Contributed talks

Sixth EACA International School
on Computer Algebra and its Applications

July 18–21, 2023

CITMAga, Santiago de Compostela, Spain

https://www.usc.es/regaca/eacaschool23/index.html

CENTRO DE INVESTIGACIÓN
Y TECNOLOGÍA MATEMÁTICA
DE GALICIA

Contributed talks

Sixth EACA International School

Contributed talks 2

Nilpotent compatible Lie algebras and their classification

Bernardo Leite da Cunha

Bernardo Leite da Cunha (bernardo.mariz@rai.usc.es)
Universidade de Santiago de Compostela
Universidade do Porto

Abstract.

In this talk we will briefly recall the notion of Lie algebra, before moving on to the concept of a compatible Lie algebra, which is a generalisation consisting of an algebra with two Lie products satisfying a certain compatibility condition. We introduce the notion of nilpotency for these structures, and we discuss a method to classify the finite-dimensional ones, for low dimensions. We will comment on the feasibility of an attempt at implementing this program using computational tools such as GAP.

References

[1] I. Z. Golubchik, V. V. Sokolov. Compatible Lie brackets and integrable equations of the principal chiral field model type. *Funct. Anal. Appl.* **36**(3) (2002), 172–181.

[2] J. Liu, Y. Sheng, Ch. Bai. Maurer-Cartan characterization and cohomologies of compatible Lie algebras. *Sci. China Math.* **66**(6) (2023), 1177–1198.

[3] T. Skjelbred, T. Sund. Sur la classification des algebres de Lie nilpotentes. *C. R. Acad. Sci. Paris Ser.* A-B, 286 (1978), A241–A242.

On a simplicial construction for the Eilenberg–Moore generalized spectral sequence

D. Miguel, A. Guidolin, A. Romero, J. Rubio

Daniel Miguel (damigutr@unirioja.es)
University of La Rioja

Abstract

Generalized spectral sequences, also named spectral systems, were introduced by Matschke in his work [1]. The author gave there several examples concerning classic spectral sequences and, in particular, presented a generalization of the Eilenberg–Moore geometric cohomology spectral sequence [2] for cubes of fibrations. From a computational perspective, the Eilenberg–Moore homology spectral sequence is implemented in the computer algebra system Kenzo [3], and it is possible to make computations on spaces of infinite type thanks to the effective homology technique [4]. This technique is based on reductions of chain complexes $C_* \Rrightarrow D_*$, which are essentially chain homotopy equivalences.

The current implementation of the Eilenberg–More spectral system in Kenzo follows the original approach [5], in which the Cobar construction plays a primary role. Therefore, in order to find within this framework a construction analogous to Matschke's, we have studied the possibility of a generalized Cobar construction. In this work, we will present some preliminary results, in the form of a new generalized filtered chain complex. Moreover, we will exhibit different issues regarding its effective homology and the convergence of its associated spectral system.

For the main problem, we are given n fibrations $f_i : E_i \to B$, $1 \leq i \leq n$. The goal is to compute the homology of the space E, defined as the pullback of all of them. To do so, we consider intermediate spaces E_I for each subset $I \subseteq \{1, ..., n\}$. E_I is defined as the pullback of the maps $\{f_i | i \in I\}$. Assuming $\pi_1(B) = 0$, it is possible to define a spectral system indexed by 4-tuples of downsets of $\mathbb{Z}^n$, which are defined as downward closed subsets of that space. This spectral system converges to the cohomology of E, $H^*(E)$, and its second page terms are given by

$$S^{pz^*}_{b^*q}((p_1, ..., p_n); n) = \mathrm{Tor}^{p_n}_{HB}(...(Tor^{p_2}_{HB}(Tor^{p_1}_{HB}(HB, HE_1), HE_2), ..., HE_n)).$$

For example, for the case $n = 3$, we have the following diagram.

$$\begin{tikzcd}
 & E \arrow[r] \arrow[dl] & E_{23} \arrow[dl] \arrow[d] \\
E_{12} \arrow[r] \arrow[d] & E_2 \arrow[d] & E_3 \arrow[dl] \\
E_1 \arrow[r] & B &
\end{tikzcd}$$

For our computational purposes, we will assume that all spaces are simplicial sets, and that all fibrations are principal twisted cartesian products. This means that each E_i can be seen as $F_i \times_{\tau_i} B$ for some twisting operator $\tau_i : B_* \to (F_i)_{*-1}$. In our work, we consider the following ideas:

- The geometric spectral system is defined by means of smash products on the category $\mathbf{(Top/B)}_*$ of pointed spaces over B. The algebraic analogue of the smash product is cotensor product. We study the different factors that make hard to give a canonical definition for multiple factors.
- We can define a generalized filtered chain complex

$$\mathrm{Cobar}^{C_*(B)}(...\mathrm{Cobar}^{C_*(B)}(\mathrm{Cobar}^{C_*(B)}(C_*(E_1), C_*(E_2)), C_*(E_3))..., C_*(E_n)).$$

  It generalizes Eilenberg and Moore's cobar chain complex, and it has effective homology. However, it is defined using trivial coproducts, so we cannot deduce anything about its homology.
- We explore several alternatives in order to modify its differential.
  - There is a coproduct for Adam's cobar construction defined by Baues ([6]). We generalize it to our framework and are able to use it to modify the previous chain complex.
  - In the case of twisted cartesian products, it is possible to use the simplicial homotopy fiber of the total spaces E_i to obtain another simplicial set equivalent to the pullback E. This construction involves the simplicial loop group GB, so it can be related to the Cobar.
  - Since we have reductions $\mathrm{Cobar}^{C_*(B)}(E_i, E_j) \Rrightarrow E_{ij}$, it is possible to define coproducts up to homotopy. We explore its definition and study its behaviour.

References

[1] B. Matschke. Successive spectral sequences. *Transactions of the American Mathematical Society*, 375:6205–6254, 2022.

[2] L. Smith. On the construction of the Eilenberg–Moore spectral sequence. *Bulletin of the American Mathematical Society*, 75:873–878, 1969.

[3] X. Dousson, J. Rubio, F. Sergeraert and Y. Siret. The Kenzo Program. `http://www-fourier.ujf-grenoble.fr/~sergerar/Kenzo`, 1999.

[4] J. Rubio and F. Sergeraert. Constructive algebraic topology. *Bulletin des Sciences Mathématiques*, 126(5):389–412, 2002.

[5] S. Eilenberg and J. C. Moore. Homology and fibrations I: Coalgebras, cotensor product and its derived functors *Commentarii Mathematici Helvetici*, 40(1):199–236, 1965.

[6] H.J. Baues. The double bar and cobar constructions. *Compositio Mathematica*, 43(3):331–341, 1981.

Rank error correction up to the Hartmann-Tzeng bound*

J. M. Muñoz

J. M. Muñoz (munoz@ugr.es)
Universidad de Granada

F. J. Lobillo (jlobillo@ugr.es)
Universidad de Granada

Abstract.

The well-known BCH bound establishes that, if $\delta - 1$ consecutive powers of a nonzero element β are roots of the generator polynomial g of a cyclic block code $\mathcal{C}$, then the minimum Hamming distance of $\mathcal{C}$ is at least δ. The set of exponents i such that $g(\beta^i) = 0$ is the β-defining set of the code $\mathcal{C}$, and the hypothesis of the BCH bound can be stated as the existence of a subset of $\delta - 1$ consecutive integers $\{b, b+1, b+2, \ldots, b+\delta-2\} = b + \{0, 1, \ldots, \delta - 2\}$ in the β-defining set. If β is in the coefficient field of g (as opposed to an extension thereof), g generates a Reed-Solomon code, which reaches the Singleton bound.

The BCH bound was generalized into the Hartmann-Tzeng bound in [4], allowing a subset of the β-defining set of the form $b + t_1\{0, 1, \ldots, \delta + 2\} + t_2\{0, 1, \ldots, r\}$, where $(n, t_1) = (n, t_2) = 1$ for n the length of the code and (a, b) being the greatest common divisor of a and b, which is shown to guarantee a minimum distance of at least $\delta + r$ for the cyclic code $\mathcal{C}$. There are known algorithms for nearest-neighbor decoding up to this bound (that is, for finding the closest codeword in the Hamming metric to one given word if the distance to the code is at most $\lfloor (\delta + r - 1)/2 \rfloor$), see e.g. [1].

*Supported by grants PRE2020-09325 from MCIN/AEI / 10.13039/501100011033 and FSE "El FSE invierte en tu futuro" and PID2019-110525GB-I00 from Agencia Estatal de Investigación (AEI / 10.13039/501100011033)

Both the BCH bound and the Hartmann-Tzeng bound have been shown to work analogously for skew cyclic (block) codes. In the context of a field F, a field automorphism $\sigma\colon F \to F$ of order n, a skew polynomial ring $F[x;\sigma]$ and some $\beta \in F$, i is in the β-defining set of a generator (skew) polynomial $g \in F[x;\sigma]$ when $x-\sigma^i(\beta)$ right divides g. As shown in [2], if $b+t_1\{0,1,\ldots,\delta+2\} + t_2\{0,1,\ldots,r\}$, where $(n,t_1) = 1$ and $(n,t_2) < \delta$, is a subset of the β-defining set of g, then the Hamming distance of the skew cyclic code $\mathcal{C}$ generated by g is at least $\delta + r$. Nearest-neighbor error correction algorithms are known for these codes in the skew Reed-Solomon case, see e.g. [3], which can readily be extended to the BCH case, that is, when $r = 0$. Further work has shown that, for K the fixed field of σ, this bound also applies to the F/K-rank metric, which is defined so that the rank weight of $v \in F^n$ is the dimension of the K-vector space spanned by the entries of v.

Our current work results in a syndrome-based error-correcting decoding algorithm for codes in a family containing the skew cyclic codes, as well as the Gabidulin codes as defined in [5], up to a Hartmann-Tzeng bound for the rank metric (and therefore for the Hamming metric) derived from a defining set related to the structure of the parity-check matrix of the code, which is reduced to the stated above for skew cyclic codes. The decoding algorithm, which generalizes the one in [6] for Gabidulin codes and can be seen as a generalization of the one in [3] for skew Reed-Solomon codes, decomposes the decoding problem into efficiently solvable linear algebra ones, notably including the skew-feedback shift-register synthesis problem.

References

[1] G.-L. Feng, K. K. Tzeng. A generalization of the Berlekamp-Massey algorithm for multisequence shift-register synthesis with applications to decoding cyclic codes. *IEEE Transactions on Information Theory* **37**(5) (1991), 1274–1287.

[2] J. Gómez-Torrecillas, F. J. Lobillo, G. Navarro. A Sugiyama-Like Decoding Algorithm for Convolutional Codes. *IEEE Transactions on Information Theory* **63**(10) (2017), 6216–6226.

[3] J. Gómez-Torrecillas, F. J. Lobillo, G. Navarro, A. Neri. Hartmann-Tzeng bound and skew cyclic codes of designed Hamming distance. *Finite Fields and Their Applications* **50** (2018), 84–112.

[4] C. R. P. Hartmann, K. K. Tzeng. Generalizations of the BCH bound. *Information and Control* **20**(5) (1972), 489–498.

[5] A. Kshevetskiy, E. Gabidulin. The new construction of rank codes. *Proceedings. International Symposium on Information Theory* (2005), 2105–2108.

[6] V. Sidorenko, L. Jiang, M. Bossert. Skew-Feedback Shift-Register Synthesis and Decoding Interleaved Gabidulin Codes. *IEEE Transactions on Information Theory* **57**(2) (2011), 621–632.

PBW Pairs in Operads and Compatible Algebras

Álex Pazos Moure

Álex Pazos Moure (alex.pazos@rai.usc.es)
Universidade de Santiago de Compostela

Abstract.
Poincaré-Birkhoff-Witt's Theorem contains a fundamental relation between a Lie algebra and its associative universal enveloping algebra. There exist numerous positive and negative results studying the identical situation in other pairs of varieties of algebras. In the present talk, we will exhibit the theoretical framework with which we approach this kind of problems in modern mathematics, generalizing it to the realm of algebraic operads and applying to those an analogue of Gröbner bases. To conclude, we will present results which show the way to go in order to identify the PBW property in any pair of varieties of algebras for which it makes sense.

References

[1] M. R. Bremner, V. Dotsenko. Algebraic Operads: An Algorithmic Companion, Chapman and Hall, New York, 2016.

[2] V. Dotsenko. Freeness Theorems for Operads via Gröbner Bases. *Sémin. Congr.* 26, Societé Mathématique de France, Paris (2013), 61–76.

[3] V. Dotsenko, P. Tamaroff. Endofunctors and Poincaré–Birkhoff–Witt Theorems. *Int. Math. Res. Not. IMRN* **2021** (16) (2021), 12670–12690.

[4] A. A. Mikhalev, I. P. Shestakov. PBW-Pairs of Varieties of Linear Algebras. *Comm. Algebra* **42** (2014), 667–687.

[5] T. Van der Linden. Non-Associative Algebras. *New perspectives in algebra, topology and categories*, Coimbra Math. Texts 1, pp. 225–258, Springer, Cham, 2021.

A study on persistent homology with integer coefficients

Javier Perera-Lago

Javier Perera-Lago (jperera@us.es)
Universidad de Sevilla

Abstract.

Homology is a tool that assigns to a topological space an abelian group for each dimension. Persistent homology is a more modern technique used to analyze the evolution of homology in a topological space that is built step by step. Both homology groups and persistent homology may be different according to the set of coefficients used in the calculations. Persistent homology with coefficients over a field such as $\mathbb{R}$ or $\mathbb{Z}_2$ has been widely studied and applied thanks to it easy computation, its stability and the existence of easy and complete invariants for its classification. Unfortunately, persistent homology over coefficients in $\mathbb{Z}$ is harder to calculate and classify, and because of that it has been barely studied. In [1], we start from the approach and concepts defined in [2] and we give some new results, we generalize the definitions and we give a partial stability result.

Let us introduce the details of [1]. The first step is to consider a topological space built on m steps, that is, an increasing sequence of m spaces. For a given dimension n, each space has its own homology groups, so we can see persistent homology as a diagram

$$0 = H_0 \xrightarrow{\rho_{0,1}} H_1 \xrightarrow{\rho_{1,2}} H_2 \xrightarrow{\rho_{2,3}} \cdots \xrightarrow{\rho_{m-1,m}} H_m \tag{1}$$

where H_i is the homology group in step i and $\rho_{i,j} = \rho_{j-1,j} \circ \cdots \circ \rho_{i,i+1}$ are group homomorphisms. From this diagram, the authors of [2] define the groups $H_{i,j} = \operatorname{Im} \rho_{i,j} = \rho_{i,j}(H_i) \subset H_j$ for $i \leq j$, then they define $H_{i,k,j} = H_{i,k} \cap (\rho_{k,j})^{-1}(H_{i-1,j}) \subset H_k$ for $i \leq k \leq j$, and finally they define the BD groups as

$$BD_{i,j} = \frac{H_{i,i,j}}{H_{i,i,j-1}} = \frac{H_{i,i+1,j}}{H_{i,i+1,j-1}} = \cdots = \frac{H_{i,j-2,j}}{H_{i,j-2,j-1}} = \frac{H_{i,j-1,j}}{H_{i-1,j-1}} \tag{2}$$

According to [2], a non trivial $BD_{i,j}$ group is meant to show that there is a topological feature (a homology class) that is born in step i and dies in step j, but it does not give a formal proof for this affirmation. All these groups can be computed by a module of Kenzo program, using the spectral sequences defined in [3].

In order to connect the $BD_{i,j}$ groups with the theory developed in [5], which always talks in terms of intervals $I = [i, j)$, we proposed in [1] the alternate notation $BD_{I,k} = \frac{H_{i,k,j}}{H_{i,k,j-1}}$ and we proved that the structure of $BD_{I,k}$ is the same for every $k \in I$. After that, we introduced the V groups defined in [5] as $V_{I,k} = V^+_{I,k}/V^-_{I,k}$, where $V^+_{I,k} = \operatorname{Im} \rho_{i,k} \cap \ker \rho_{k,j}$ and $V^-_{I,k} = (\operatorname{Im} \rho_{i,k} \cap \ker \rho_{k,j-1}) + (\operatorname{Im} \rho_{i-1,k} \cap \ker \rho_{k,j})$. To provide intuition to the spaces $V_{I,k}$, notice that when working with field coefficients, it is proved in [4] that the dimension of $V_{I,k}$ shows how many homology classes are born in step i and die at step j.

In [1], we also introduced a new definition of BD groups for infinite intervals $I = [i, \infty)$, given by $BD_I = \frac{H_{i,m}}{H_{i-1,m}}$, and we proved that this formula is equivalent to $BD_{[i,m+1),k} = \frac{H_{i,k,m+1}}{H_{i,k,m}}$ if we include a last term $H_{m+1} = 0$ and a null homomorphism $\rho_{m,m+1}$ in the equation 1.

Recall that BD groups were defined in [2] for persistent homology with integer coefficients, while V groups were defined in [5] for field coefficients. In [1], we provided a proof that, when working with field coefficients, $BD_{I,k}$ and $V_{I,k}$ are isomorphic. We also proved that, independently on the choice of coefficients, $V^+_{I,k} \subset H_{i,k,j}$ and $V^-_{I,k} \subset H_{i,k,j-1}$. Finally, a proof or a counterexample for the isomorphism between $BD_{I,k}$ and $V_{I,k}$ when working with integer coefficients was left as future work.

After that, we wanted to prove some stability results for BD and V groups, so we needed to extend their definitions for a more general framework. Observe that Equation 1 can be generalized by having a homology group H_i for each $i \in \mathbb{R}$ and linear maps $\rho_{i,j} : H_i \to H_j$ for $i \leq j$, satisfying that $\rho_{i,j} = \rho_{k,j} \circ \rho_{i,k}$ when $i \leq k \leq j$. In this general framework, the author of [5] gave an extended definition for V groups. The extended definition for BD groups was stated by us in [1]. In both cases, we proved in [1] that these last definitions are indeed a good generalization to those used in the more simple framework with only m steps. After that, we proved that, in this general framework, $BD_{I,k}$ and $V_{I,k}$ are also isomorphic when working with field coefficients. We left as future work the proof in case of working with integer coefficients.

Finally, inspired in the theory developed in [6], we gave first steps to stability considering the 1_ε functor, which induces an ε perturbation into persistent homology. We proved how this perturbation affects our BD definition and V groups: by transforming $BD_{(i,j+\varepsilon),k+\varepsilon}$ into $BD_{(i,j),t}$ and $V_{(i,j+\varepsilon),k+\varepsilon}$ into $V_{(i,j),t}$. We do not explore more stability results, but we consider this a good starting point to completely prove that persistent homology is also stable when working with integer coefficients.

In summary, although persistent homology is more difficult to study and apply when using integer coefficients, we show that it is possible to make some connections with the more developed theory for field coefficients, and we think that it is possible to go beyond.

References

[1] Perera-Lago, Javier Un estudio sobre la homología persistente con coeficientes enteros, *2021*, Advisor: Prof. Rocio Gonzalez-Diaz `https://idus.us.es/handle/11441/130282`.

[2] Romero, Ana and Heras, Jónathan and Rubio, Julio and Sergeraert, Francis Defining and computing persistent Z-homology in the general case, *2014* `https://arxiv.org/abs/1403.7086`

[3] Romero, Ana and Rubio, Julio and Sergeraert, Francis Computing spectral sequences, *2006* `https://www.sciencedirect.com/science/article/pii/S0747717106000460`

[4] Carlsson, Gunnar and De Silva, Vin Zigzag persistence, *2010* `https://link.springer.com/article/10.1007/s10208-010-9066-0`

[5] Crawley-Boevey, William Decomposition of pointwise finite-dimensional persistence modules, *2015* `https://www.worldscientific.com/doi/abs/10.1142/S0219498815500668?casa_token=ztFcpRY50T4AAAAA:rd96PvT-f6KrBofvPGAfHhK-VLs7FmoJ0nzD1YRTe_Jxpl0r46FPcoaN8Q2UvhVIhOKDPtgNlg`

[6] Chazal, Frédéric and De Silva, Vin and Glisse, Marc and Oudot, Steve The structure and stability of persistence modules, *2016* `https://link.springer.com/content/pdf/10.1007/978-3-319-42545-0.pdf`

On the subalgebra lattice of evolution algebras

Andrés Pérez Rodríguez

Andrés Pérez Rodríguez (andresperez.rodriguez@usc.es)
Universidade de Santiago de Compostela

Abstract.

The relationship between a group and the structure of its lattice of subgroups is highly developed and has aroused enormous interest among many leading algebraists ([4]). In addition, since lattices appear frequently in mathematics, in the literature we can find how this study has been transferred to the right ideals of a ring ([1]) or to the submodules of a module ([2]), among others. Above all, the study of the subalgebra lattice in some non-associative structures stands out, such as in Lie algebras ([3]) or in Leibniz algebras ([5]). However, this relationship is not well known in genetic algebras and in evolution algebras has not been studied yet.

An evolution algebra is an algebra provided with a basis $B = \{e_i : i \in I\}$, called natural basis, such that $e_ie_j = 0$ when $i \neq j$. Indeed, evolution algebras are a new type of commutative but nonassociative algebras introduced by J. P. Tian in 2008 in [6] that arise with the purpose of modeling non-Mendelian genetics, which is the basic language of molecular biology. In addition, these non-associative algebras with dynamic nature also have numerous connections with other fields of mathematics such as graph theory, stochastic processes, group theory or dynamic systems.

The main objective of this talk is to develop the relationship between an evolution algebra and its subalgebra lattice, emphasizing two of its main properties: distributivity and modularity. At the beginning, some problems encountered throughout our investigation will be presented, such as the fact that evolution algebras are not closed under subalgebras or the difficulty to prove the existence of subalgebras in general. Subsequently, the distributivity in the nilpotent evolution algebras will be characterized and it will end by commenting on some results for modularity in the solvable case.

References

[1] H. Brungs. Rings with a distributive lattice of right ideals. *Journal of Algebra* **40** (1976), 392–400.

[2] V. Camillo. Distributive modules. *Journal of Algebra* **36** (1975), 16–25.

[3] B. Kolman. Semi-modular Lie algebras, *J. Sci. Hiroshima Univ. Ser. A-I Math.* **29** (1965), 149–163.

[4] R. Schmidt. Subgroup Lattices of Groups. De Gruyter Exp. Math., 14 Walter de Gruyter & Co., Berlin, 1994.

[5] S. Siciliano, D. A. Towers. On the subalgebra lattice of a Leibniz algebra. *Comm. Algebra* **50** (2021), 255–267.

[6] J. P. Tian. Evolution Algebras and their Applications. Lecture Notes in Math., 1921, Springer, Berlin, 2008.

A categorical isomorphism for Hopf braces

Brais Ramos Pérez

Brais Ramos Pérez (braisramos.perez@usc.es)
Universidade de Santiago de Compostela

Abstract.
Hopf braces are recent algebraic objects introduced by I. Angiono, C. Galindo and L. Vendramin in [1] throughout 2017. This particular kind of objects consist on a pair of Hopf algebras over the same object and with the same underlying coalgebra structure that satisfy a complex relation between the products. What makes these objects particularly interesting is not only their algebraic properties, but also that they induce solutions of the Quantum Yang-Baxter equation.

The aim of this talk will be introduce the concept of Brace triple. These obejcts give rise to a new category that is isomorphic to the category of Hopf braces under cocommutativity assumptions.

References

[1] I. Angiono, C. Galindo, L. Vendramin. Hopf braces and Yang-Baxter operators. *Proceedings of the American Mathematical Society* **145**(5) (2017), 1981–1995.

[2] R. González Rodríguez. The fundamental theorem of Hopf modules for Hopf braces. *Linear and Multilinear Algebra* **70**(20) (2022), 5146–5156.

[3] J. A. Guccione, J. J. Guccione, L. Vendramin. Yang-Baxter operators in symmetric categories. *Comm. Algebra* **46**(7) (2018), 2811–2845.

[4] Y. Li, Y. Sheng, R. Tang. Post-Hopf algebras, relative Rota-Baxter operators and solutions of the Yang-Baxter equation. (2022). arXiv:2203.12174.

Skew-derivations on Oscillator real Lie algebras

Javier Rández

Javier Rández (javier.randez@unirioja.es)
Universidad de La Rioja

Pilar Benito (pilar.benito@unirioja.es)
Universidad de La Rioja

Abstract.

In 1985, Hilgert and Hofmann [1] introduced oscillator algebras as split extensions of Heisenberg algebras (see [2] for a formal definition). These algebras are solvable, non-nilpotent, and quadratic, and can be constructed as a double extension of Hilbert spaces. Furthermore, they can be doubly extended into mixed quadratic algebras. To achieve the last assertion, it is necessary to understand the algebra of derivations of the oscillator variety. In particular, the subalgebra of skew-derivations. The general structure of these derivation algebras is described in [3, Proposition 4.3], albeit without proof. In the talk, we will provide a proof with an explicit matrix description of these derivations. This result extends the one given in [4, Theorem 2].

References

[1] J. Hilgert, K. H. Hofmann. Lorentzian cones in real Lie algebras. *Monatshefte für Mathematik*, **100**(3) (1985), 183–210.

[2] K. H. Neeb. Invariant subsemigroups of Lie groups. *American Mathematical Soc.*, 1993.

[3] A. Medina, P. Revoy. Algèbres de Lie et produit scalaire invariant. *Annales scientifiques de l'École normale supérieure* **18**(3) (1985), 553–561.

[4] P. Benito, J. Roldán-López. Metrics related to Oscillator algebras. arXiv:2212.12600, 2022.

The ring of invariant polynomials on two matrices of degree 4

Rustam Turdibaev

Rustam Turdibaev (rustam.turdibaev@usc.es)
Universidade de Santiago de Compostela

Abstract.
The invariant theory of $n \times n$ matrices studies the algebra of invariant of the general linear group GL_n acting on the direct product of square matrices of size n by simultaneous conjugation. The problem consists of two parts: determining a minimum set of generators of the algebra of such polynomials and finding the polynomial relations between them. It is well-known that this algebra is generated by the traces of monomials in generic matrices and all relations are deduced from the Cayley-Hamilton theorem. However, a minimal generating set of this algebra and exact relations among them remain largely open problems. In this talk we present a solution for the case $n = 4$.

References

[1] V. Drensky, R. La Scala. Defining relations of low degree of invariants of two 4×4 matrices. *Internat. J. Algebra Comput.* **19**(1) (2009), 107–127.

[2] Z. Normatov, R. Turdibaev. Calogero-Moser spaces and the invariants of two matrices of degree 3. To appear in *Transform. Groups*, 2022.

[3] C. Procesi. The invariant theory of $n \times n$ matrices. *Advances in Math.* **19** (3) (1976), 306–381.

[4] Ju. P. Razmyslov. Identities with trace in full matrix algebras over a field of characteristic zero. *Izv. Akad. Nauk SSSR Ser. Mat.* **38** (1974), 723–756.

The Univalent Program and its semantics

Javier Villar Ortega

Javier Villar Ortega (javillo@unirioja.es)
Universidad de La Rioja

Abstract.
The Univalent Program, the research program that constitutes the main body of work on the field of Homotopy Type Theory, refers to a series of conjectures, proposals and tools regarding the properties of certain Martin-Löf type theories.

These provide a bridge between fields like Categorical Logic, Homotopy Theory, and Theoretical Computing; with the aim of providing a new proposal for the foundation of Mathematics, with an underlying constructivist-oriented type theory that takes into consideration its computational content, in a way useful for the design of proof assistants.

In this talk, we give an overview of the theory, with some historical hints on its development, the general current state of affairs on several of its main questions, and a brief explanation of its connections with the theory of locally Cartesian-closed categories (LCC).

References

[1] The Univalent Foundations Program. Homotopy Type Theory: Univalent Foundations of Mathematics. (2013) arXiv:1308.0729. [1308.0729].

[2] E. Riehl. On the ∞-topos semantics of Homotopy Type Theory. (2022). arXiv:2212.06937. [2212.06937].

[3] S. Awodey, N. Gambino, K. Sojakova. Homotopy-initial algebras in type theory. *Journal of the ACM* **63**(6) (2017), Article 51, pp. 1–45.

[4] D. Annenkov, P. Capriotti, N. Kraus, C. Sattler. Homotopy-initial algebras in type theory. *Mathematical Structures in Computer Science*, Special Issue: Homotopy Type Theory, 2023, pp. 1–56.

[5] P. L. Lumsdaine, M. Shulman. Semantics of higher inductive types. *Mathematical Proceedings of the Cambridge Philosophical Society* **169** (2020), 159–208.

[6] C. Cohen, T. Coquand, S. Huber, A. Mörtberg. Cubical Type Theory: a constructive interpretation of the univalence axiom. (2016). arXiv:1611.02108. [1611.02108].

Sponsors

- Galician Centre for Mathematical Research and Technology - CITMAga
  https://citmaga.gal/es/

- Ministerio de Ciencia e Innovación (AEI) https://www.aei.gob.es/
  "Redes de Investigación" RED2022-134220-T

- Red **EACA**: Red Temática de Cálculo Simbólico, Álgebra Computacional y Aplicaciones
  http://www.unirioja.es/dptos/dmc/RedEACA/

- Facultade de Matemáticas (USC)
  https://www.usc.gal/es/centro/facultad-matematicas

- Consellería de Cultura, Educación, Formación Profesional y Universidades
  https://culturaeducacion.xunta.gal/portada

Rede: ED341D R2016/022

- Research Group in Mathematics (GiMAT) https://www.usc.es/regaca/gimat/

- Universidade de Santiago de Compostela https://www.usc.gal/es