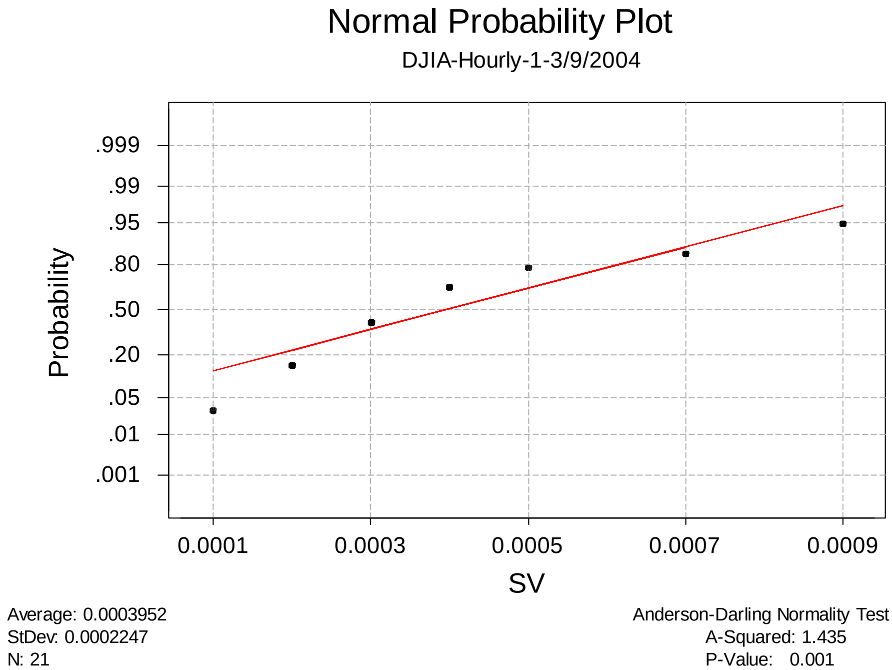


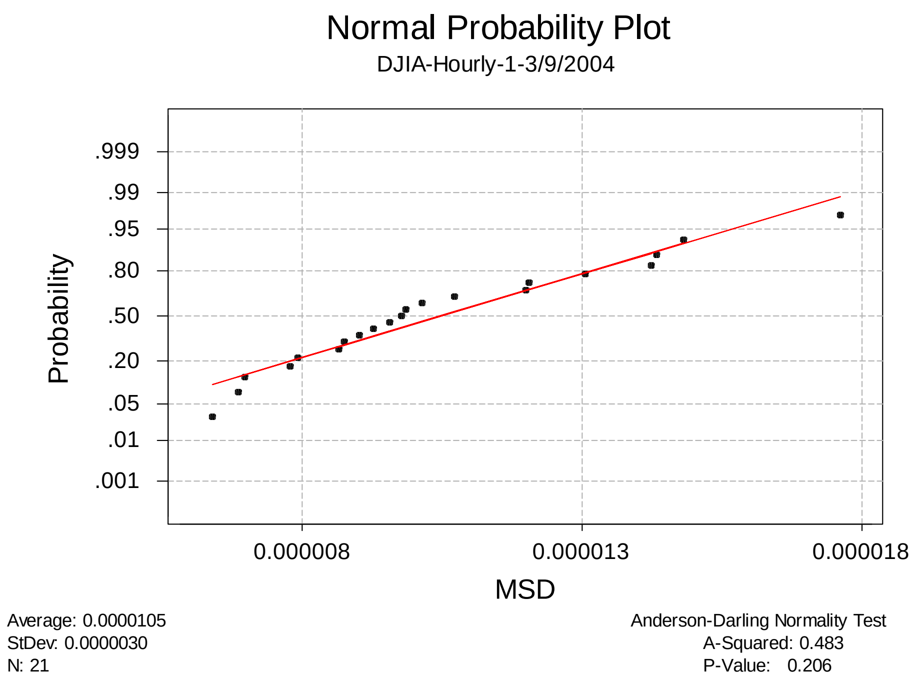
Figure 4. Risk measures and the mean returns of the annual S&P500 data.



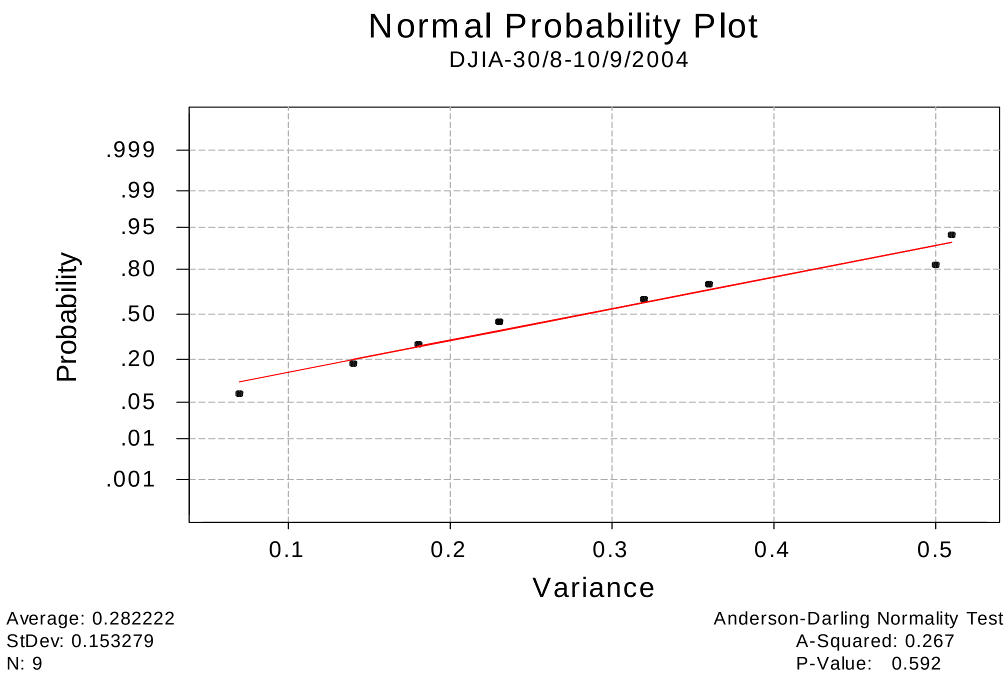

Normal Probability Plot
DJIA-Hourly-1-3/9/2004
Probability
.999
.99
.95
.80
.50
.20
.05
.01
.001
4.00E-10
9.00E-10
1.40E-09
1.90E-09
Variance
Average: 0.0000000
StDev: 0.0000000
N: 21
Anderson-Darling Normality Test
A-Squared: 1.007
P-Value: 0.009

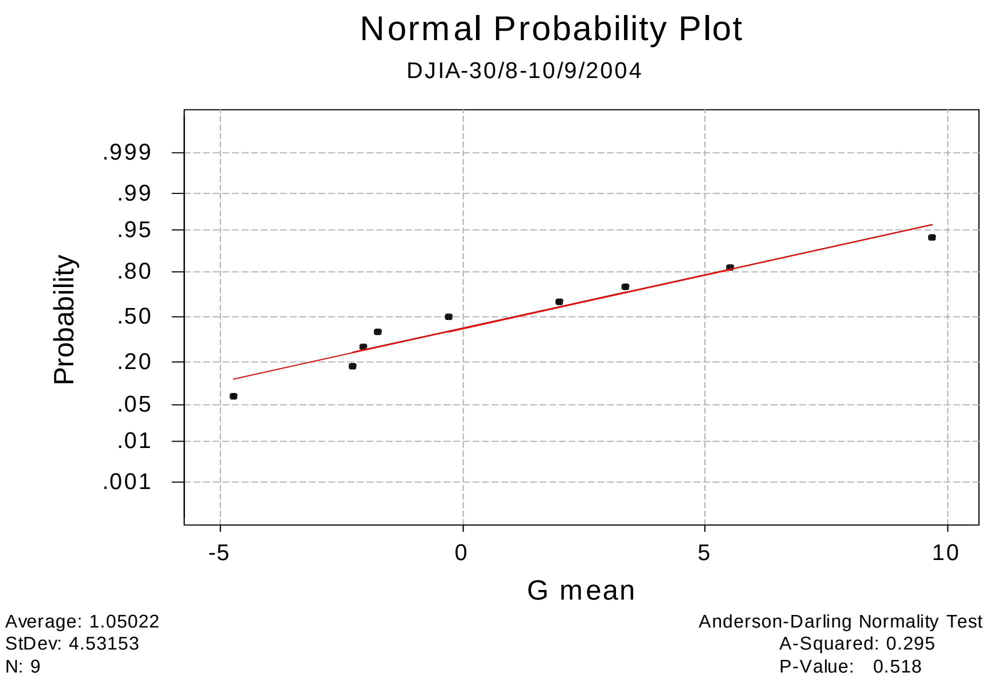

Normal Probability Plot
DJIA-Hourly-1-3/9/2004
Probability
.999
.99
.95
.80
.50
.20
.05
.01
.001
-4.0E-06
-2.0E-06
0.000000
0.000002
G mean
Average: 0.0000003
StDev: 0.0000016
N: 21
Anderson-Darling Normality Test
A-Squared: 0.350
P-Value: 0.438

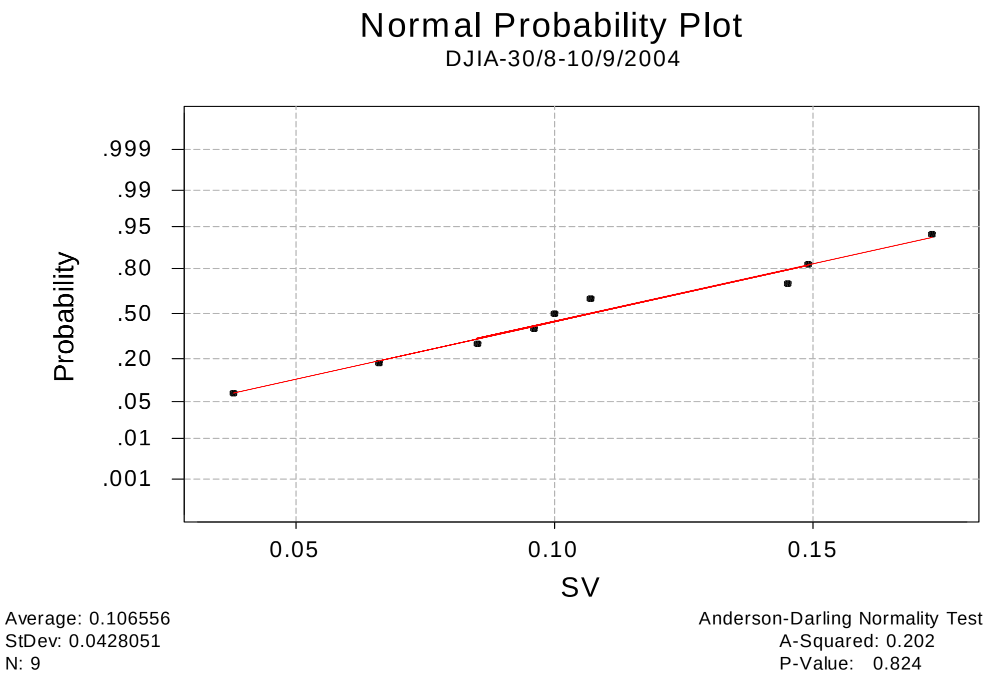

Normal Probability Plot
DJIA-Hourly-1-3/9/2004
Probability
.999
.99
.95
.80
.50
.20
.05
.01
.001
0.0001
0.0003
0.0005
0.0007
0.0009
SV
Average: 0.0003952
StDev: 0.0002247
N: 21
Anderson-Darling Normality Test
A-Squared: 1.435
P-Value: 0.001

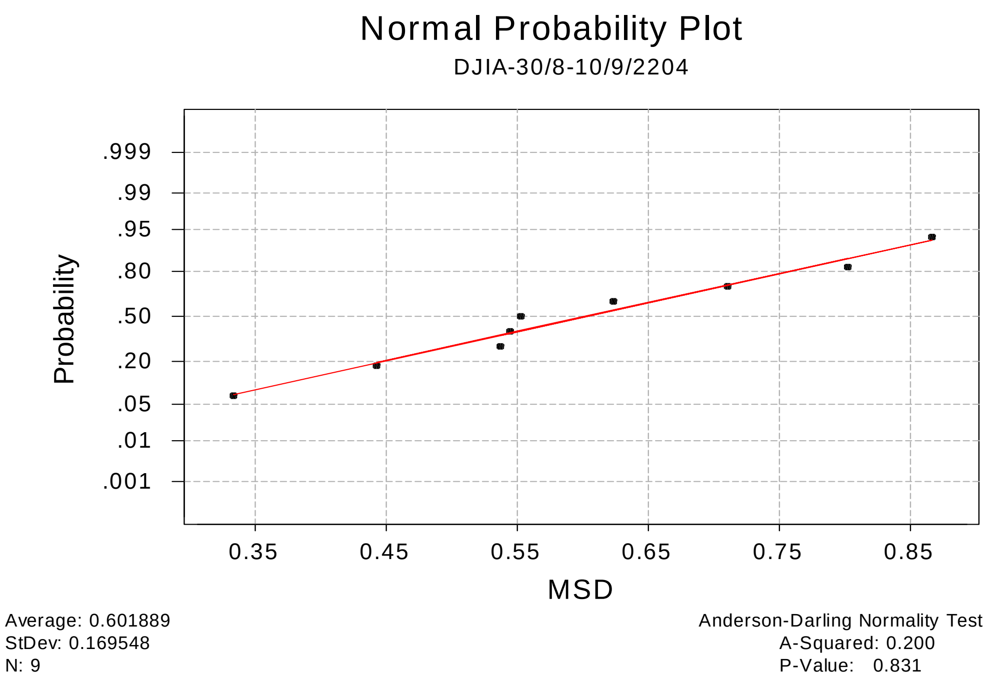

Normal Probability Plot
DJIA-Hourly-1-3/9/2004
Probability
.999
.99
.95
.80
.50
.20
.05
.01
.001
0.000008
0.000013
0.000018
MSD
Average: 0.0000105
StDev: 0.0000030
N: 21
Anderson-Darling Normality Test
A-Squared: 0.483
P-Value: 0.206

Daily Data

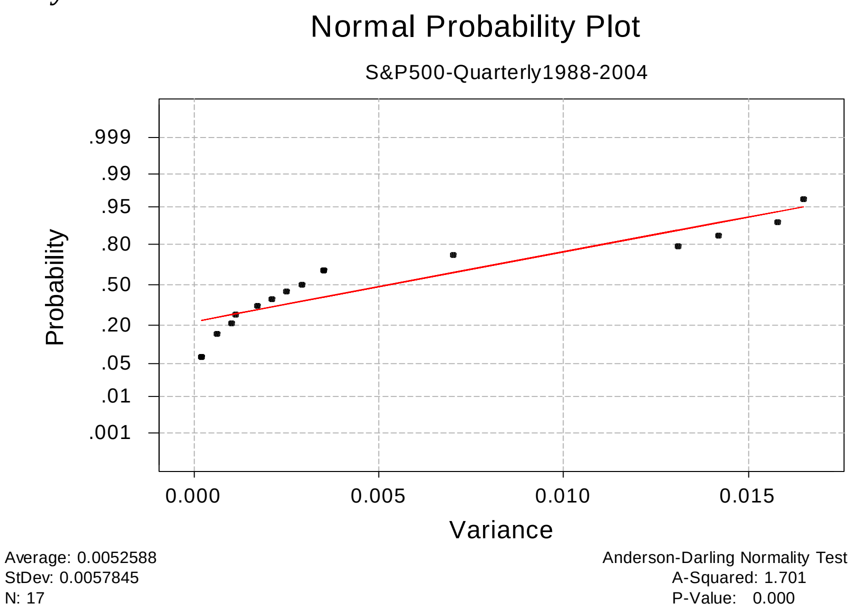

Normal Probability Plot
DJIA-30/8-10/9/2004
Probability
.999
.99
.95
.80
.50
.20
.05
.01
.001
0.1
0.2
0.3
0.4
0.5
Variance
Average: 0.282222
StDev: 0.153279
N: 9
Anderson-Darling Normality Test
A-Squared: 0.267
P-Value: 0.592

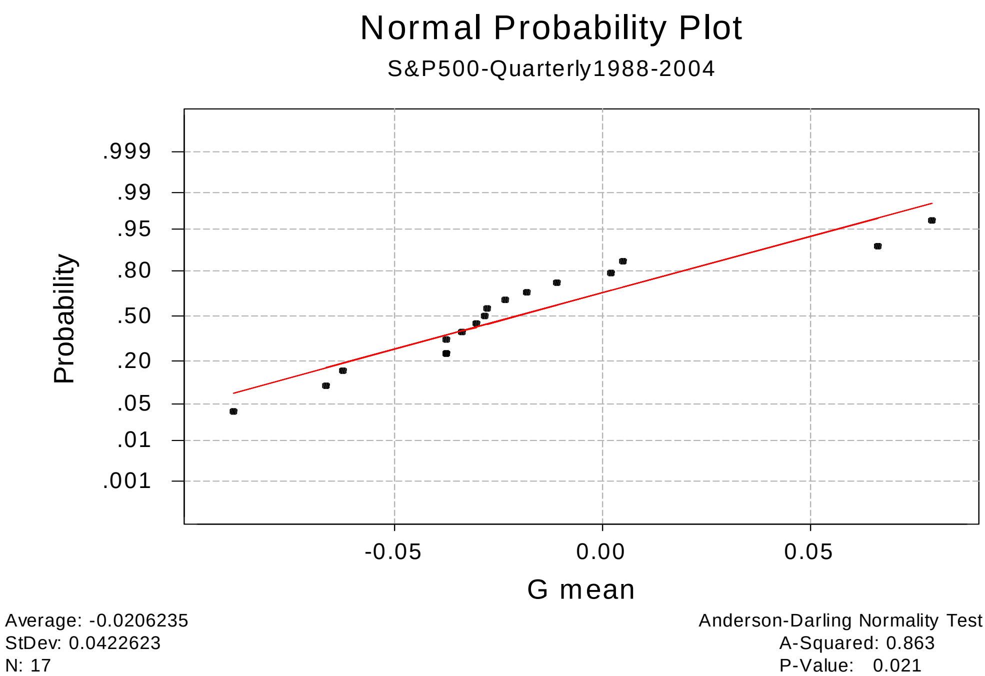

Normal Probability Plot
DJIA-30/8-10/9/2004
Probability
.999
.99
.95
.80
.50
.20
.05
.01
.001
-5
0
5
10
G mean
Average: 1.05022
StDev: 4.53153
N: 9
Anderson-Darling Normality Test
A-Squared: 0.295
P-Value: 0.518

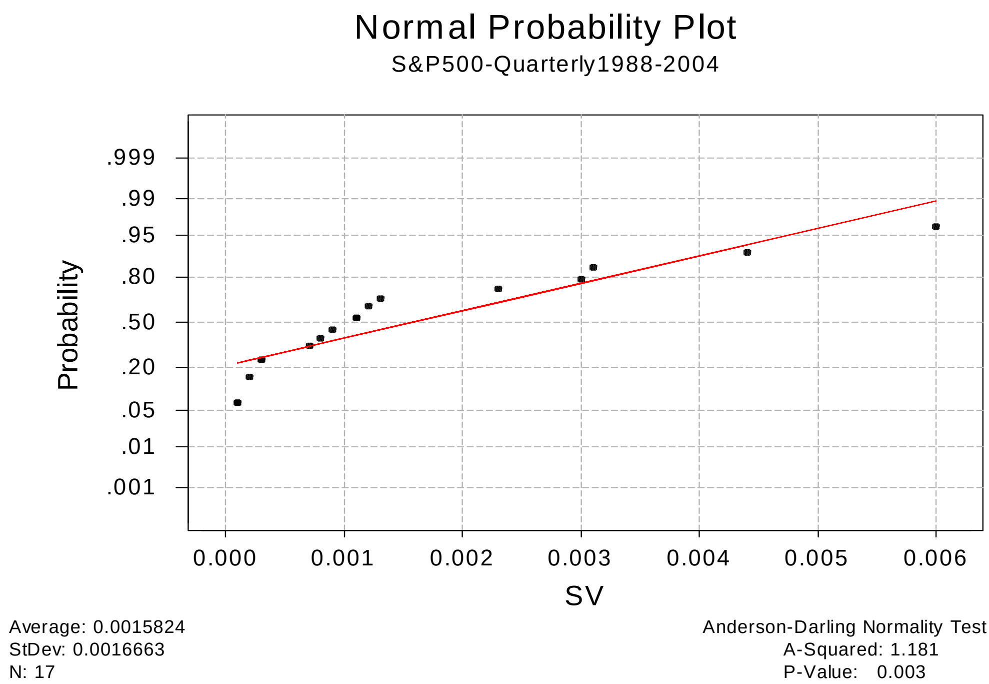

Normal Probability Plot
DJIA-30/8-10/9/2004
Probability
.999
.99
.95
.80
.50
.20
.05
.01
.001
0.05
0.10
0.15
SV
Average: 0.106556
StDev: 0.0428051
N: 9
Anderson-Darling Normality Test
A-Squared: 0.202
P-Value: 0.824

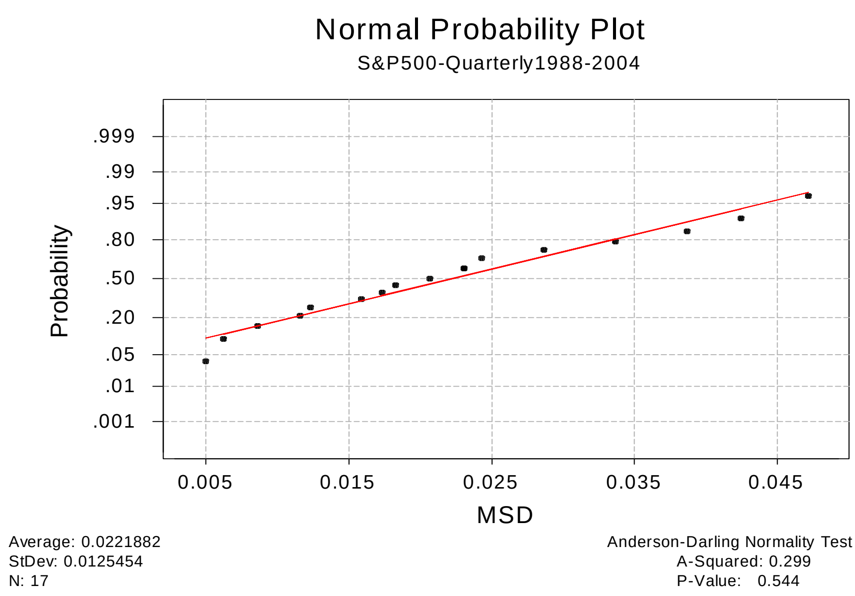

Normal Probability Plot
DJIA-30/8-10/9/2204
Probability
.999
.99
.95
.80
.50
.20
.05
.01
.001
0.35
0.45
0.55
0.65
0.75
0.85
MSD
Average: 0.601889
StDev: 0.169548
N: 9
Anderson-Darling Normality Test
A-Squared: 0.200
P-Value: 0.831

Quarterly Data

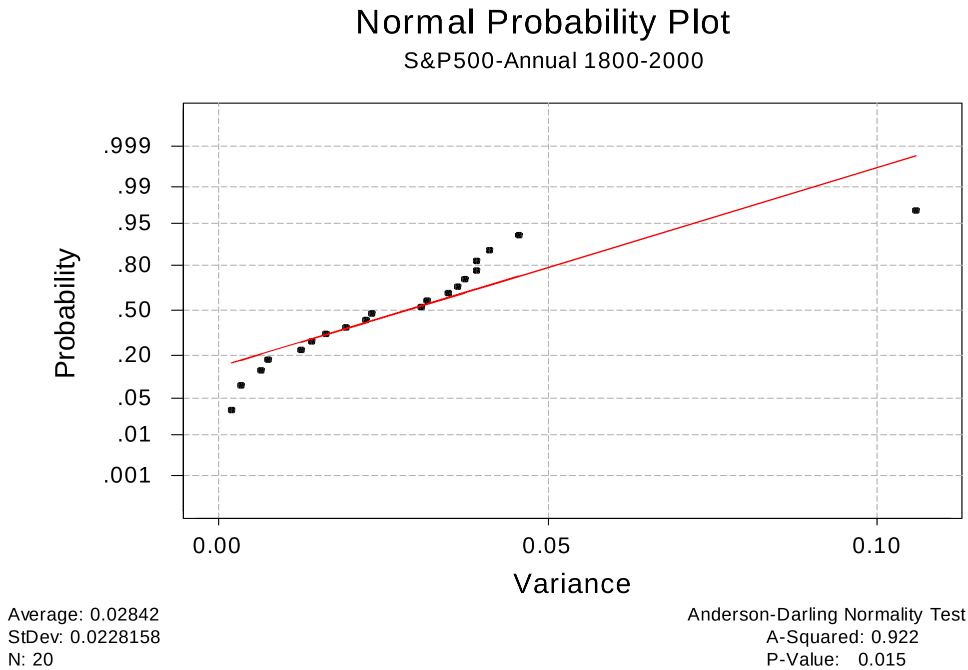


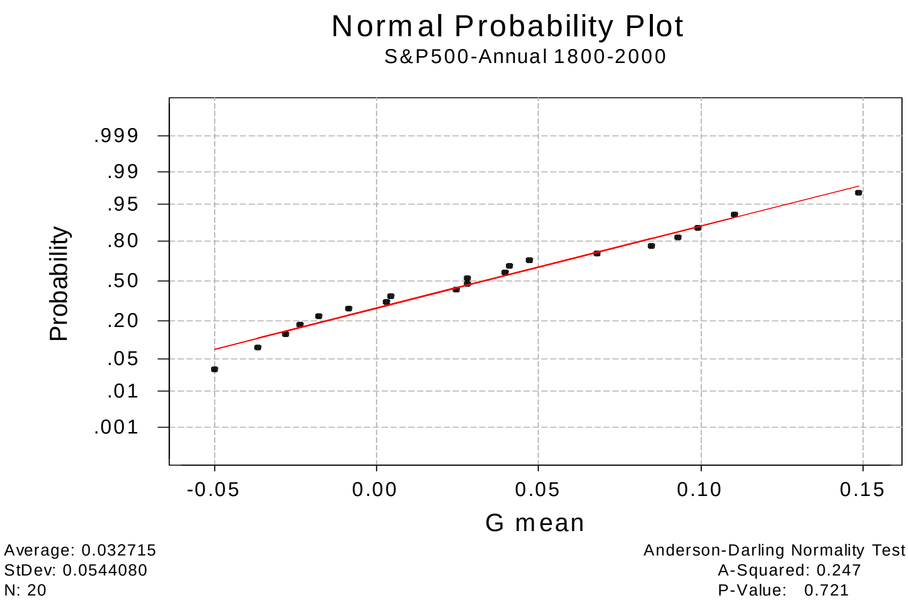


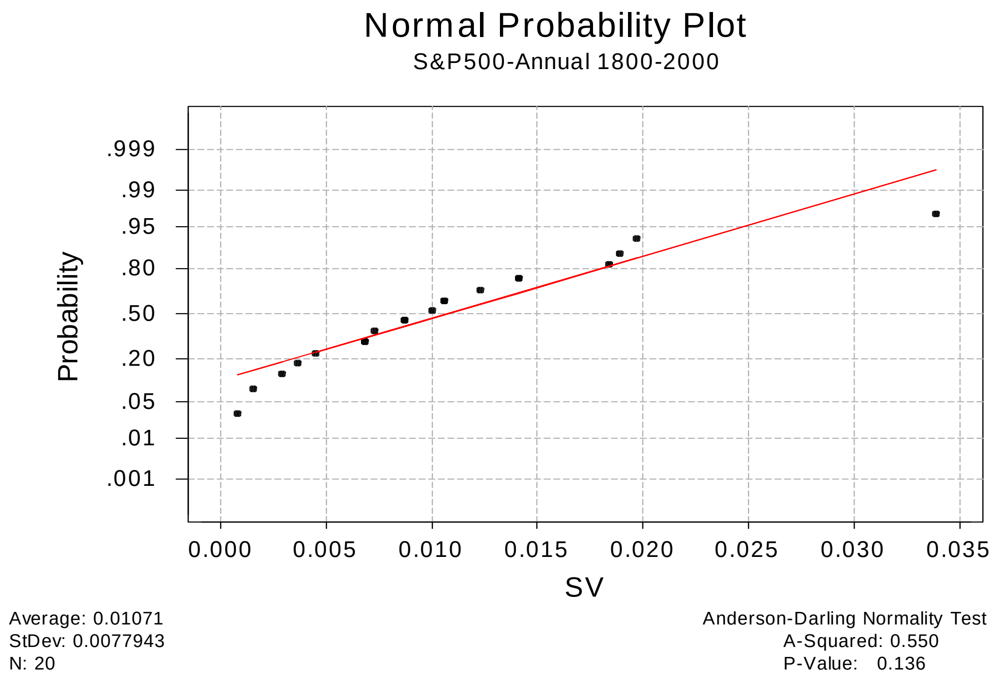

Normal Probability Plot
S&P500-Quarterly1988-2004
Probability
.999
.99
.95
.80
.50
.20
.05
.01
.001
0.000
0.001
0.002
0.003
0.004
0.005
0.006
SV
Average: 0.0015824
StDev: 0.0016663
N: 17
Anderson-Darling Normality Test
A-Squared: 1.181
P-Value: 0.003


Normal Probability Plot
S&P500-Quarterly1988-2004
Probability
.999
.99
.95
.80
.50
.20
.05
.01
.001
0.005
0.015
0.025
0.035
0.045
MSD
Average: 0.0221882
StDev: 0.0125454
N: 17
Anderson-Darling Normality Test
A-Squared: 0.299
P-Value: 0.544

Annual Data


Normal Probability Plot
S&P500-Annual 1800-2000
Probability
.999
.99
.95
.80
.50
.20
.05
.01
.001
0.00
0.05
0.10
Variance
Average: 0.02842
StDev: 0.0228158
N: 20
Anderson-Darling Normality Test
A-Squared: 0.922
P-Value: 0.015


Normal Probability Plot
S&P500-Annual 1800-2000
Probability
.999
.99
.95
.80
.50
.20
.05
.01
.001
-0.05
0.00
0.05
0.10
0.15
G mean
Average: 0.032715
StDev: 0.0544080
N: 20
Anderson-Darling Normality Test
A-Squared: 0.247
P-Value: 0.721


Normal Probability Plot
S&P500-Annual 1800-2000
Probability
.999
.99
.95
.80
.50
.20
.05
.01
.001
0.000
0.005
0.010
0.015
0.020
0.025
0.030
0.035
SV
Average: 0.01071
StDev: 0.0077943
N: 20
Anderson-Darling Normality Test
A-Squared: 0.550
P-Value: 0.136

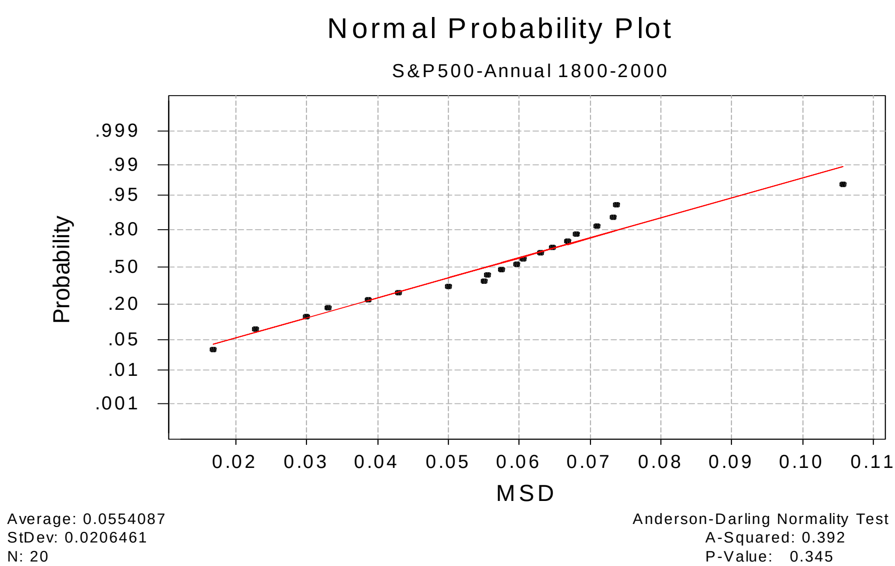

Normal Probability Plot
S&P500-Annual 1800-2000
Probability
.999
.99
.95
.80
.50
.20
.05
.01
.001
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
0.10
0.11
MSD
Average: 0.0554087
StDev: 0.0206461
N: 20
Anderson-Darling Normality Test
A-Squared: 0.392
P-Value: 0.345

Table A. Descriptive Statistics of the DJIA hourly Data.

	Skew	Kurt	AD test (*p*-value)
Period 1	-1.1892	16.6952	0.0000
Period 2	-0.20754	3.00451	0.0000
Period 3	-0.37386	5.9281	0.0000
Period 4	0.35372	4.16554	0.0000
Period 5	-0.02428	3.63098	0.0000
Period 5	-0.26975	2.57861	0.0000
Period 7	0.0054	1.6359	0.0000
Period 8	1.5163	20.1302	0.0000
Period 9	-0.02124	2.8187	0.0000
Period 10	0.50217	6.47718	0.0000
Period 11	0.07774	1.27488	0.0000
Period 12	0.86476	4.65025	0.0000
Period 13	0.50267	3.6256	0.0000
Period 14	0.7177	2.9258	0.0000
Period 15	1.3377	10.4374	0.0000
Period 16	0.20367	2.46024	0.0000
Period 17	0.62067	3.24858	0.0000
Period 18	-0.23534	2.41478	0.0000
Period 19	21.787	693.69	0.0000
Period 20	0.33645	4.81147	0.0000
Period 21	-0.66908	7.26015	0.0000

Table B. Descriptive Statistics of S&P Daily Data.
S&P 500-Daily Composite Price Index

	Skew	Kurt	AD test (*p*-value)
8/30/2004	-0.665	-0.62338	0.317
8/31/2004	1.1472	0.97214	0.027
9/1/2004	1.5981	3.75608	0.046
9/2/2004	0.206	0.935549	0.681
9/3/2004	0.0531	0.315172	0.896
9/7/2004	1.4715	2.6591	0.125
9/8/2004	0.2382	0.821171	0.624
9/9/2004	-0.1845	-0.80588	0.926
9/10/2004	-0.8335	0.834168	0.280

Table (C): Descriptive Statistics of S&P quarterly Data.
S&P 500 - Quarterly Stock Returns

	Skew	Kurt	AD test (*p*-value)
1988	-1.37	1.50	0.2740
1989	-0.19	-4.63	0.2820
1990	-0.48	-2.01	0.6330
1991	1.46	2.17	0.268
1992	0.48	-2.35	0.5650
1993	0.33	0.47	0.8200
1994	0.13	1.47	0.5070
1995	0.57	-1.71	0.6100
1996	-1.19	0.44	0.2070
1997	-0.37	-3.90	0.2150
1998	-0.70	-1.65	0.4970
1999	1.56	2.17	0.1180
2000	0.37	-3.90	0.2150
2001	-0.20	-3.20	0.5870
2002	1.68	3.11	0.1190
2003	0.48	-2.35	0.5650
2004	0.63	-1.70	0.5530

Table D. Descriptive Statistics of S&P annual Data.
S&P 500-Annual Composite Price Index

	Skew	Kurt	AD test (*p*-value)
Period 1	-0.2150	-0.6731	0.9080
Period 2	-0.3054	0.4300	0.8130
Period 3	0.3327	-1.0186	0.3610
Period 4	-0.2967	-0.8964	0.1050
Period 5	1.0463	1.2593	0.4790
Period 6	-0.4424	-0.9007	0.5450
Period 7	1.6578	2.0697	0.0050
Period 8	1.1296	1.8817	0.3390
Period 9	0.4490	0.3996	0.9380
Period 10	-0.8036	0.7635	0.4820
Period 11	-0.3792	-0.3203	0.8170
Period 12	-0.0962	-0.6229	0.9220
Period 13	-0.6214	-0.0437	0.8400
Period 14	0.0091	-1.2601	0.4880
Period 15	-0.3479	-0.6158	0.8150
Period 16	0.4411	-0.7300	0.8740
Period 17	-0.2822	-1.5808	0.2620
Period 18	-0.5990	-0.6275	0.569
Period 19	-0.2113	-1.0622	0.537
Period 20	-0.5164	-1.0192	0.447