# Demostración automática de teoremas geométricos con GeoGebra

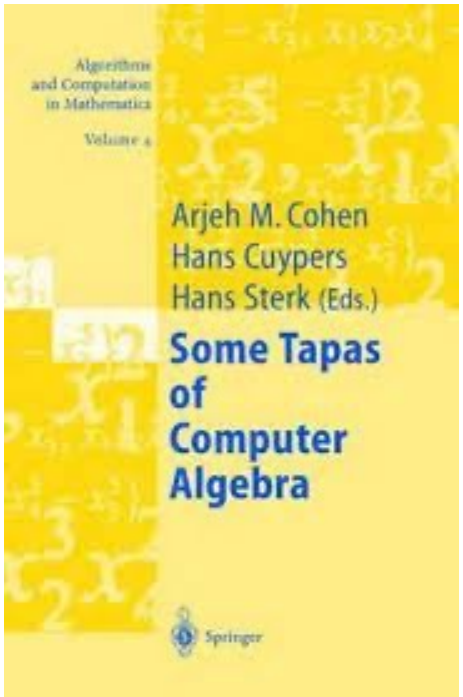
Tomas Recio
http://www.recio.tk
Universidad Antonio de Nebrija

EACA Tapas Seminar on Computer Algebra
6 de Mayo 2021

**Some Tapas of Computer Algebra**. Eds: Arjeh M. Cohen, Hans Cuypers, Hans Sterk

Algorithms and Computation in Mathematics, Vol 4, 1999. Springer-Verlag.
https://www.springer.com/gp/book/9783540634805#aboutBook

Taormina, 1998




- Front MatterPages I-XIV Gröbner Bases, an Introduction, Arjeh M. Cohen
- Symbolic Recipes for Polynomial System Solving, Laureano Gonzalez-Vega, Fabrice Rouillier, Marie-Françoise Roy
- Lattice Reduction Frits Beukers
- Factorisation of Polynomials Frits Beukers
- Computations in Associative and Lie Algebras Gábor Ivanyos, Lajos Rónyai
- Symbolic Recipes for Real Solutions Laureano Gonzalez-Vega, Fabrice Rouillier, Marie-Françoise Roy, Guadalupe Trujillo
- Gröbner Bases and Integer Programming Günter M. Ziegler
- Working with Finite Groups Hans Cuypers, Leonard H. Soicher, Hans Sterk
- Symbolic Analysis of Differential Equations Marius van der Put
- Gröbner Bases for Codes Mario de Boer, Ruud Pellikaan
- Gröbner Bases for Decoding Mario de Boer, Ruud Pellikaan
- Automatic Geometry Theorem Proving Tomas Recio, Hans Sterk, M. Pilar Vélez
- The Birkhoff Interpolation Problem Maria-Jose Gonzalez-Lopez, Laureano Gonzalez-Vega
- The Inverse Kinematics Problem in Robotics Maria-Jose Gonzalez-Lopez, Laureano Gonzalez-Vega
- Quaternion Algebras Gábor Ivanyos, Lajos Rónyai
- Explorations with the Icosahedral Group Arjeh M. Cohen, Hans Cuypers, Remko Riebeek
- The Small Mathieu Groups Hans Cuypers, Leonard H. Soicher, Hans Sterk
- The Golay Codes Mario de Boer, Ruud Pellikaan

M.P. Vélez Melón (U. Nebrija)
F. Botana (U. Vigo)
A. Montes (UPC)
M.A. Abánades (URJC)
Zoltán Kovács (The Private University College of Education of the Diocese of Linz)
C. Solyom-Gecse (JKU, Linz)
J. R. Sendra (UAH)
C. Villarino (UAH)
M. Ladra, P. Páez (U. Santiago)
R. Vajda (U. Szeged)

P. Richard (U. Montreal)
B. Hauer, M. Hohenwarter, (JKU Linz) ,
C. Abar (Pontificia Universidade Sao Paulo)
S. Van Vaerenbergh (UC)
…….

R. Losada, J.L. Valcarce, N. De Lucas…
F. Etayo (UC), A. Martínez-Sevilla (UGR)

MTM2017-88796-P
GeoGebra como EPO

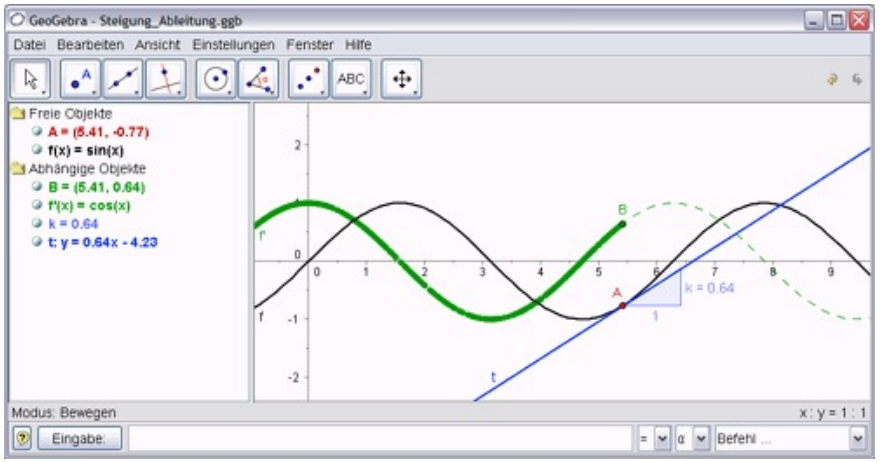
GeoGebra

Dynamic Mathematics for Everyone

# GEOGEBRA

**THE GRAPHING CALCULATOR FOR FUNCTIONS, GEOMETRY, ALGEBRA, CALCULUS, STATISTICS AND 3D MATH!**

## DYNAMIC MATHEMATICS FOR LEARNING AND TEACHING



Traducción algebraica. Variables libres, dimensión, componentes degeneradas

Refutación, eliminación no-libres hipótesis + negación tesis,
no cero => añadir condiciones de no-degeneración, generalmente verdad

Eliminación no-libres hipótesis + tesis, no cero => añadir restricciones, generalmente falso

Test número finito de casos

Verdad parcial

Caso real (MEP…)

Condiciones de no-degeneración (comprehensivas, Rabinowitch), caso cero dimensional…

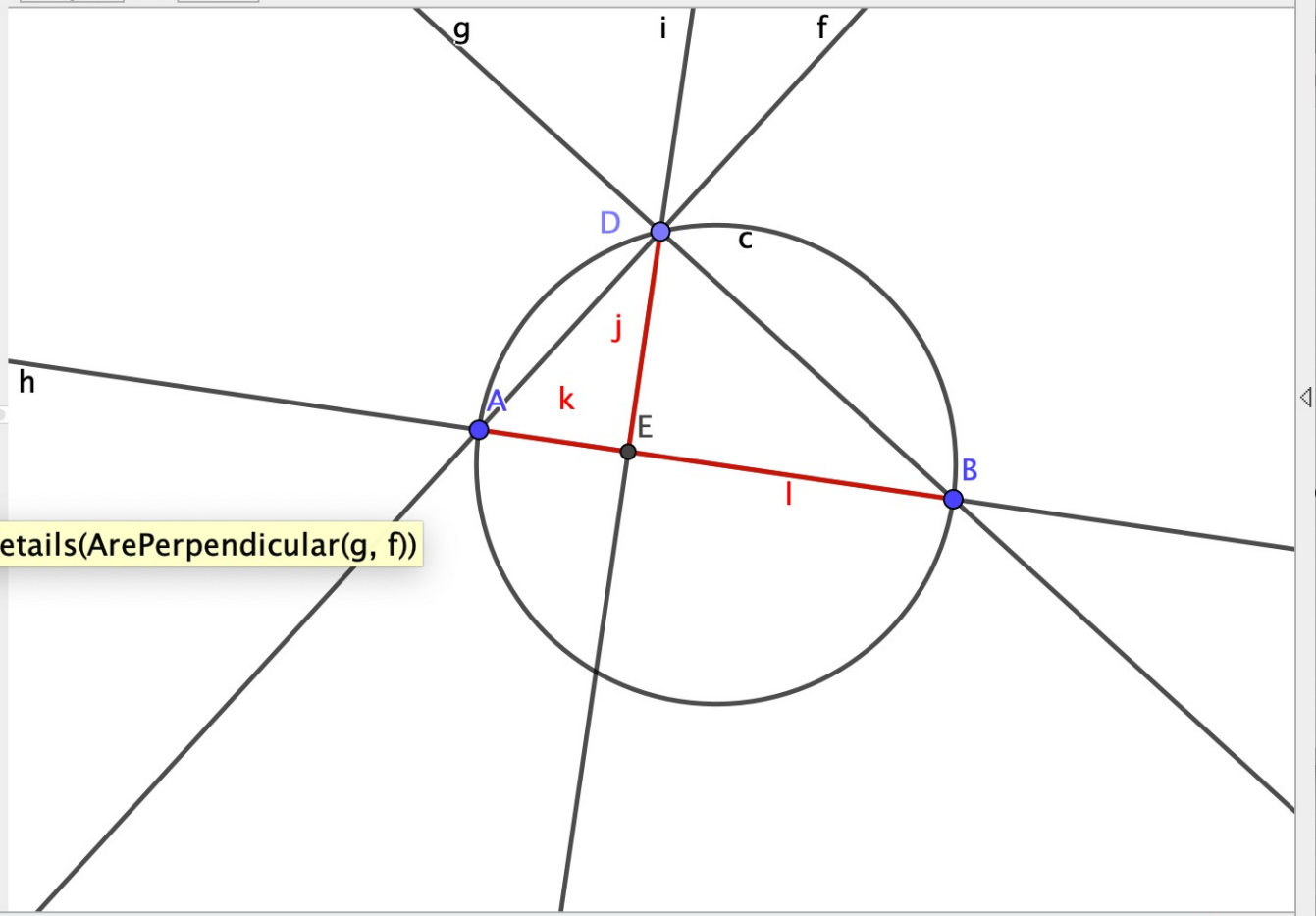
GeoGebra Classic 5

Algebra

- A = (0.72, 1.8)
- B = (5.78, 1.06)
- C = (3.25, 1.43)
- c: $(x - 3.25)^2 + (y - 1.43)^2 = 6.54$
- D = (2.66, 3.92)
- f: −2.12x + 1.94y = 1.96
- g: 2.86x + 3.12y = 19.82
- h: −0.74x − 5.06y = −9.64
- i: 5.06x − 0.74y = 10.54
- E = (2.31, 1.57)
- j = 2.37
- k = 1.61
- l = 3.5
- l1 = {true, {“AreEqual[A,B]”}}

List l1: ProveDetails(ArePerpendicular(g, f))

Graphics




Input:

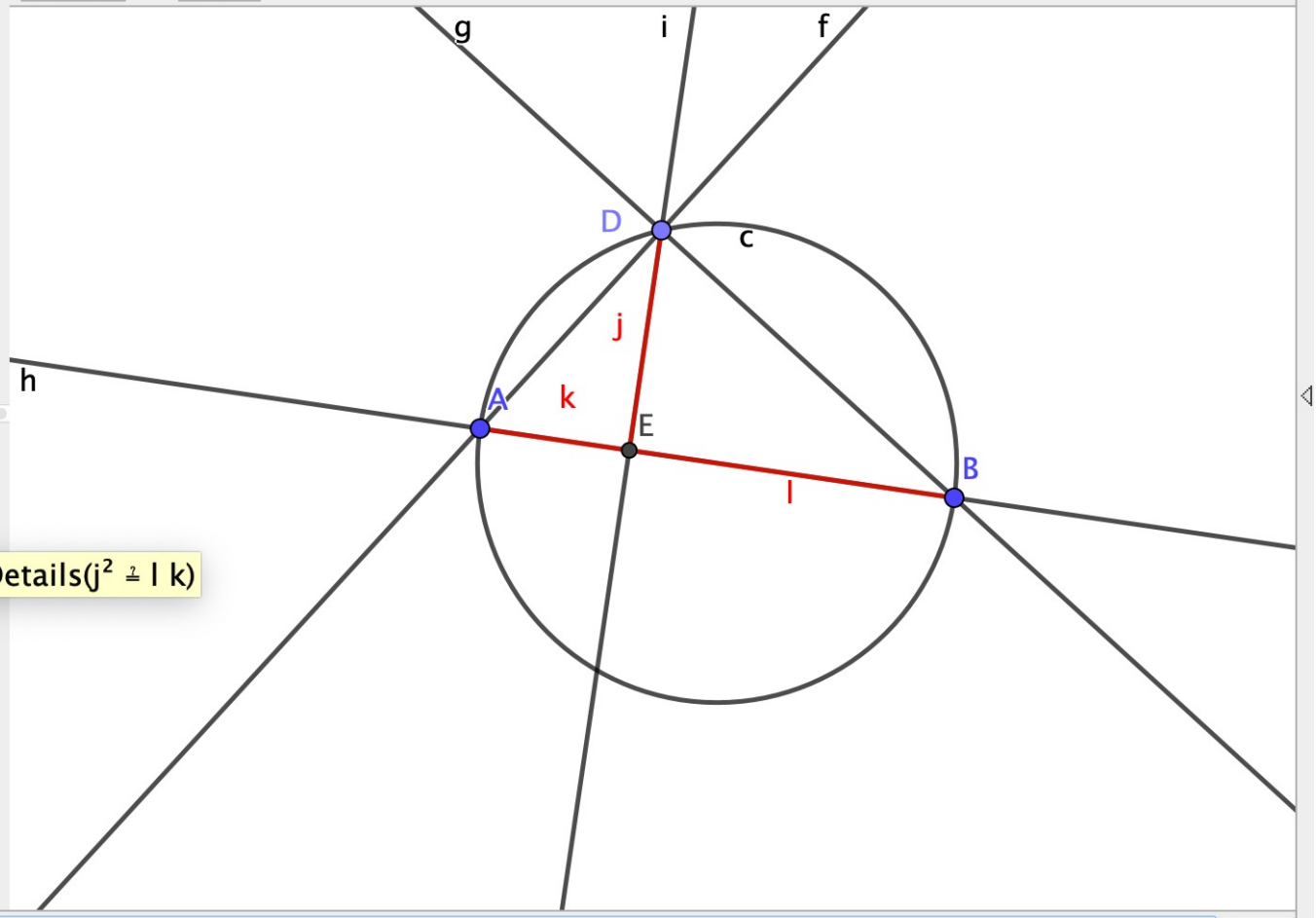
GeoGebra Classic 5

Algebra

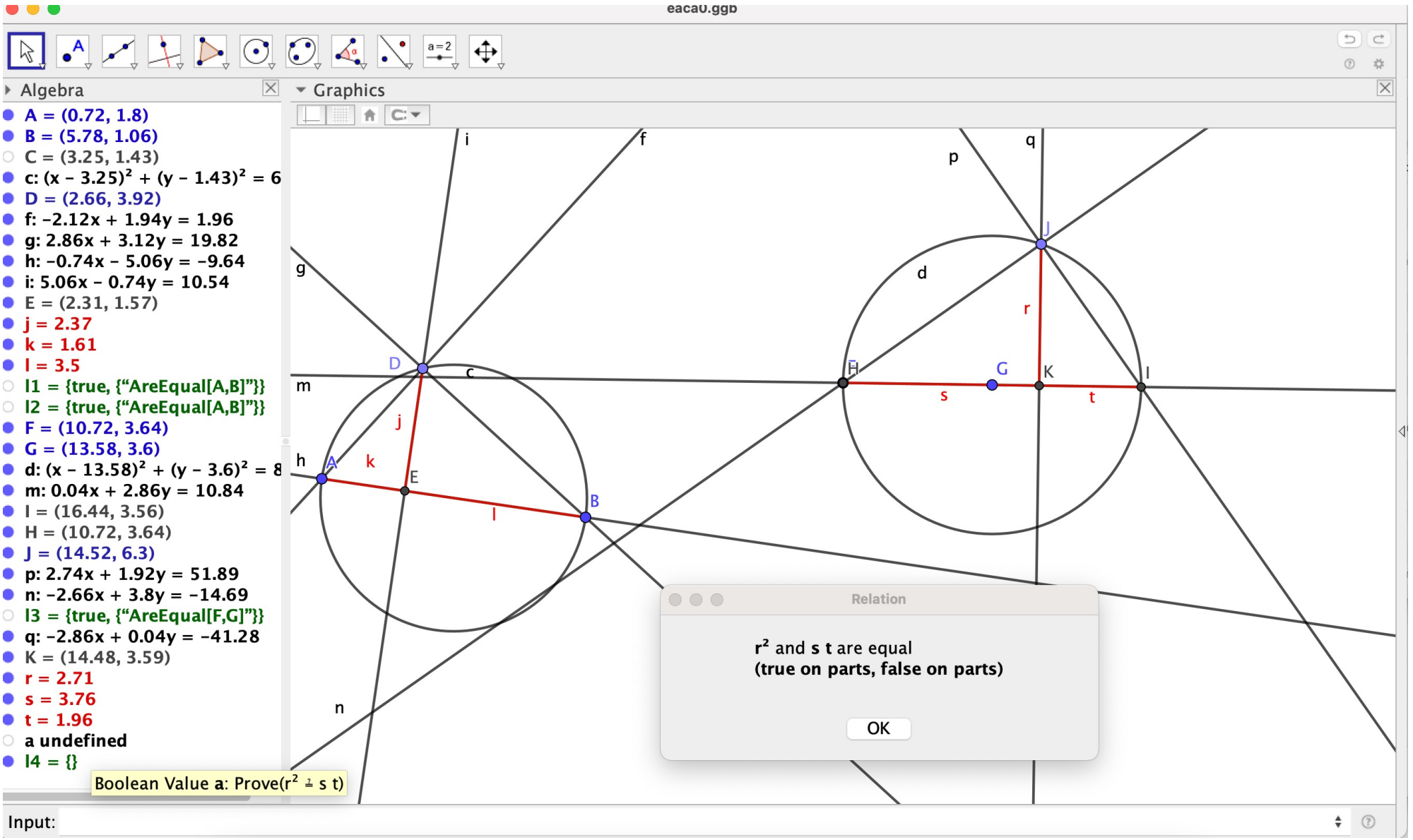
- A = (0.72, 1.8)
- B = (5.78, 1.06)
- C = (3.25, 1.43)
- c: $(x - 3.25)^2 + (y - 1.43)^2 = 6.54$
- D = (2.66, 3.92)
- f: $-2.12x + 1.94y = 1.96$
- g: $2.86x + 3.12y = 19.82$
- h: $-0.74x - 5.06y = -9.64$
- i: $5.06x - 0.74y = 10.54$
- E = (2.31, 1.57)
- j = 2.37
- k = 1.61
- l = 3.5
- l1 = {true, {"AreEqual[A,B]"}}
- l2 = {true, {"AreEqual[A,B]"}}

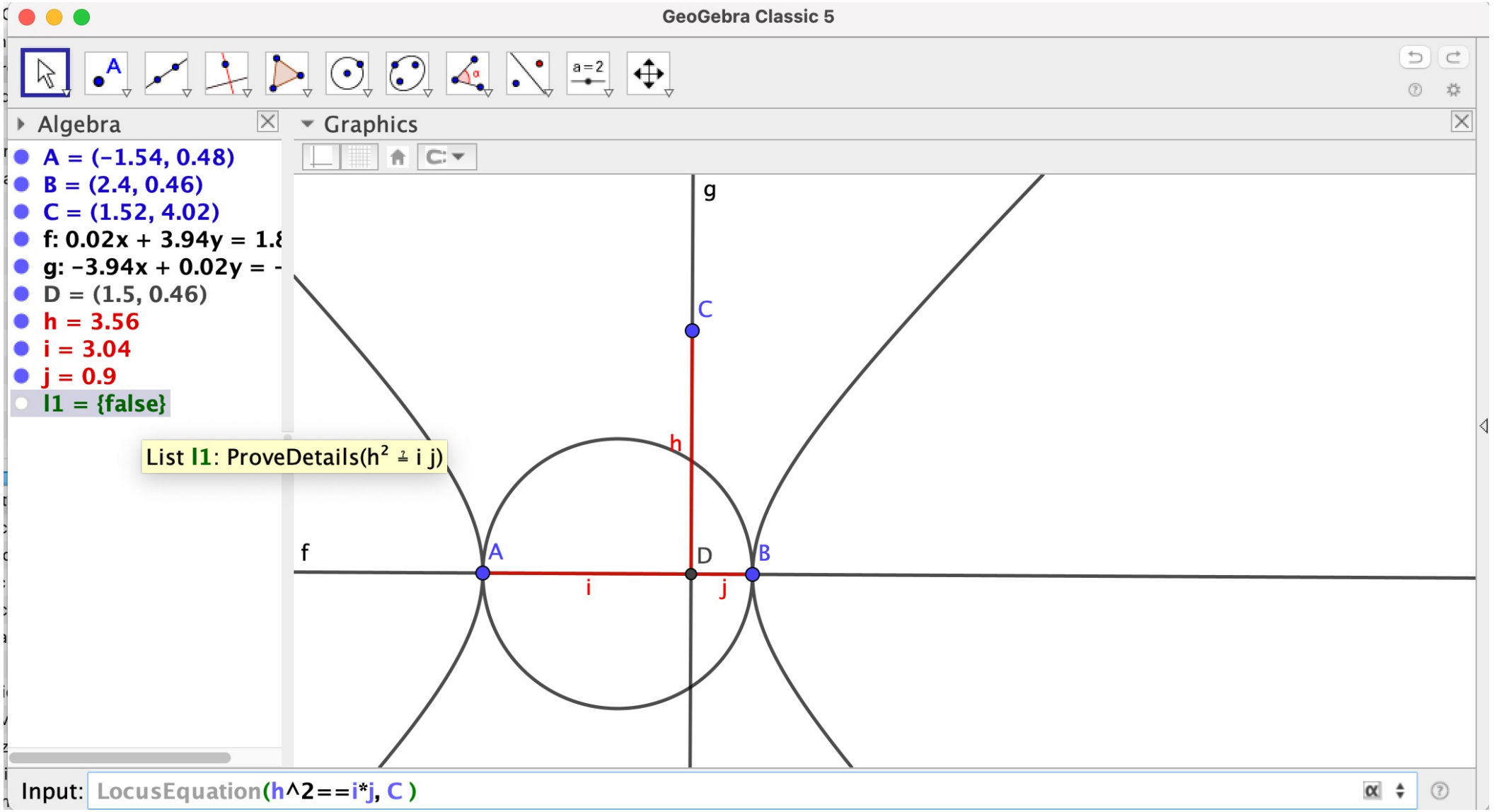
List l2: ProveDetails($j^2 = l\ k$)

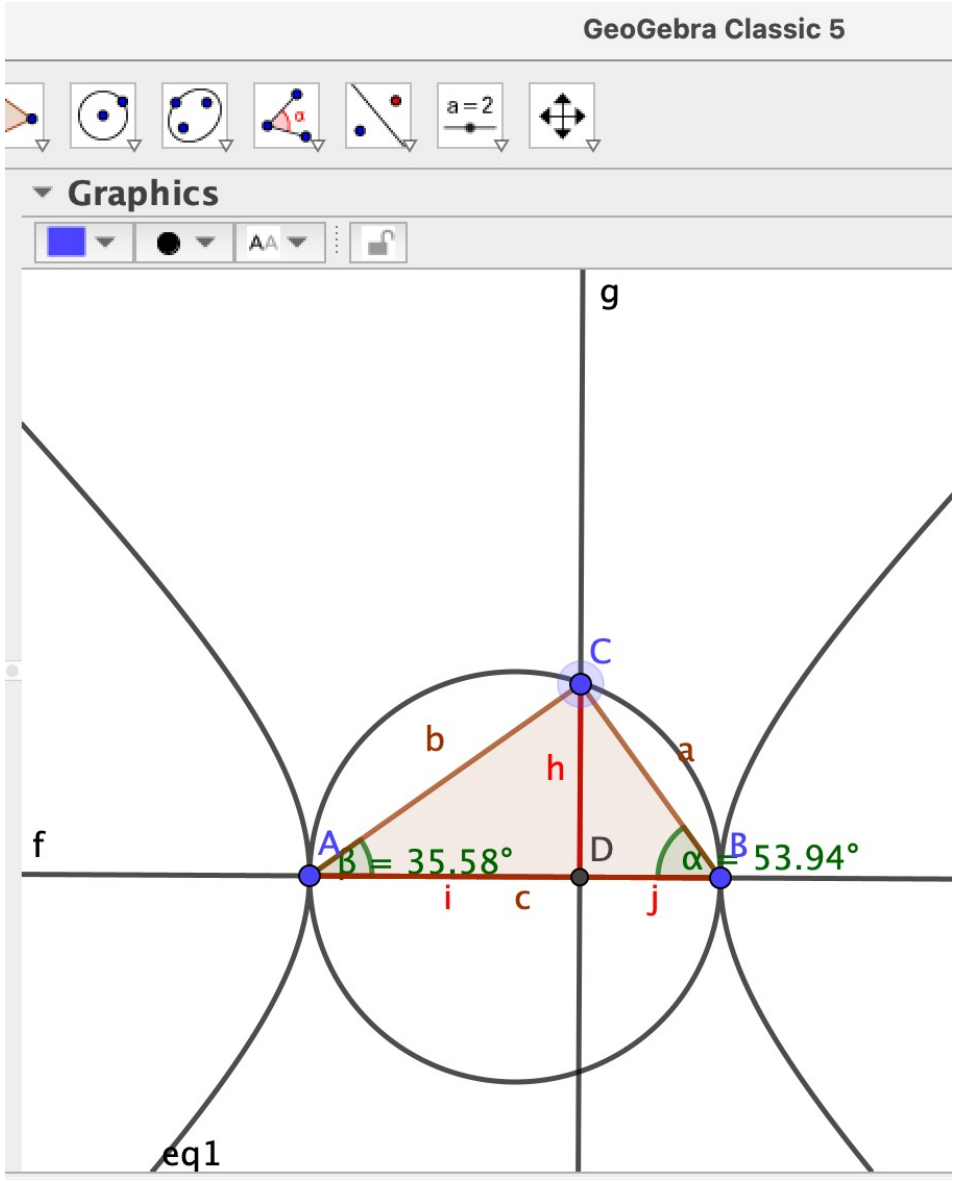
Graphics

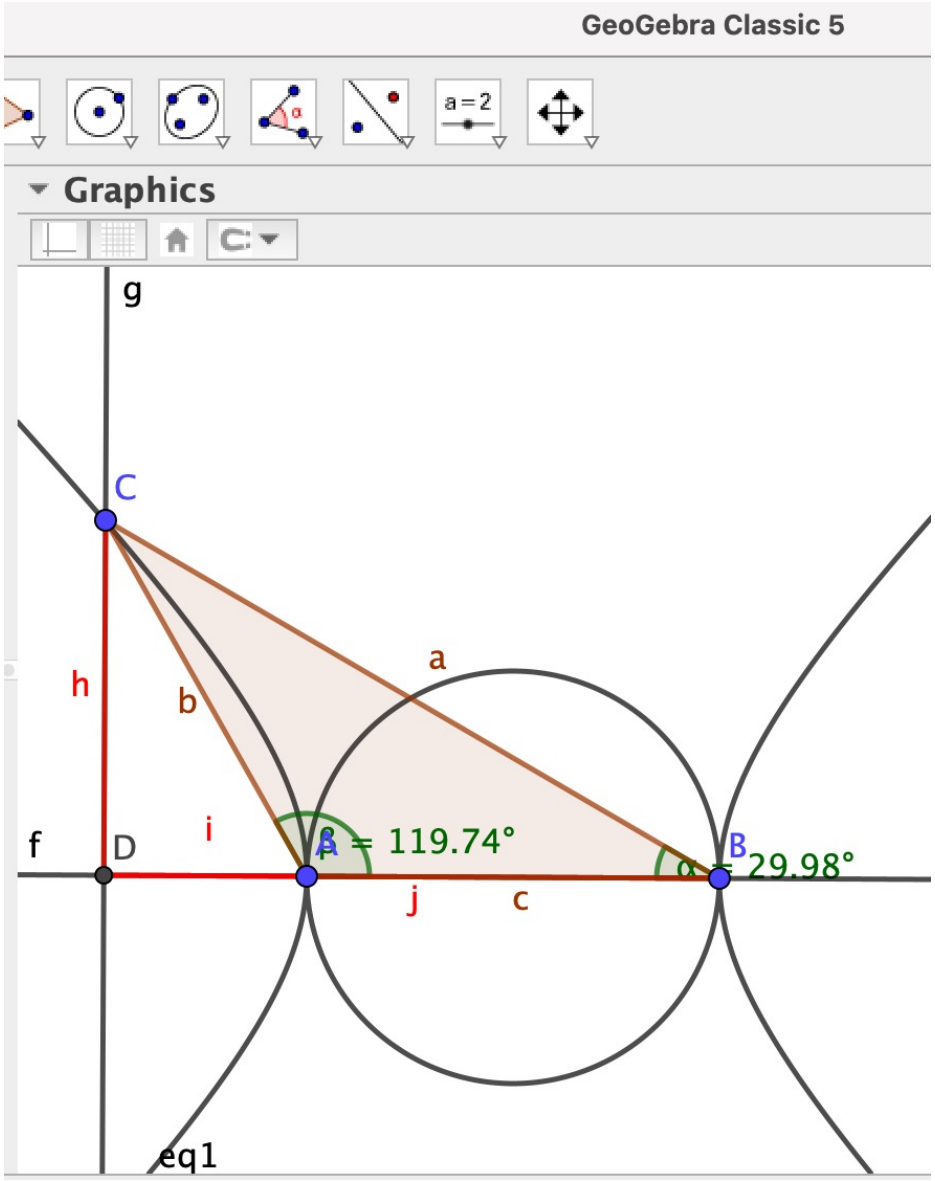



Input:


eacau.ggb
Algebra
A = (0.72, 1.8)
B = (5.78, 1.06)
C = (3.25, 1.43)
c: (x – 3.25)² + (y – 1.43)² = 6
D = (2.66, 3.92)
f: –2.12x + 1.94y = 1.96
g: 2.86x + 3.12y = 19.82
h: –0.74x – 5.06y = –9.64
i: 5.06x – 0.74y = 10.54
E = (2.31, 1.57)
j = 2.37
k = 1.61
l = 3.5
l1 = {true, {"AreEqual[A,B]"}}
l2 = {true, {"AreEqual[A,B]"}}
F = (10.72, 3.64)
G = (13.58, 3.6)
d: (x – 13.58)² + (y – 3.6)² = 8
m: 0.04x + 2.86y = 10.84
I = (16.44, 3.56)
H = (10.72, 3.64)
J = (14.52, 6.3)
p: 2.74x + 1.92y = 51.89
n: –2.66x + 3.8y = –14.69
l3 = {true, {"AreEqual[F,G]"}}
q: –2.86x + 0.04y = –41.28
K = (14.48, 3.59)
r = 2.71
s = 3.76
t = 1.96
a undefined
l4 = {}
Boolean Value a: Prove(r² = s t)
Graphics
Relation
r² and s t are equal
(true on parts, false on parts)
OK
Input:


GeoGebra Classic 5
Algebra
Graphics
A = (-1.54, 0.48)
B = (2.4, 0.46)
C = (1.52, 4.02)
f: 0.02x + 3.94y = 1.8
g: -3.94x + 0.02y = -
D = (1.5, 0.46)
h = 3.56
i = 3.04
j = 0.9
l1 = {false}
List l1: ProveDetails(h² = i j)
g
C
h
f
A
D
B
i
j
Input: LocusEquation(h^2==i*j, C )


GeoGebra Classic 5
Graphics
g
C
b
h
a
f
A
β = 35.58°
D
α = 53.94°
B
i
c
j
eq1


GeoGebra Classic 5
Graphics
g
C
h
b
a
f
D
i
β = 119.74°
B
α = 29.98°
j
c
eq1

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- Kovács, Z.; R.T. and Sólyom-Gecse C.: *Rewriting input expressions in complex algebraic geometry provers*. Ann Math Artif Intell. April 2019, Volume 85, Issue 2–4, pp 73–87.
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Implementación, divulgación

Aplicación: Realidad Aumentada, Paseos Matemáticos, Educación…

GeoGebra

Dynamic Mathematics for Everyone

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## DYNAMIC MATHEMATICS FOR LEARNING AND TEACHING

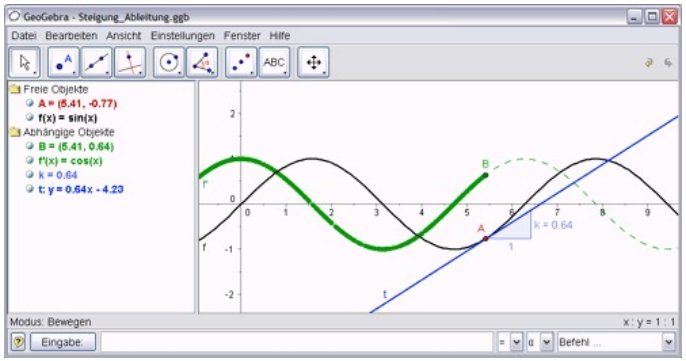
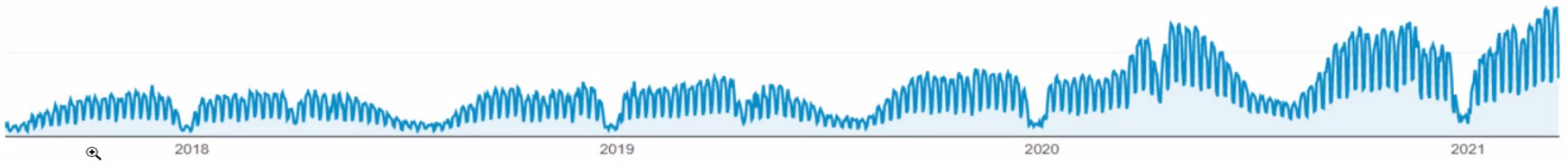



image.p

Markus Hohenwarter Today at 12:02



De Markus Hohenwarter a Todos 5:47

Past year: March 2020 - March 2021
pageviews: 435m (+88% prev year)
users: 77m (+49% prev year)

Continents: Americas 44%, Europe 35%, Asia 18%

Languages: Spanish 30%, Englisch 29%, German 7%, French 6%, Chinese 5%, Italian 4%, Portugese 3%

Top 10 Countries: USA 13%, Mexico 10%, Germany 6%, Colombia 5%, Italy 4%, France 4%, China 4%, Spain 4%, Argentina 3%, Brazil 3%

GeoGebra Community Gathering, March 25, 2021

https://geogebra.zoom.us/rec/play/y26WqWb4FIfDXg94MvQ3qEraAI7SkseD8E8g6sjn7qV3ICxUNXBV5ah72l-r3EzljDXpQQ3b7aW402s8.qpSjiuB1vSl9J-jO

- Botana, F.; Hohenwarter, M.; Janicic, J.; Kovacs, Z.; Petrovic, I.;R.T.; Weitzhofer S.: *Automated Theorem Proving in GeoGebra: Current Achievements*. Journal of Automated Reasoning, Volume 55, Issue 1, pp 39-59, 2015.

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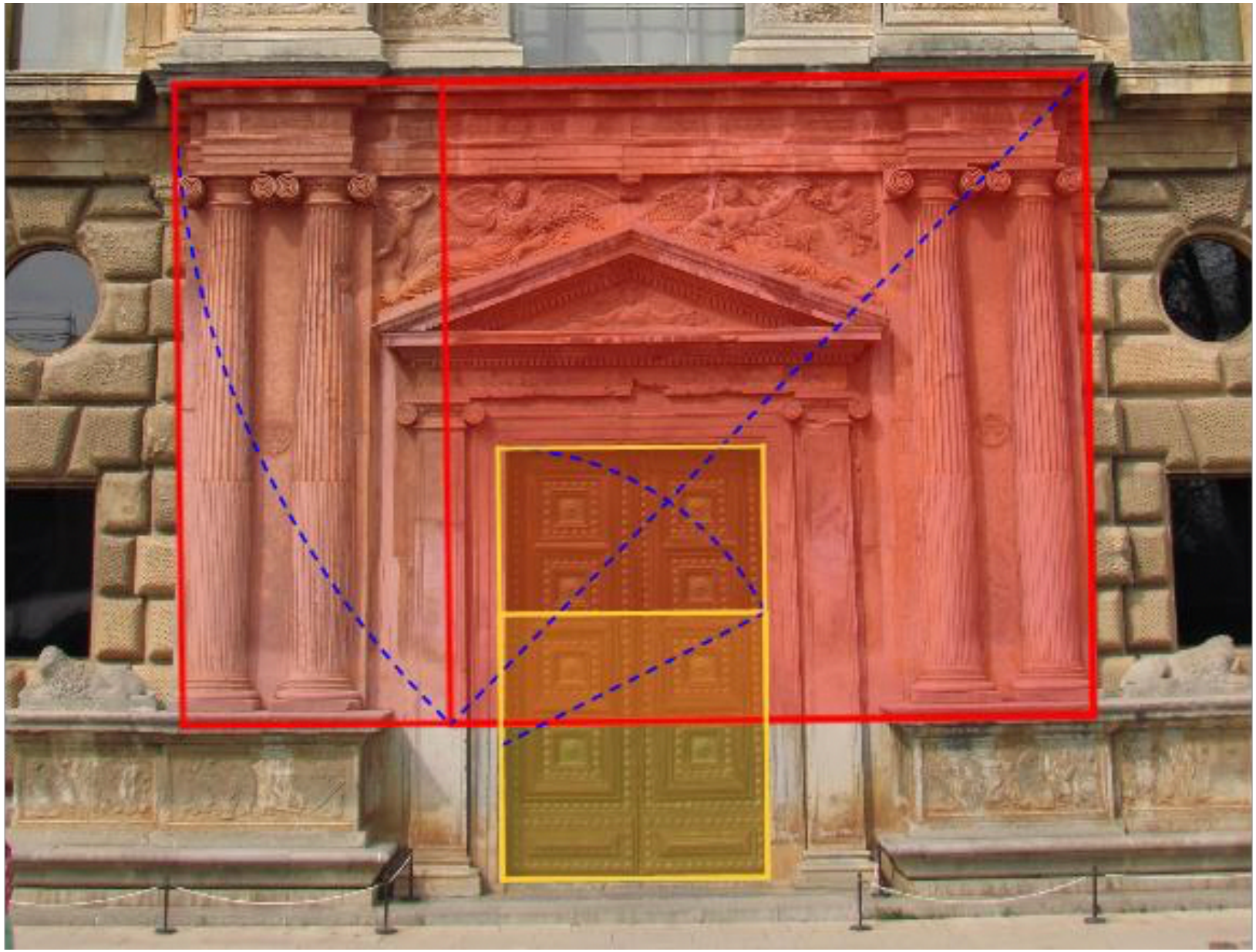
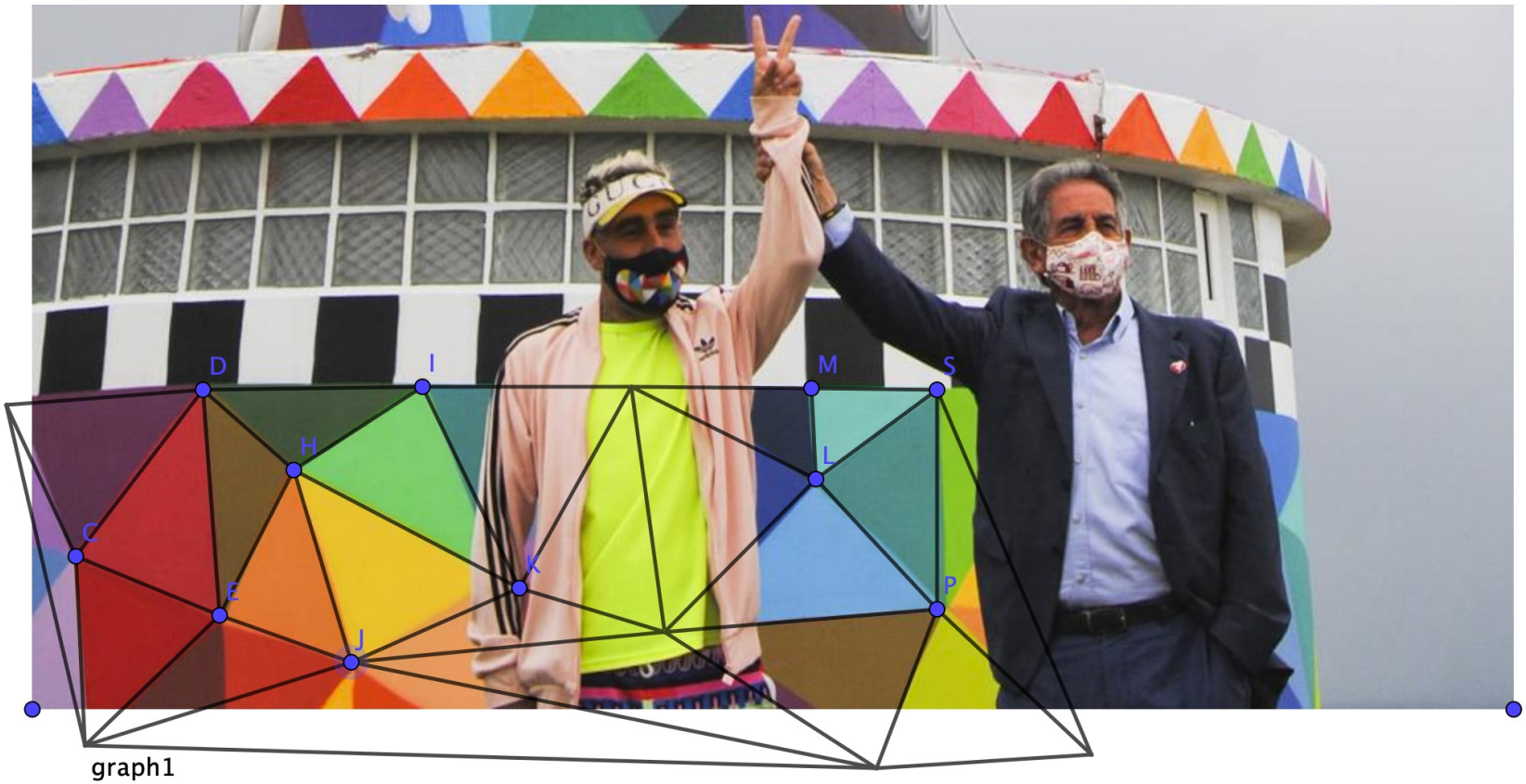


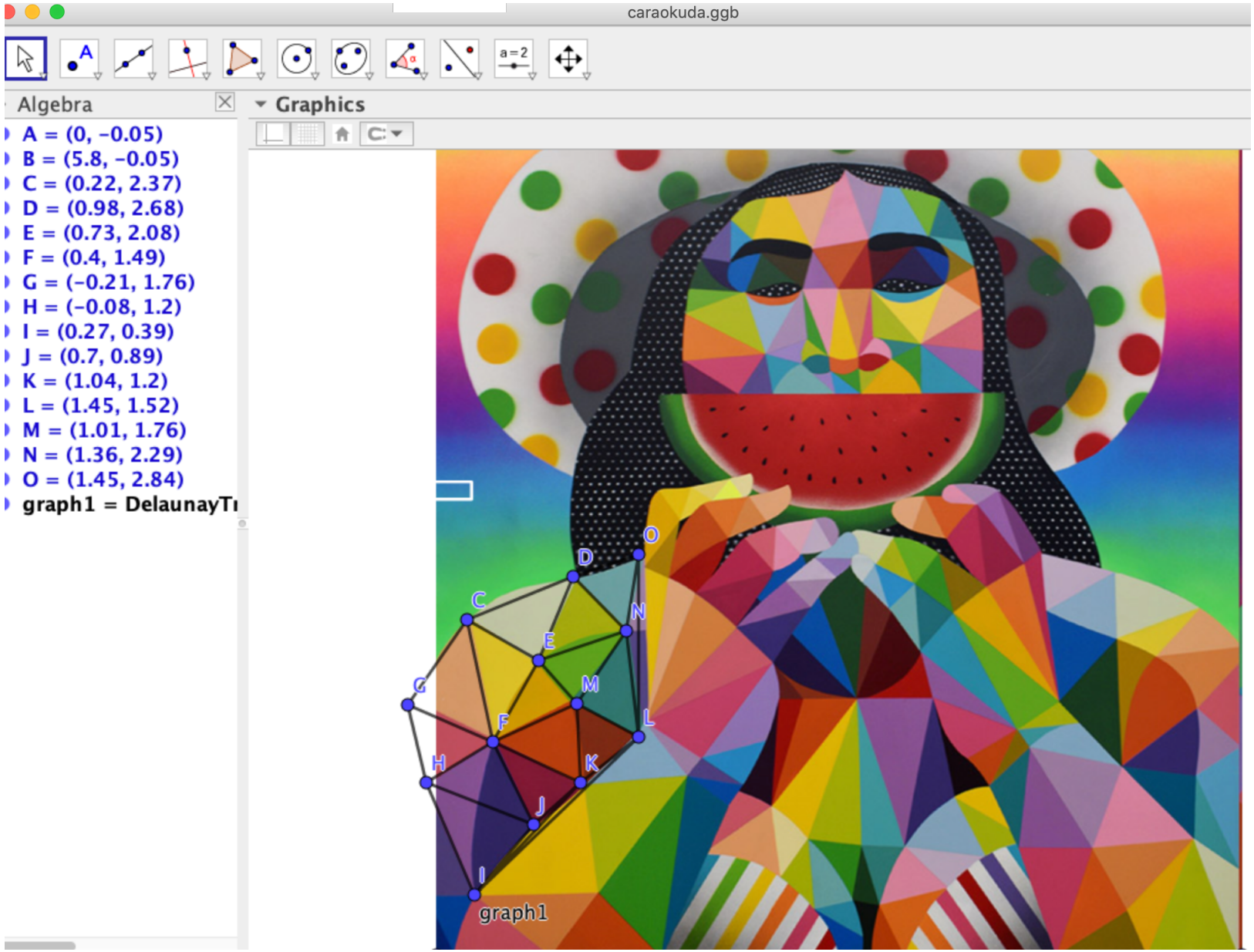
GeoGebra Classic 5

Algebra

- A = (-0.19, 1.64)
- B = (5.61, 1.64)
- C = (-0.02, 2.24)
- D = (0.48, 2.89)
- E = (0.54, 2.01)
- F = (0.01, 1.5)
- G = (-0.29, 2.83)
- H = (0.83, 2.58)
- I = (1.34, 2.91)
- J = (1.06, 1.83)
- K = (1.72, 2.12)
- L = (2.88, 2.54)
- M = (2.86, 2.9)
- N = (2.15, 2.91)
- O = (2.28, 1.95)
- P = (3.35, 2.03)
- Q = (3.11, 1.41)
- R = (3.96, 1.47)
- S = (3.35, 2.89)
- graph1 = DelaunayTı

Graphics




caraokuda.ggb
Algebra
A = (0, -0.05)
B = (5.8, -0.05)
C = (0.22, 2.37)
D = (0.98, 2.68)
E = (0.73, 2.08)
F = (0.4, 1.49)
G = (-0.21, 1.76)
H = (-0.08, 1.2)
I = (0.27, 0.39)
J = (0.7, 0.89)
K = (1.04, 1.2)
L = (1.45, 1.52)
M = (1.01, 1.76)
N = (1.36, 2.29)
O = (1.45, 2.84)
graph1 = DelaunayTr
Graphics
graph1

- Botana, F.,  Kovács, Z., Martínez-Sevilla, A.: R.T.: *Automatically Augmented Reality with GeoGebra.* In: Theodosia Prodromou (Ed.),  Augmented Reality in Educational Settings, Brill-Sense. Nov. 2019.

- Botana F.; Kovács Z. and Recio T.: *Automatically Augmented Reality for Outdoor Mathematics*. In: Research on Outdoor STEM Education in the digiTal Age. Proceedings of the ROSETA Online Conference in June 2020. Matthias Ludwig, Simone Jablonski, Amélia Caldeira and Ana Moura (Editors). WTM – Verlag für wissenschaftliche Texte und Medien, Münster 2020. Conference Proceedings in Mathematics Education (6), pages 71-78.

Trabajo en curso…

AG: Geómetra mecánico, autómata

Relación /Compara / Descubre

Valoración de resultados:

H-sencilla vs T-sencilla

H-sencilla vs T-compleja

H-compleja vs T-sencilla

H-compleja vs T-compleja

- https://github.com/kovzol/geogebra/releases

Versiones GeoGebra 5 Discovery y GeoGebra 6 Discovery off-line

- http://www.autgeo.online/geogebra-discovery/

GeoGebra 6 Discovery on-line

- http://www.autgeo.online/ag/automated-geometer.html?offline=1

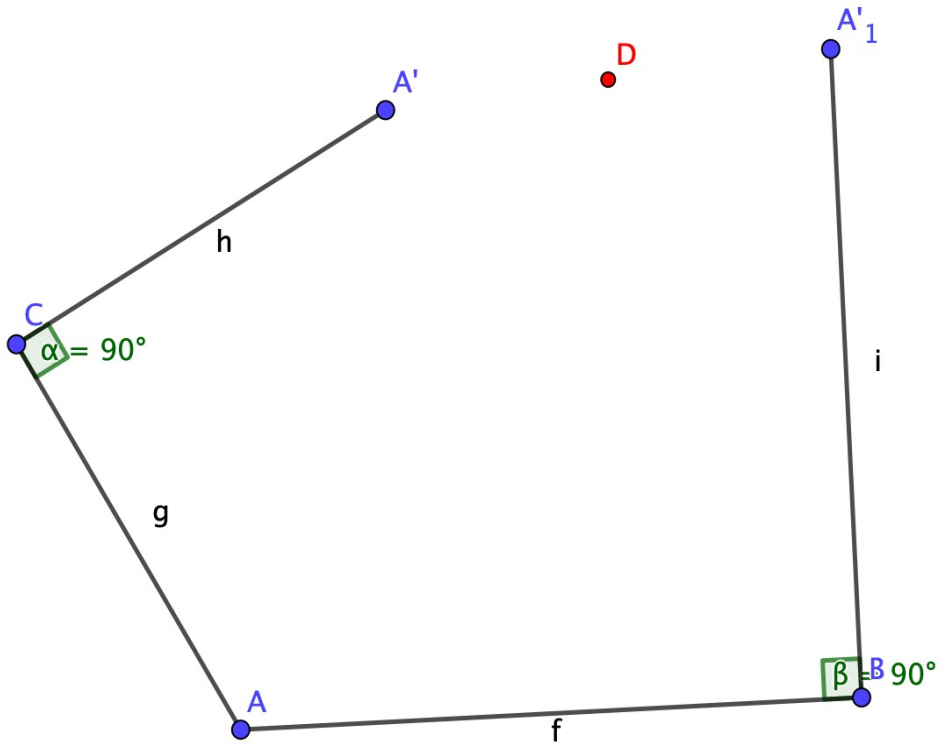
Automated Geometer

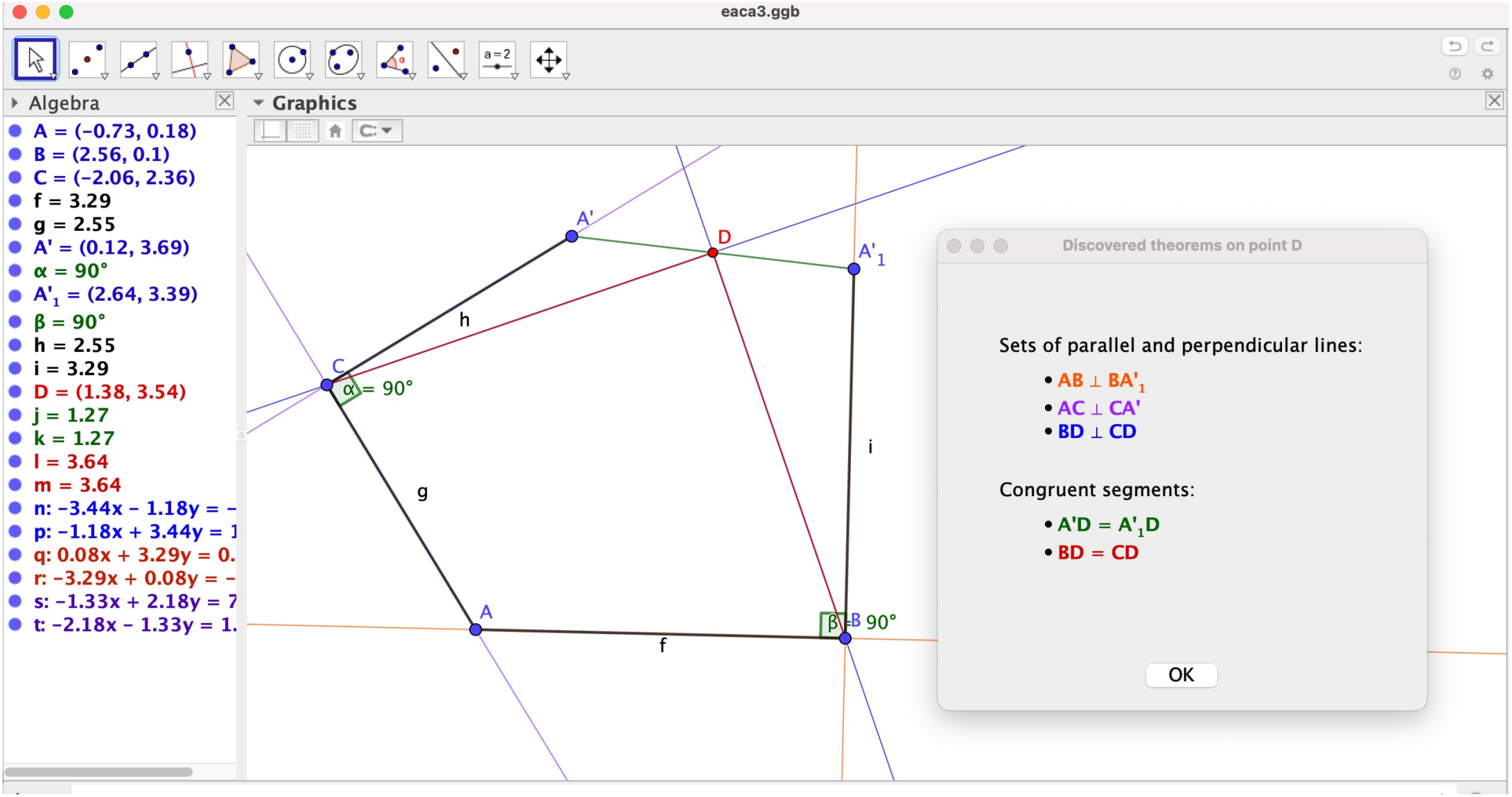
eaca3.ggb

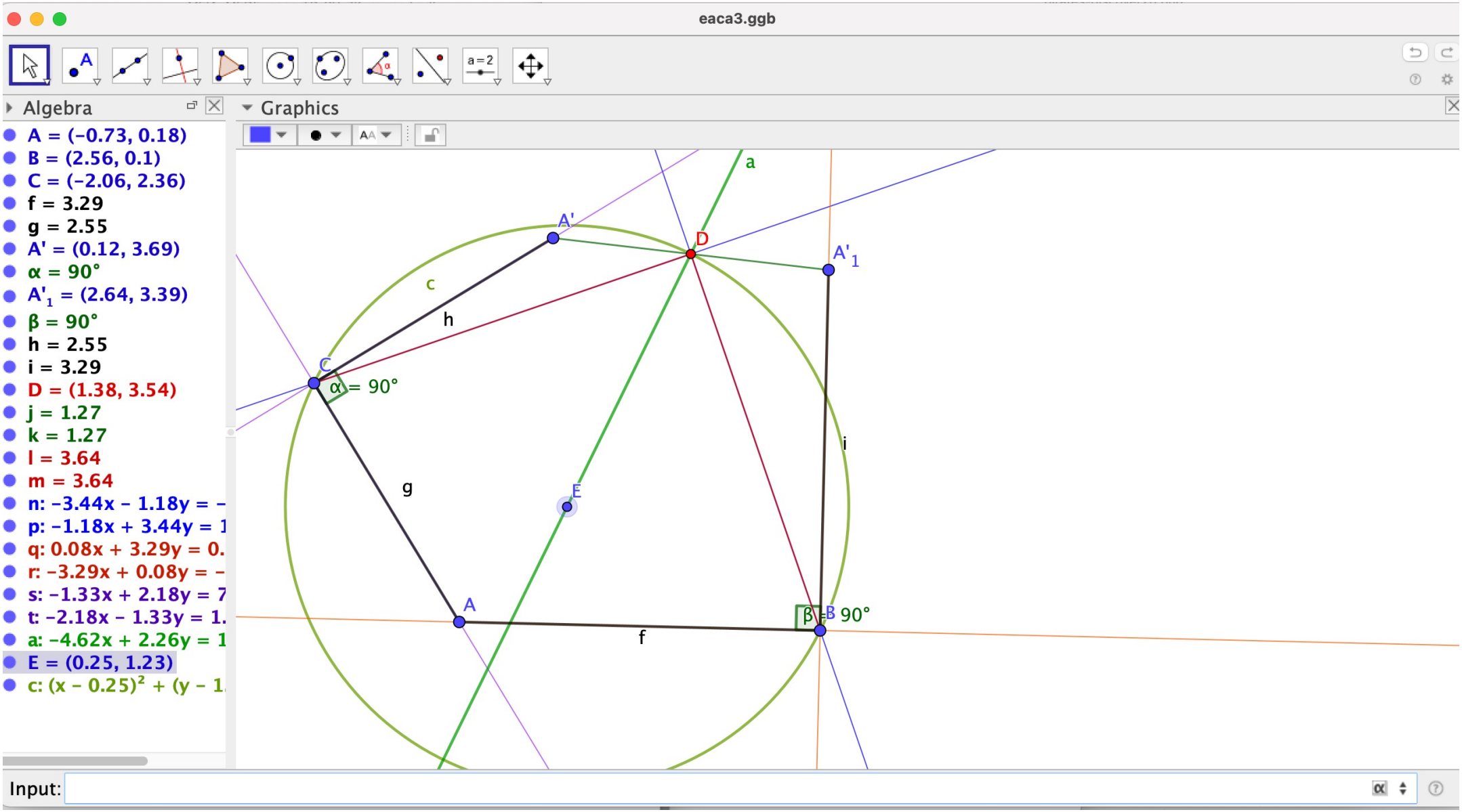
Algebra

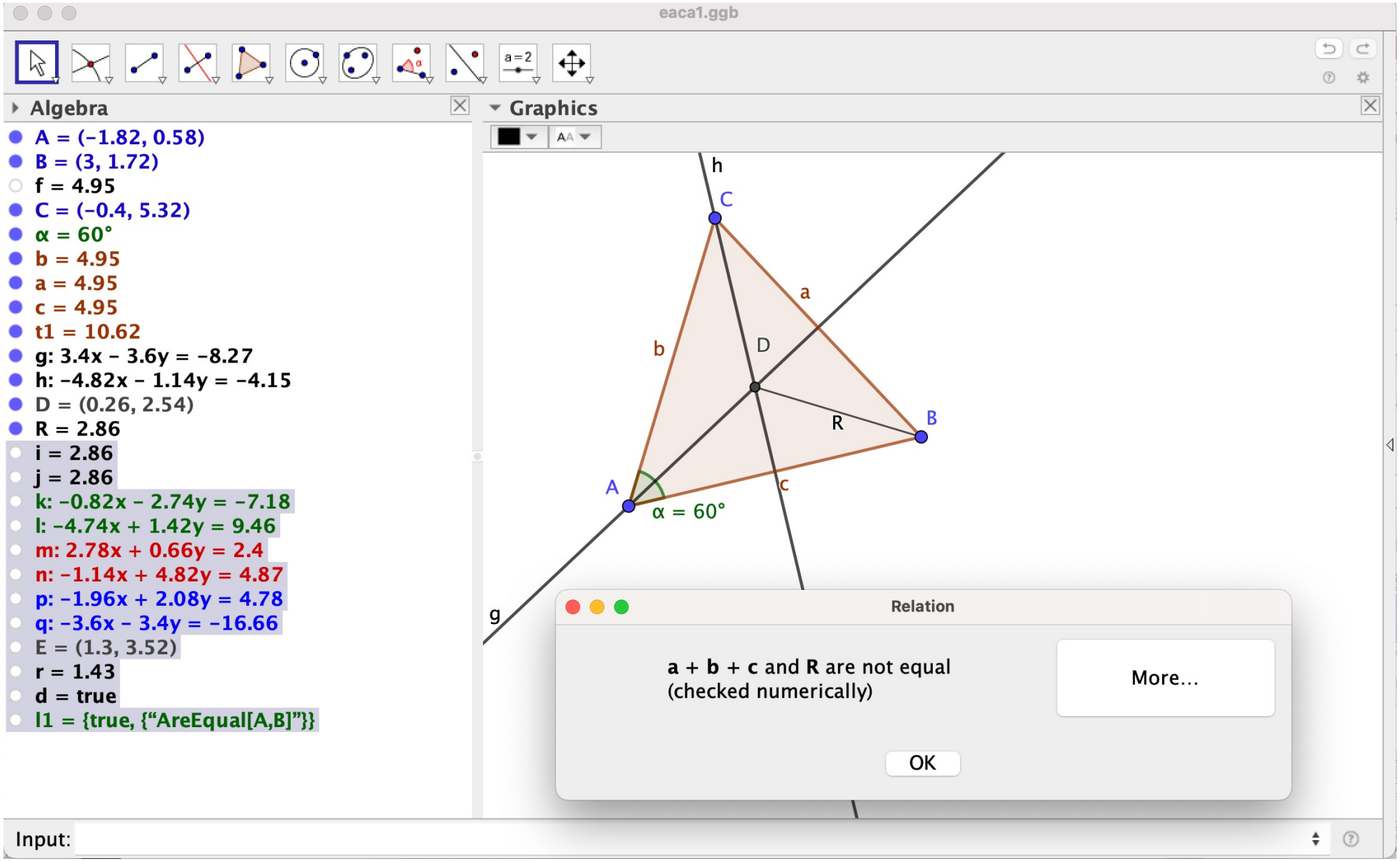
- A = (-0.73, 0.18)
- B = (2.94, 0.36)
- C = (-2.06, 2.36)
- f = 3.68
- g = 2.55
- A' = (0.12, 3.69)
- α = 90°
- $A'_1$ = (2.76, 4.03)
- β = 90°
- h = 2.55
- i = 3.68
- D = (1.44, 3.86)
- j = 1.33
- k = 1.33
- l = 3.81
- m = 3.81
- n: -3.5x - 1.5y = -10
- p: -1.5x + 3.5y = 11.
- q: -0.18x + 3.67y = (
- r: -3.67x - 0.18y = -
- s: -1.33x + 2.18y = 7
- t: -2.18x - 1.33y = 1.
- a: -5x + 2y = 0.53
- E = (0.44, 1.36)
- c: $(x - 0.44)^2 + (y - 1$

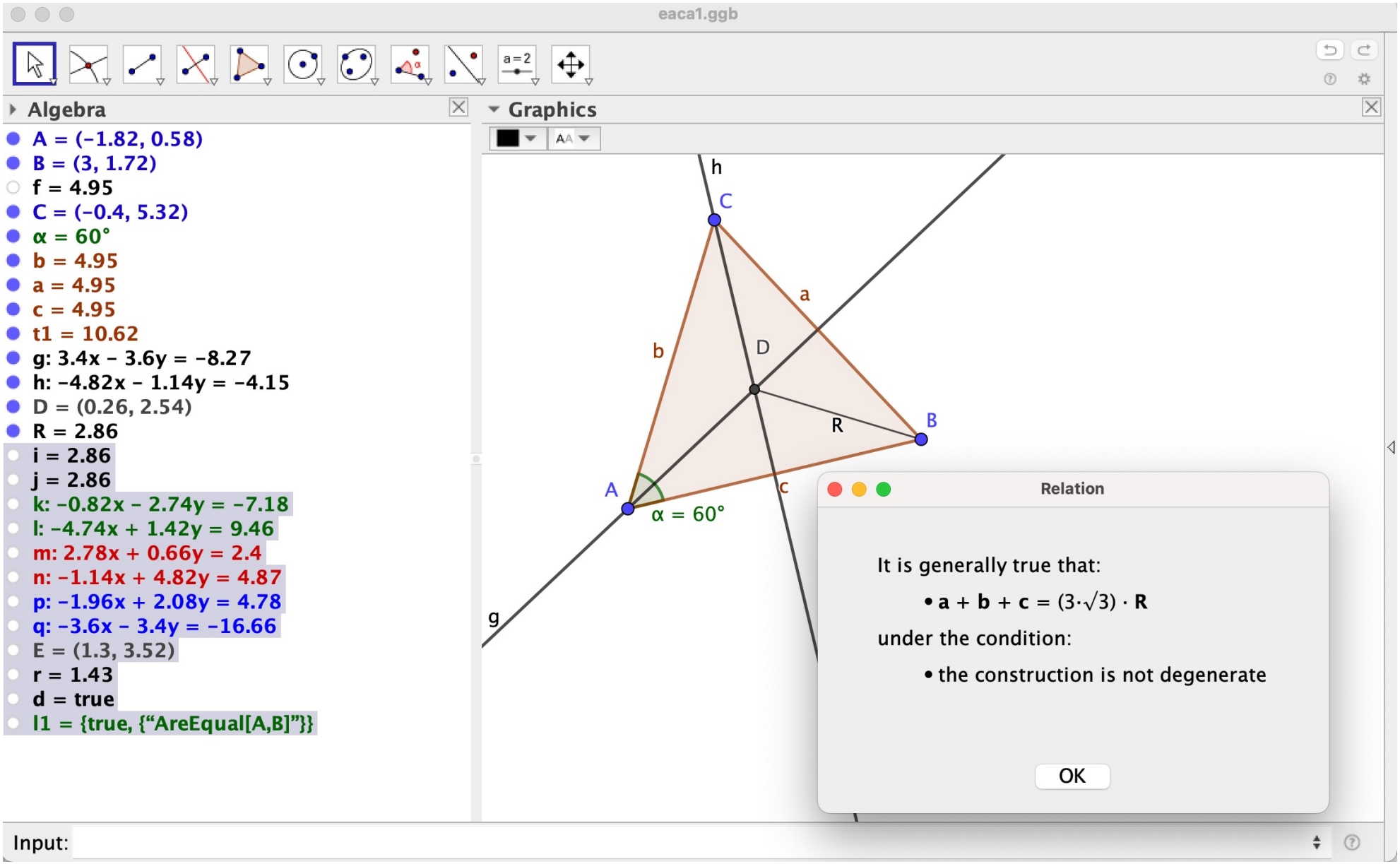
Graphics




eaca3.ggb
Algebra
A = (-0.73, 0.18)
B = (2.56, 0.1)
C = (-2.06, 2.36)
f = 3.29
g = 2.55
A' = (0.12, 3.69)
α = 90°
A'1 = (2.64, 3.39)
β = 90°
h = 2.55
i = 3.29
D = (1.38, 3.54)
j = 1.27
k = 1.27
l = 3.64
m = 3.64
n: -3.44x - 1.18y = -
p: -1.18x + 3.44y = 1
q: 0.08x + 3.29y = 0.
r: -3.29x + 0.08y = -
s: -1.33x + 2.18y = 7
t: -2.18x - 1.33y = 1.
Graphics
A'
D
A'1
h
C
α = 90°
i
g
A
β = 90°
f
Discovered theorems on point D
Sets of parallel and perpendicular lines:
AB ⊥ BA'1
AC ⊥ CA'
BD ⊥ CD
Congruent segments:
A'D = A'1D
BD = CD
OK


eaca3.ggb
Algebra
Graphics
A = (-0.73, 0.18)
B = (2.56, 0.1)
C = (-2.06, 2.36)
f = 3.29
g = 2.55
A' = (0.12, 3.69)
α = 90°
A'1 = (2.64, 3.39)
β = 90°
h = 2.55
i = 3.29
D = (1.38, 3.54)
j = 1.27
k = 1.27
l = 3.64
m = 3.64
n: -3.44x - 1.18y = -
p: -1.18x + 3.44y = 1
q: 0.08x + 3.29y = 0.
r: -3.29x + 0.08y = -
s: -1.33x + 2.18y = 7
t: -2.18x - 1.33y = 1.
a: -4.62x + 2.26y = 1
E = (0.25, 1.23)
c: (x - 0.25)² + (y - 1.
a
A'
D
A'1
c
h
C
α = 90°
i
g
E
A
B
β = 90°
f
Input:


eaca1.ggb
Algebra
A = (-1.82, 0.58)
B = (3, 1.72)
f = 4.95
C = (-0.4, 5.32)
α = 60°
b = 4.95
a = 4.95
c = 4.95
t1 = 10.62
g: 3.4x - 3.6y = -8.27
h: -4.82x - 1.14y = -4.15
D = (0.26, 2.54)
R = 2.86
i = 2.86
j = 2.86
k: -0.82x - 2.74y = -7.18
l: -4.74x + 1.42y = 9.46
m: 2.78x + 0.66y = 2.4
n: -1.14x + 4.82y = 4.87
p: -1.96x + 2.08y = 4.78
q: -3.6x - 3.4y = -16.66
E = (1.3, 3.52)
r = 1.43
d = true
l1 = {true, {"AreEqual[A,B]"}}
Graphics
h
C
a
b
D
R
B
A
c
α = 60°
g
Relation
a + b + c and R are not equal
(checked numerically)
More...
OK
Input:

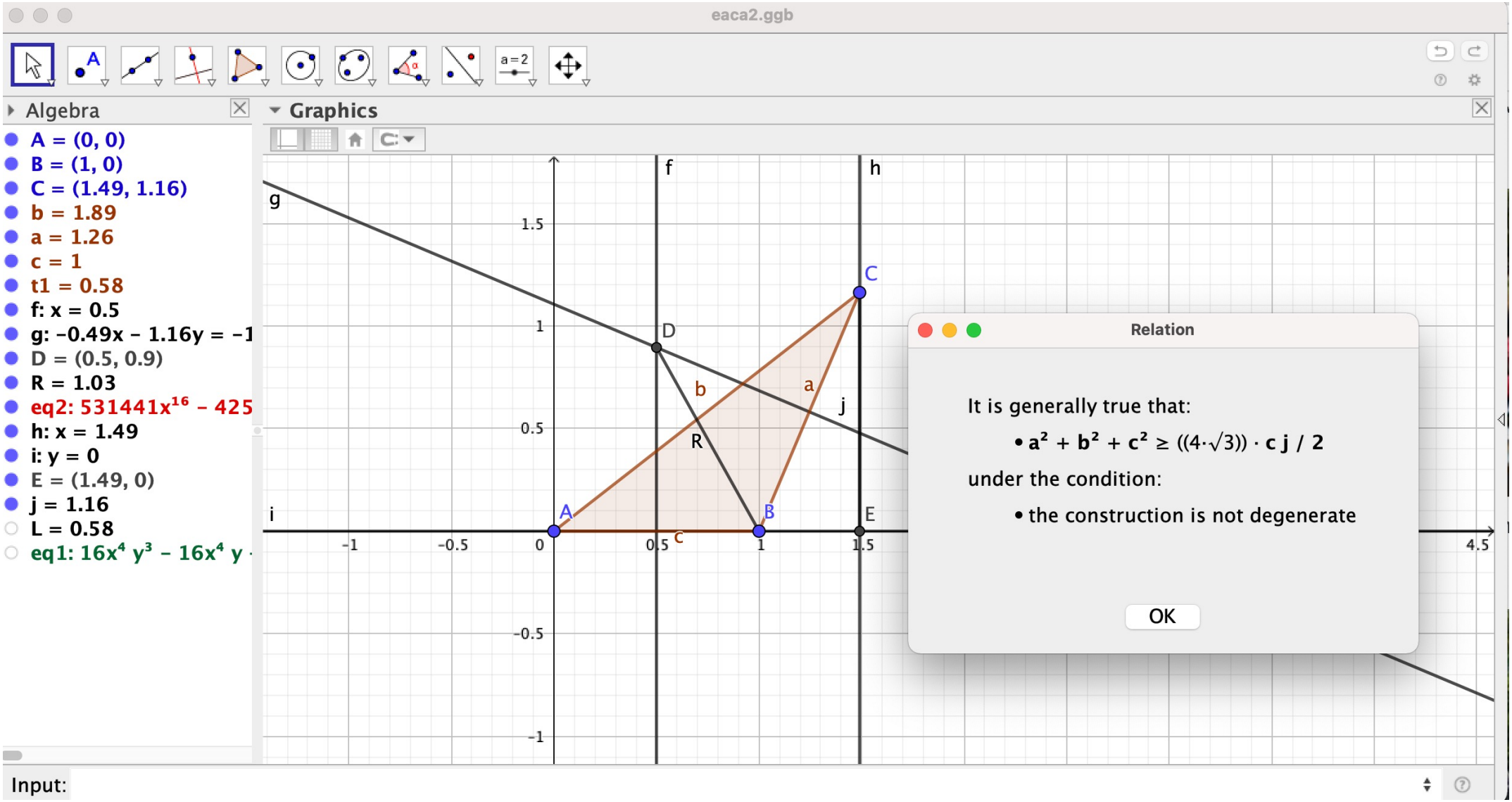

eaca1.ggb
Algebra
A = (-1.82, 0.58)
B = (3, 1.72)
f = 4.95
C = (-0.4, 5.32)
α = 60°
b = 4.95
a = 4.95
c = 4.95
t1 = 10.62
g: 3.4x - 3.6y = -8.27
h: -4.82x - 1.14y = -4.15
D = (0.26, 2.54)
R = 2.86
i = 2.86
j = 2.86
k: -0.82x - 2.74y = -7.18
l: -4.74x + 1.42y = 9.46
m: 2.78x + 0.66y = 2.4
n: -1.14x + 4.82y = 4.87
p: -1.96x + 2.08y = 4.78
q: -3.6x - 3.4y = -16.66
E = (1.3, 3.52)
r = 1.43
d = true
l1 = {true, {"AreEqual[A,B]"}}
Graphics
h
C
a
b
D
R
B
A
c
α = 60°
g
Relation
It is generally true that:
• a + b + c = (3·√3) · R
under the condition:
• the construction is not degenerate
OK
Input:

https://www.jstage.jst.go.jp/article/tmj1911/25/0/25_0_115/_pdf/-char/en

Prof. R. Weitzenböck has proved in the Math. Zeitschrift 5 that

$$\frac{F}{a^2+b^2+c^2} \leqq \frac{1}{4\sqrt{3}},$$

where the equality holds good when and only when the triangle is regular.

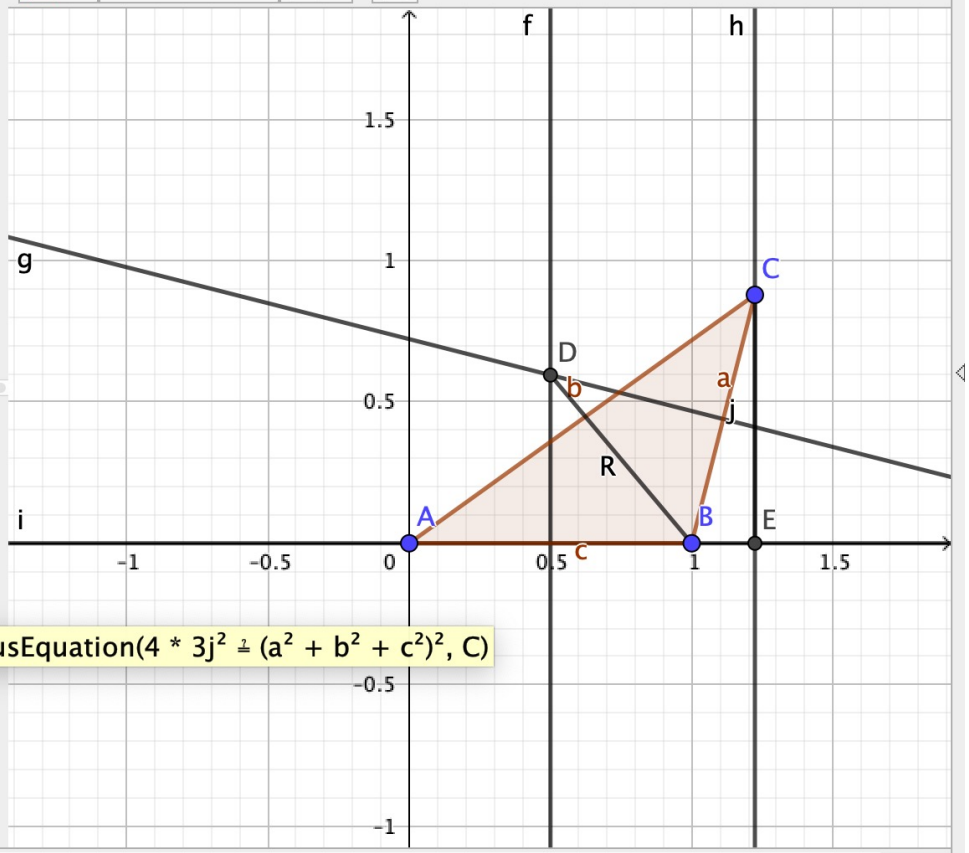

eaca2.ggb
Algebra
A = (0, 0)
B = (1, 0)
C = (1.49, 1.16)
b = 1.89
a = 1.26
c = 1
t1 = 0.58
f: x = 0.5
g: -0.49x - 1.16y = -1
D = (0.5, 0.9)
R = 1.03
eq2: 531441x^16 - 425
h: x = 1.49
i: y = 0
E = (1.49, 0)
j = 1.16
L = 0.58
eq1: 16x^4 y^3 - 16x^4 y
Graphics
Relation
It is generally true that:
• a² + b² + c² ≥ ((4·√3)) · c j / 2
under the condition:
• the construction is not degenerate
OK
Input:

eaca2.ggb

Algebra

- A = (0, 0)
- B = (1, 0)
- C = (1.22, 0.88)
- b = 1.51
- a = 0.91
- c = 1
- t1 = 0.44
- f: x = 0.5
- g: -0.22x - 0.88y = -0.64
- D = (0.5, 0.6)
- R = 0.78
- eq2: $531441x^{16} - 4251528x^{15} + 3621672x^{14}y^2 + 14880348x^{14} - 2535170$
- h: x = 1.22
- i: y = 0
- E = (1.22, 0)
- j = 0.88
- L = 0.44
- eq1: $16x^4y^3 - 16x^4y - 32x^3y^3 + 32x^3y + 24x^2y^3 - 16x^2y - 8xy^3 - 16y^5 +$
- eq3: $x^4 - 2x^3 + 2x^2y^2 + 3x^2 - 2xy^2 - 2x + y^4 - y^2 = -1$
- $d(x,y) = \left(\left(x - \frac{1}{2}\right)^2 + \left(y - \frac{\sqrt{3}}{2}\right)^2\right)\left(\left(x - \frac{1}{2}\right)^2 + \left(y + \frac{\sqrt{3}}{2}\right)^2\right)$
- $e(x,y) = x^4 - 2x^3 + 2x^2y^2 + 3x^2 - 2xy^2 - 2x + y^4 - y^2 + 1$

Graphics




Implicit Curve eq3: LocusEquation(4 * 3j² = (a² + b² + c²)², C)

Input:

# GEOMETRIC INEQUALITIES

BY

O. BOTTEMA
*Delft, The Netherlands*

R. Ž. DJORDJEVIĆ
*Belgrade, Yugoslavia*

R. R. JANIĆ
*Belgrade, Yugoslavia*

D. S. MITRINOVIĆ
*Belgrade, Yugoslavia*

P. M. VASIĆ
*Belgrade, Yugoslavia*

## THE SIDES AND THE RADII OF A TRIANGLE

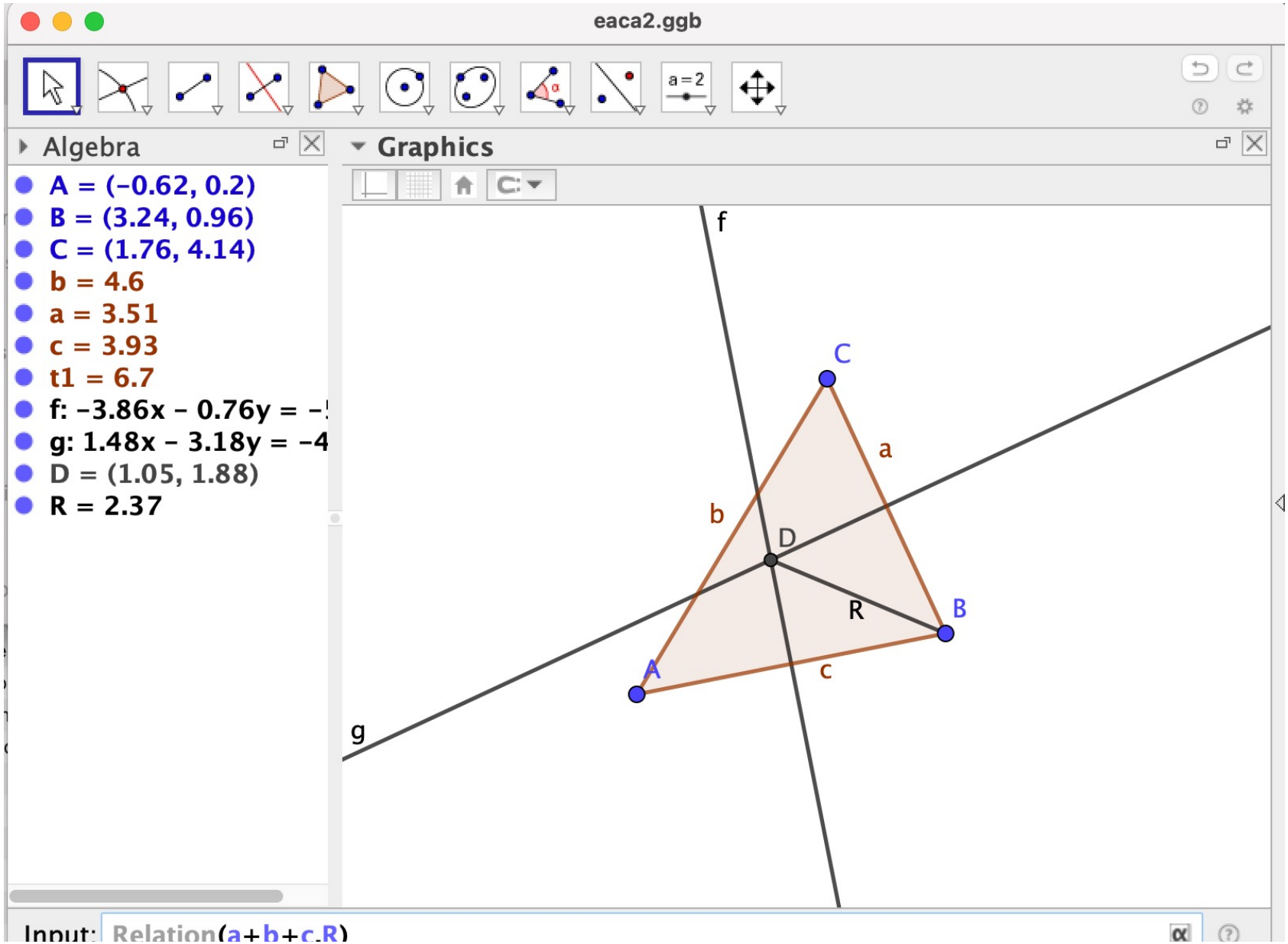
**5.3** $a+b+c \leqslant 3R\sqrt{3}$.

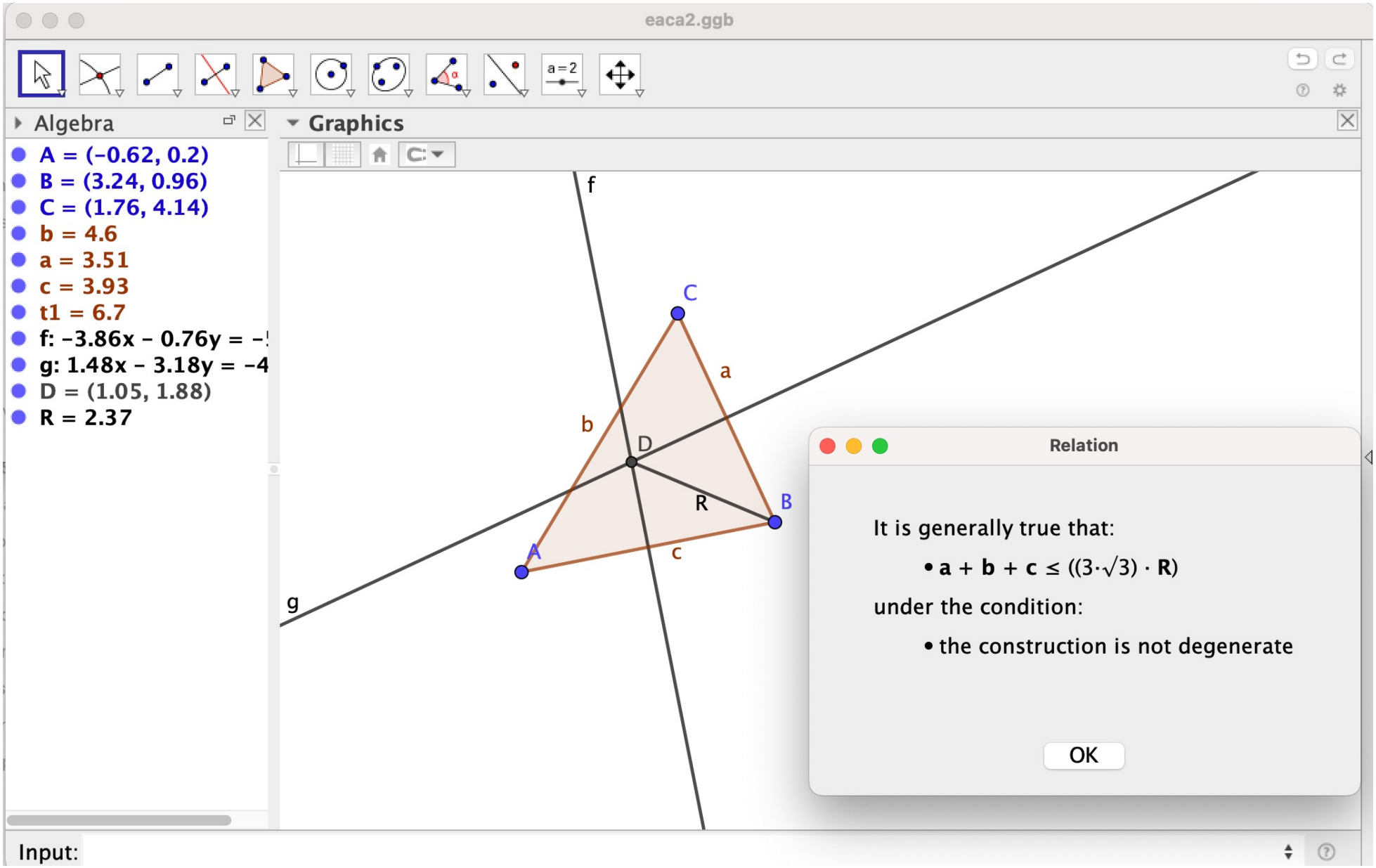
Equality holds if and only if $a = b = c$.

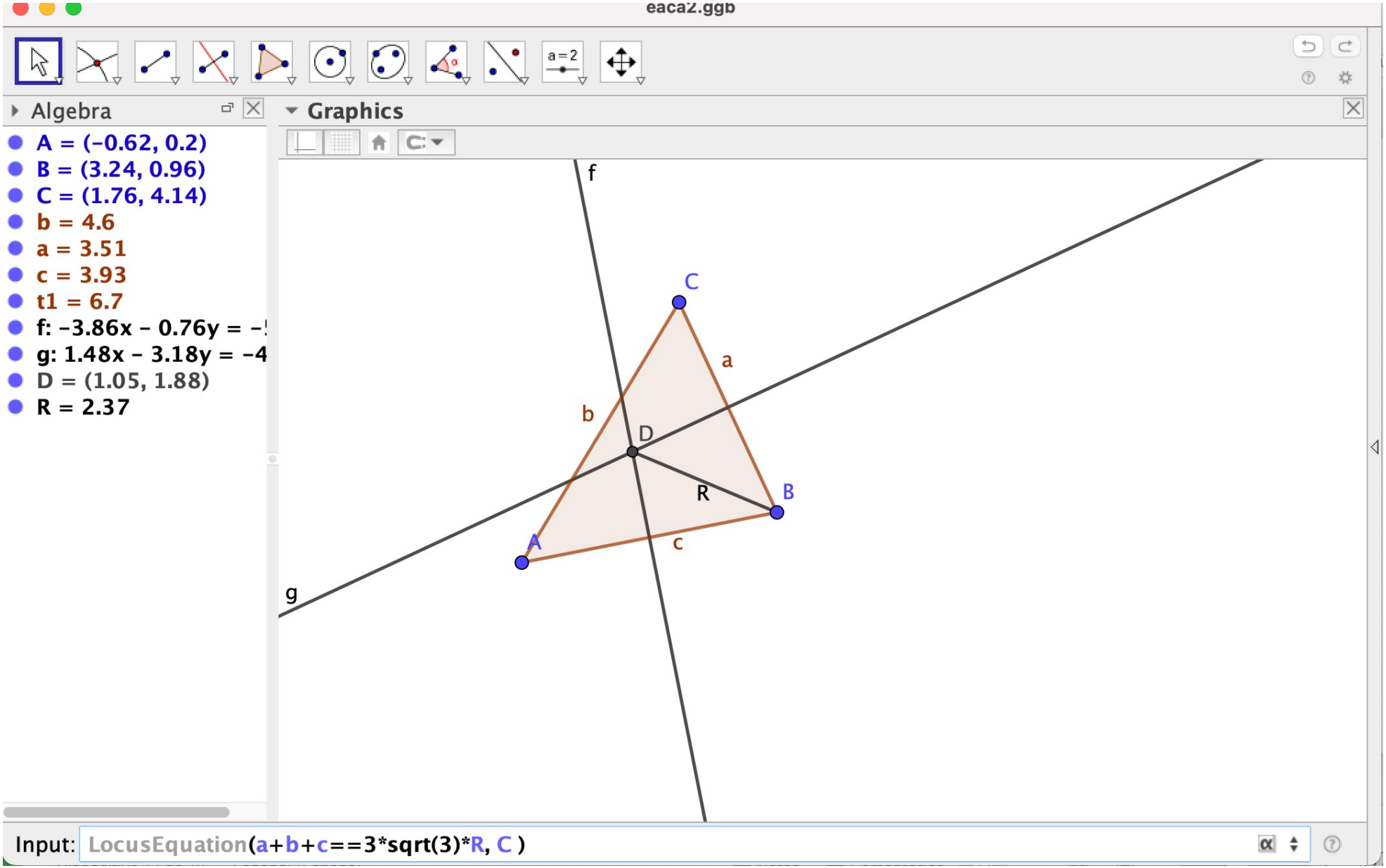
S. Nakajima, Tôhoku Math. J. 25 (1925), 115–121

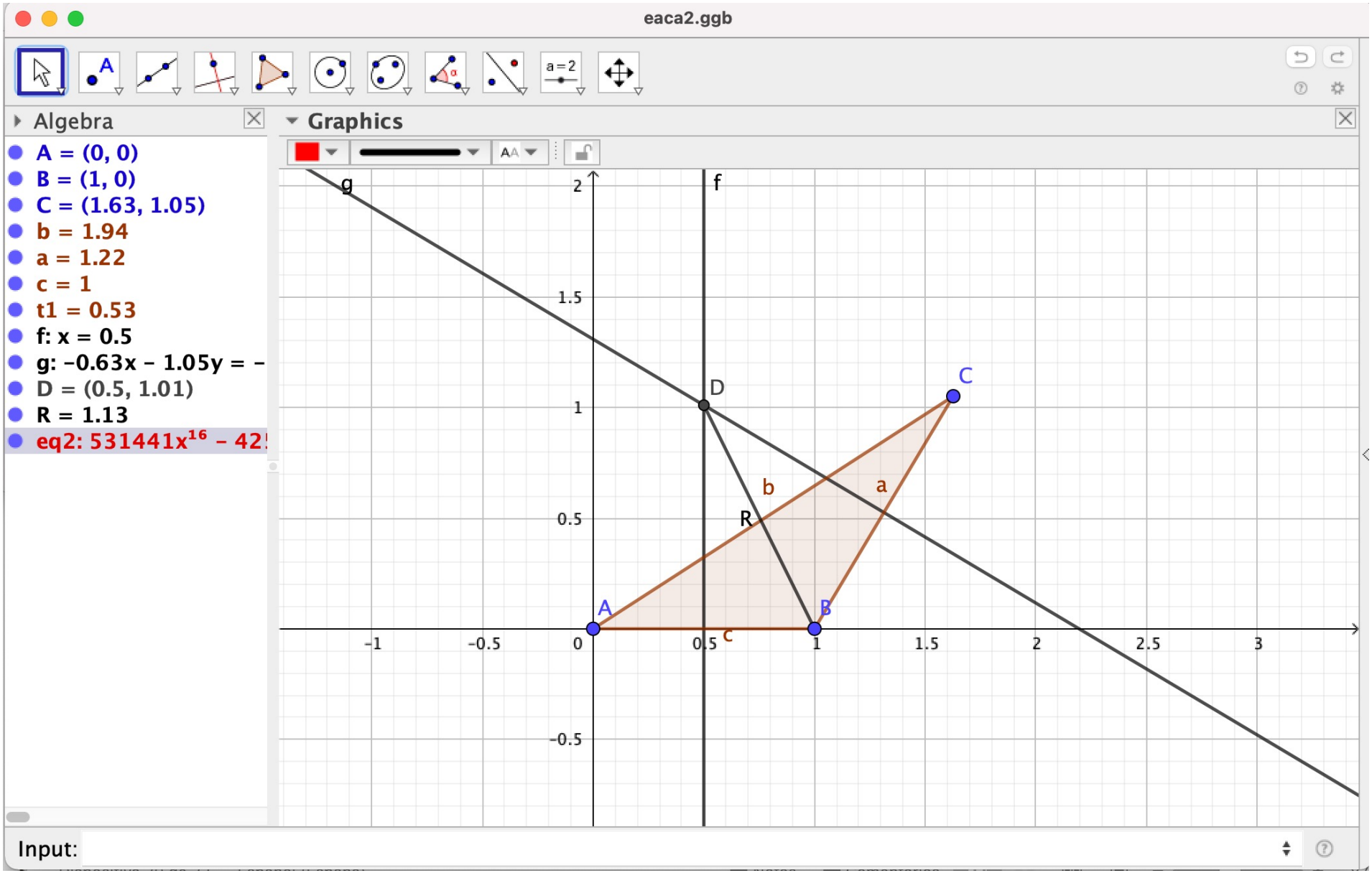
A. Padoa, Period. Mat. (4) 5 (1925), 80–85.

https://www.isinj.com/mt-usamo/Geometric%20Inequalities%20-%20Bottema,%20et.%20al.%20(1968).pdf


eaca2.ggb
Algebra
A = (-0.62, 0.2)
B = (3.24, 0.96)
C = (1.76, 4.14)
b = 4.6
a = 3.51
c = 3.93
t1 = 6.7
f: -3.86x - 0.76y = -
g: 1.48x - 3.18y = -4
D = (1.05, 1.88)
R = 2.37
Graphics
f
C
a
b
D
R
B
A
c
g
Input:
Relation(a+b+c,R)


eaca2.ggb
Algebra
A = (-0.62, 0.2)
B = (3.24, 0.96)
C = (1.76, 4.14)
b = 4.6
a = 3.51
c = 3.93
t1 = 6.7
f: -3.86x - 0.76y = -
g: 1.48x - 3.18y = -4
D = (1.05, 1.88)
R = 2.37
Graphics
f
g
C
a
b
D
R
B
A
c
Relation
It is generally true that:
• a + b + c ≤ ((3·√3) · R)
under the condition:
• the construction is not degenerate
OK
Input:


eaca2.ggb
Algebra
A = (-0.62, 0.2)
B = (3.24, 0.96)
C = (1.76, 4.14)
b = 4.6
a = 3.51
c = 3.93
t1 = 6.7
f: -3.86x - 0.76y = -
g: 1.48x - 3.18y = -4
D = (1.05, 1.88)
R = 2.37
Graphics
f
g
C
a
b
D
R
B
A
c
Input: LocusEquation(a+b+c==3*sqrt(3)*R, C )


eaca2.ggb
Algebra
A = (0, 0)
B = (1, 0)
C = (1.63, 1.05)
b = 1.94
a = 1.22
c = 1
t1 = 0.53
f: x = 0.5
g: -0.63x - 1.05y = -
D = (0.5, 1.01)
R = 1.13
eq2: 531441x^16 - 42
Graphics
g
f
D
C
b
a
R
A
B
c
Input:

- eq1: 67895369965279750000000000000000000000000000000000000000000000x$^{16}$ + 31194438930844390000000000000000000000000000000000000000000000x$^{15}$ y - 14411797290521530000000000000000000000000000000000000000000000x$^{15}$ + 47059951913352050000000000000000000000000000000000000000000000x$^{14}$ y$^{2}$ - 11588661671859760000000000000000000000000000000000000000000000x$^{14}$ y + 12493206906688777000000000000000000000000000000000000000000000x$^{14}$ + 20086836722444138000000000000000000000000000000000000000000000x$^{13}$ y$^{3}$ - 89803061398792930000000000000000000000000000000000000000000000x$^{13}$ y$^{2}$ + 13903309193642640000000000000000000000000000000000000000000000x$^{13}$ y - 53832692569772870000000000000000000000000000000000000000000000x$^{13}$ + 14108933063437447000000000000000000000000000000000000000000000x$^{12}$ y$^{4}$ - 66940607645497250000000000000000000000000000000000000000000000x$^{12}$ y$^{3}$ + 72462203218936740000000000000000000000000000000000000000000000x$^{12}$ y$^{2}$ - 77406678807776980000000000000000000000000000000000000000000000x$^{12}$ y + 99117472073965060000000000000000000000000000000000000000000000x$^{12}$ + 55012698579891600000000000000000000000000000000000000000000000x$^{11}$ y$^{5}$ - 23774611034540523000000000000000000000000000000000000000000000x$^{11}$ y$^{4}$ + 71159112947968780000000000000000000000000000000000000000000000x$^{11}$ y$^{3}$ - 30748666260372003000000000000000000000000000000000000000000000x$^{11}$ y$^{2}$ + 20399356956530064000000000000000000000000000000000000000000000x$^{11}$ y + 55678171151014770000000000000000000000000000000000000000000000x$^{11}$ + 23849106543142050000000000000000000000000000000000000000000000x$^{10}$ y$^{6}$ - 16226818942375814000000000000000000000000000000000000000000000x$^{10}$ y$^{5}$ + 17543366158422122000000000000000000000000000000000000000000000x$^{10}$ y$^{4}$ - 36068389772830790000000000000000000000000000000000000000000000x$^{10}$ y$^{3}$ + 66510829565884990000000000000000000000000000000000000000000000x$^{10}$ y$^{2}$ - 14673089909929755000000000000000000000000000000000000000000000x$^{10}$ y - 46868803420819330000000000000000000000000000000000000000000000x$^{10}$ + 82941478320751330000000000000000000000000000000000000000000000x$^{9}$ y$^{7}$ - 34609753591338590000000000000000000000000000000000000000000000x$^{9}$ y$^{6}$ + 14995326325129030000000000000000000000000000000000000000000000x$^{9}$ y$^{5}$ - 70966128481978910000000000000000000000000000000000000000000000x$^{9}$ y$^{4}$ + 94028392234202490000000000000000000000000000000000000000000000x$^{9}$ y$^{3}$ - 39057914318683185000000000000000000000000000000000000000000000x$^{9}$ y$^{2}$ - 36854144484159060000000000000000000000000000000000000000000000x$^{9}$ y + 18946617672208996000000000000000000000000000000000000000000000x$^{9}$ + 24794609485184870000000000000000000000000000000000000000000000x$^{8}$ y$^{8}$ - 21283499095142360000000000000000000000000000000000000000000000x$^{8}$ y$^{7}$ + 22784580711197690000000000000000000000000000000000000000000000x$^{8}$ y$^{6}$ - 67853662750040960000000000000000000000000000000000000000000000x$^{8}$ y$^{5}$ + 16194292171421384000000000000000000000000000000000000000000000x$^{8}$ y$^{4}$ - 10701552439934930000000000000000000000000000000000000000000000x$^{8}$ y$^{3}$ - 11279559404511952000000000000000000000000000000000000000000000x$^{8}$ y$^{2}$ + 50916715079808550000000000000000000000000000000000000000000000x$^{8}$ y + 11449851460035295000000000000000000000000000000000000000000000x$^{8}$ + 74195125675016650000000000000000000000000000000000000000000000x$^{7}$ y$^{9}$ - 29857736300709610000000000000000000000000000000000000000000000x$^{7}$ y$^{8}$ + 16617378186072595000000000000000000000000000000000000000000000x$^{7}$ y$^{7}$ - 84582290019839940000000000000000000000000000000000000000000000x$^{7}$ y$^{6}$ + 16822638923853406000000000000000000000000000000000000000000000x$^{7}$ y$^{5}$ - 18145854247984460000000000000000000000000000000000000000000000x$^{7}$ y$^{4}$ - 19758952205967827000000000000000000000000000000000000000000000x$^{7}$ y$^{3}$ + 17041041210884982000000000000000000000000000000000000000000000x$^{7}$ y$^{2}$ - 36501585137152076000000000000000000000000000000000000000000000x$^{7}$ y - 38764093043674717000000000000000000000000000000000000000000000x$^{7}$ + 16177609261892428000000000000000000000000000000000000000000000x$^{6}$ y$^{10}$ - 16186760396296636000000000000000000000000000000000000000000000x$^{6}$ y$^{9}$ + 16903021429835228000000000000000000000000000000000000000000000x$^{6}$ y$^{8}$ - 64869544215773790000000000000000000000000000000000000000000000x$^{6}$ y$^{7}$ + 18819560145670416000000000000000000000000000000000000000000000x$^{6}$ y$^{6}$ - 22442047692586068000000000000000000000000000000000000000000000x$^{6}$ y$^{5}$ + 10359587680251027000000000000000000000000000000000000000000000x$^{6}$ y$^{4}$ + 13587947826848620000000000000000000000000000000000000000000000x$^{6}$ y$^{3}$ + 10796335875167770000000000000000000000000000000000000000000000x$^{6}$ y$^{2}$ - 26937684938468340000000000000000000000000000000000000000000000x$^{6}$ y - 20787730807257910000000000000000000000000000000000000000000000x$^{6}$ + 39269263817569183000000000000000000000000000000000000000000000x$^{5}$ y$^{11}$ - 15220979911408365000000000000000000000000000000000000000000000x$^{5}$ y$^{10}$ + 10184325764948416000000000000000000000000000000000000000000000x$^{5}$ y$^{9}$ - 54649335961373170000000000000000000000000000000000000000000000x$^{5}$ y$^{8}$ + 14486653141621111000000000000000000000000000000000000000000000x$^{5}$ y$^{7}$ - 24819900723820910000000000000000000000000000000000000000000000x$^{5}$ y$^{6}$ + 12439408548874280000000000000000000000000000000000000000000000x$^{5}$ y$^{5}$ + 18605397716479308000000000000000000000000000000000000000000000x$^{5}$ y$^{4}$ - 22844041748316096000000000000000000000000000000000000000000000x$^{5}$ y$^{3}$ - 27255276114625857000000000000000000000000000000000000000000000x$^{5}$ y$^{2}$ - 44626733121238865000000000000000000000000000000000000000000000x$^{5}$ y - 56326844979361380000000000000000000000000000000000000000000000x$^{5}$ + 64374357821878260000000000000000000000000000000000000000000000x$^{4}$ y$^{12}$ - 70525920844535080000000000000000000000000000000000000000000000x$^{4}$ y$^{11}$ + 69776712821820840000000000000000000000000000000000000000000000x$^{4}$ y$^{10}$ - 32151014644986047000000000000000000000000000000000000000000000x$^{4}$ y$^{9}$ + 10827695490883443000000000000000000000000000000000000000000000x$^{4}$ y$^{8}$ - 19425328856451680000000000000000000000000000000000000000000000x$^{4}$ y$^{7}$ + 17942637832127898000000000000000000000000000000000000000000000x$^{4}$ y$^{6}$ + 20287790563237602000000000000000000000000000000000000000000000x$^{4}$ y$^{5}$ - 10431497739006597000000000000000000000000000000000000000000000x$^{4}$ y$^{4}$ - 4708304461594359000000000000000000000000000000000000000000000x$^{4}$ y$^{3}$ - 1424103311992425000000000000000000000000000000000000000000000x$^{4}$ y$^{2}$ - 469908181726052740000000000000000000000000000000000000000000000x$^{4}$ y + 1904234557599107300000000000000000000000000000000000000000000x$^{4}$ + 1134048407670945900000000000000000000000000000000000000000000x$^{3}$ y$^{13}$ - 4228288849250314000000000000000000000000000000000000000000000x$^{3}$ y$^{12}$ + 3260251314396366500000000000000000000000000000000000000000000x$^{3}$ y$^{11}$ - 1797260003114585300000000000000000000000000000000000000000000x$^{3}$ y$^{10}$ + 5918551387599348000000000000000000000000000000000000000000000x$^{3}$ y$^{9}$ - 1371607398251384000000000000000000000000000000000000000000000x$^{3}$ y$^{8}$ + 1576604360927031200000000000000000000000000000000000000000000x$^{3}$ y$^{7}$ - 4348853648285894700000000000000000000000000000000000000000000x$^{3}$ y$^{6}$ - 2549110532365970000000000000000000000000000000000000000000000x$^{3}$ y$^{5}$ - 1313402560663748000000000000000000000000000000000000000000000x$^{3}$ y$^{4}$ - 581984540557164500000000000000000000000000000000000000000000x$^{3}$ y$^{3}$ + 2862440288262450000000000000000000000000000000000000000000000x$^{3}$ y$^{2}$ - 1359629179044523200000000000000000000000000000000000000000000x$^{3}$ y + 2221047277460767000000000000000000000000000000000000000000000x$^{3}$ + 1418210642228223700000000000000000000000000000000000000000000x$^{2}$ y$^{14}$ - 1597268065703999300000000000000000000000000000000000000000000x$^{2}$ y$^{13}$ + 1399847424218532600000000000000000000000000000000000000000000x$^{2}$ y$^{12}$ - 7210106063326817000000000000000000000000000000000000000000000x$^{2}$ y$^{11}$ + 2680893910907260600000000000000000000000000000000000000000000x$^{2}$ y$^{10}$ - 6592355956769027000000000000000000000000000000000000000000000x$^{2}$ y$^{9}$ + 1138231650709056000000000000000000000000000000000000000000000x$^{2}$ y$^{8}$ - 7663226335468069000000000000000000000000000000000000000000000x$^{2}$ y$^{7}$ - 4005854696577414000000000000000000000000000000000000000000000x$^{2}$ y$^{6}$ - 2580005741046406000000000000000000000000000000000000000000000x$^{2}$ y$^{5}$ + 1758968258910710700000000000000000000000000000000000000000000x$^{2}$ y$^{4}$ - 1759256720100650400000000000000000000000000000000000000000000x$^{2}$ y$^{3}$ + 3529539038024575500000000000000000000000000000000000000000000x$^{2}$ y$^{2}$ - 1196393069597538600000000000000000000000000000000000000000000x$^{2}$ y + 1099339070932119700000000000000000000000000000000000000000000x$^{2}$ + 1370173363937503400000000000000000000000000000000000000000000x y$^{15}$ - 4907762709263573000000000000000000000000000000000000000000000x y$^{14}$ + 4235577858241282000000000000000000000000000000000000000000000x y$^{13}$ - 2304822611028430000000000000000000000000000000000000000000000x y$^{12}$ + 8906853482927100000000000000000000000000000000000000000000000x y$^{11}$ - 2564635822731223000000000000000000000000000000000000000000000x y$^{10}$ + 5288892921890439000000000000000000000000000000000000000000000x y$^{9}$ - 8261269084176926000000000000000000000000000000000000000000000x y$^{8}$ + 4925395631043681000000000000000000000000000000000000000000000x y$^{7}$ + 3759369641661203400000000000000000000000000000000000000000000x y$^{6}$ - 9145378654556376000000000000000000000000000000000000000000000x y$^{5}$ + 2195925767865276000000000000000000000000000000000000000000000x y$^{4}$ - 1414784875245959300000000000000000000000000000000000000000000x y$^{3}$ + 1519922483395765300000000000000000000000000000000000000000000x y$^{2}$ - 4514638568484311000000000000000000000000000000000000000000000x y + 2703832605179044000000000000000000000000000000000000000000000x + 1309896081349673000000000000000000000000000000000000000000000y$^{16}$ - 1395076274168309500000000000000000000000000000000000000000000y$^{15}$ + 8708338787131326000000000000000000000000000000000000000000000y$^{14}$ - 4026947761266860000000000000000000000000000000000000000000000y$^{13}$ + 1374696007464531200000000000000000000000000000000000000000000y$^{12}$ - 3989791381958786000000000000000000000000000000000000000000000y$^{11}$ + 999863823524972500000000000000000000000000000000000000000000y$^{10}$ - 2099548143131912700000000000000000000000000000000000000000000y$^{9}$ + 435689609446171600000000000000000000000000000000000000000000y$^{8}$ - 56702437251036960000000000000000000000000000000000000000000y$^{7}$ + 8345514942941506000000000000000000000000000000000000000000y$^{6}$ - 675555635757630000000000000000000000000000000000000000000y$^{5}$ + 68363042530844100000000000000000000000000000000000000000y$^{4}$ - 3515195607157063000000000000000000000000000000000000000y$^{3}$ + 240680605345652950000000000000000000000000000000000000y$^{2}$ - 64171127924352310000000000000000000000000000000000000y = -27180732239823723000000000000000000000000000000000000

- eq2: $88209y^8 + 204228y^{10} + 236134y^{12} + 204228y^{14} + 88209y^{16} - 962280y^8 x - 2237112y^{10} x - 2237112y^{12} x - 962280y^{14} x + 609444y^6 x^2 + 6281388y^8 x^2 + 10694484y^{10} x^2 + 6689844y^{12} x^2 + 962280y^{14} x^2 - 6018624y^6 x^3 - 26125848y^8 x^3 - 28362960y^{10} x^3 - 8905464y^{12} x^3 + 1485702y^4 x^4 + 30035016y^6 x^4 + 69737274y^8 x^4 + 42802020y^{10} x^4 + 4452732y^{12} x^4 - 13401936y^4 x^5 - 92158128y^6 x^5 - 118283976y^8 x^5 - 34344648y^{10} x^5 + 1495908y^2 x^6 + 56867832y^4 x^6 + 183225240y^6 x^6 + 123219252y^8 x^6 + 11448216y^{10} x^6 - 12597120y^2 x^7 - 144481968y^4 x^7 - 236640096y^6 x^7 - 71821080y^8 x^7 + 531441x^8 + 47790324y^2 x^8 + 235946682y^4 x^8 + 191561004y^6 x^8 + 17955270y^8 x^8 - 4251528x^9 - 105973272y^2 x^9 - 250455240y^4 x^9 - 88267320y^6 x^9 + 14880348x^{10} + 149197140y^2 x^{10} + 167328828y^4 x^{10} + 17653464y^6 x^{10} - 29760696x^{11} - 135733968y^2 x^{11} - 63947880y^4 x^{11} + 37200870x^{12} + 77551020y^2 x^{12} + 10657980y^4 x^{12} - 29760696x^{13} - 25351704y^2 x^{13} + 14880348x^{14} + 3621672y^2 x^{14} - 4251528x^{15} + 531441x^{16} = 0$

$$2800673127487897369859632256043733525105640331422639718325373923494151956525783252206788220804103453316154488155627351925217044557987840*x^{72}*(-1+x)^{72}*(221709312*x^{10}-1108546560*x^{9}+5547202304*x^{8}-15537529856*x^{7}+23360155968*x^{6}-20355008960*x^{5}+10731606224*x^{4}-3448222560*x^{3}+658900764*x^{2}-70266636*x+3485889)^{2}*(4*x^{2}+23)^{4}*(4*x^{2}-8*x+27)^{4}*(2*x-1)^{12}$$

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- Kovács Z.; R.T.: *GeoGebra reasoning tools for humans and for automatons*. Electronic Proceedings of the 25th Asian Technology Conference in Mathematics, December 14-16, 2020. ISSN 1940-4204 (online version). http://atcm.mathandtech.org/EP2020/invited/21786.pdf

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GRACIAS

http://www.recio.tk