102nd Peripatetic Seminar on Sheaves and Logic

Santiago de Compostela, 13th-14th January, 2018.

Schedule

	Saturday 13th	**Sunday 14th**
09:20 - 09:50	Registration	Alan Cigoli
09:50 - 10:30	Peter Johnstone	Jonathan Weinberger
10:30 - 11:10	Fernando Lucatelli Nunes	Pilar Páez Guillán
11:10 - 11:40	Coffee Break	Coffee Break
11:40 - 12:20	Ramón González Rodríguez	Andrea Montoli
12:20 - 13:00	Leovigildo Alonso	Igor Bakovic
15:00 - 15:40	Cyrille Sandry Simeu	
15:40 - 16:20	Alejandro Fernández Fariña	
16:20 - 16:50	Coffee Break	
16:50	Free time for discussions	
21:00	Conference Dinner	

Abstracts

Remarks on sub-open inclusions

Peter Johnstone

University of Cambridge

We present some new results, and some old but apparently little-known ones, on the question of when the associated sheaf functor for an inclusion of toposes preserves the subobject classifier.

Non-canonical isomorphisms

Fernando Lucatelli Nunes

Universidade de Coimbra

There are several examples of categorical axioms asserting that a canonically defined natural transformation is invertible. In most of these situations, the existence of a suitable invertible (non-canonical) transformation suffices. The first example is the preservation of colimits studied by Caccamo-Winskel [1]. Three other examples are given by Lack [2]: distributive categories, semi-additive categories and normal functors between braided monoidal categories. We briefly talk about such situations and, then, we show how a characterization of pseudomorphisms between pseudoalgebras encompasses the examples above [3].

[1] M. Caccamo and G. Winskel. Limit preservation from naturality. In Proceedings of the 10th Conference on Category Theory in Computer Science (CTCS 2004), volume 122 of Electron. Notes Theor. Comput. Sci., pages 3–22. Elsevier Sci. B. V., Amsterdam, 2005.
[2] S. Lack. Non-canonical isomorphisms. J. Pure Appl. Algebra, 216(3):593–597, 2012.
[3] Lucatelli Nunes. Pseudoalgebras and non-canonical isomorphisms, Arxiv: 1711.02051, 2017.

Fundamental theorems of Doi-Hopf modules in a non-associative setting

Ramón González Rodríguez

Universidade de Vigo

In this talk we introduce the notion of weak non-asssociative Doi-Hopf module and give the Fundamental Theorem of Hopf modules in this setting. Also we prove that there exists a categorical equivalence that admits as particular instances the ones constructed in the literature for Hopf algebras, weak Hopf algebras, Hopf quasigroups, and weak Hopf quasigroups.

A categorical point of view for quasi-coherent sheaves

Leovigildo Alonso

Universidade de Santiago de Compostela

The notion of quasi-coherent sheaf plays a basic role in algebraic geometry. The category of quasi-coherent sheaves has better geometric properties than the whole category of sheaves of modules, e. g. in the affine case it corresponds to modules over the ring of global sections. On other geometric contexts these sheaves behave differently, as is the case of formal schemes. Motivated by the case of algebraic stacks whose associated small topos are not functorial with respect to stack morphisms we looked at quasi-coherent sheaves on the small flat topos. We will show that the notion is equivalent to the purely categorical notion of Cartesian sheaf under some reasonable conditions on the stack. We will also see that under the same conditions, it is further equivalent to descent data on a presentation of the stack and moreover they are also characterized as comodules over the corresponding Hopf algebroid. These characterizations restore the functoriality with respect to stack morphisms.

Higher extensions in exact Mal'tsev categories: distributivity of congruences and 3^n-Lemmas

Cyrille Sandry Simeu

Université Catholique de Louvain

The aim of this talk is to better understand the correspondence between n-fold extensions and 3^n-diagrams, which may be seen as non-abelian Yoneda extensions, useful in (co)homology of non-abelian algebraic structures.
We study a higher-dimensional version of the coequaliser/kernel pair adjunction, which relates n-fold reflexive graphs with n-fold arrows in any exact Mal'tsev category.
We first ask ourselves how this adjunction restricts to an equivalence of categories. This leads to the concept of an *effective n-fold equivalence relation*, corresponding to the n-fold regular epimorphisms. We characterise those in terms of what Bourn calls *parallelistic* higher equivalence relations.
We then further restrict the equivalence, with the aim of characterising the n-fold extensions. We find a congruence distributivity condition, resulting in a *denormalised 3^n-Lemma* valid in exact Mal'tsev categories. We deduce a 3^n-Lemma for short exact sequences in semi-abelian categories, which involves a distributivity condition between joins and meets of normal subobjects.
Joint work with Tim Van der Linden

[1] F. Borceux and D. Bourn, *Mal cev, protomodular, homological and semi-abelian categories*, Math. Appl., vol. 566, Kluwer Acad. Publ., 2004.
[2] D. Bourn, 3×3-*Lemma and protomodularity*, J. Algebra 236 (2001), 778-795.
[3] D. Bourn , *The denormalized* 3×3-*lemma*, J. Pure Appl. Algebra 177 (2003), 113-129.
[4] D. Bourn, *On the direct image of intersections in exact homological categories*, J. Pure Appl. Algebra 196 (2005), 39-52.
[5] A.Carboni, G. M. Kelly, and M. C. Pedicchio, *Some remarks on Maltsev and Goursat categories*, Appl. Categ. Structures 1 (1993), 385-421.
[6] T.Everaert, J. Goedecke, T. Janelidze-Gray, and T. Van der Linden, *Relative Mal tsev categories*, Theory Appl. Categ. 28 (2013), no. 29, 1002-1021.

[7] T. Everaert, J. Goedecke, and T. Van der Linden, *Resolutions, higher extensions and the relative Mal tsev axiom*, J. Algebra 371 (2012), 132-155.
[8] T. Everaert, M. Gran, and T. Van der Linden, *Higher Hopf formulae for homology via Galois Theory*, Adv. Math. 217 (2008), no. 5, 2231-2267.
[9] D. Rodelo and T. Van der Linden, *Higher central extensions and cohomology*, Adv. Math. 287 (2016), 31-108.

Perfect braided crossed modules of Lie algebras

Alejandro Fernández Fariña

Universidade de Santiago de Compostela

The non-abelian tensor product of groups, given by Brown-Loday, and the non-abelian tensor product of Lie algebras, given by Ellis, determine the universal central extensions of perfect groups and perfect Lie algebras, respectively. We will talk about the extension theory of braided crossed modules of Lie algebras. Moreover, we will prove the braided version of Casas-Ladra theorem and Ellis theorem.

A Beck-Chevalley condition for the direction functor

Alan Cigoli

Université Catholique de Louvain

We characterize internal opfibrations in the 2-category of fibrations over a fixed category, and we relate this notion to some existing notions, such as two-sided fibration and regular span in the sense of Yoneda. We focus then on the case where the base category is strongly semi-abelian and the opfibration in question is obtained by means of the direction functors sending, for each object A, crossed n-fold extensions of A to the corresponding A-modules. The fact that those functors gather together coherently is encoded by a Beck-Chevalley condition, and gives us an internal opfibration in $Fib(A)$.

Fibrations in Type Theory with Shapes

Jonathan Weinberger

TU Darmstadt

In order to develop a synthetic theory of $(\infty, 1)$-categories recently Riehl and Shulman introduced an intensional type theory extended by certain (pre-)types of shapes, modelled in Reedy fibrant bisimplicial sets. Besides defining analogues of ∞-groupoids and $(\infty, 1)$-categories in their type theory they also give various notions of type families, or fibrations. We show how these can be seen as synthetic versions of different kinds of fibrations known from $(\infty, 1)$-category theory à la Joyal and Lurie. In particular we discuss various closure properties and the validity of type-theoretic Grothendieck constructions. This talk is based on joint work with Ulrik Buchholtz.

Non abelian tensor product of restricted Lie superalgebras

Pilar Páez Guillán

Universidade de Santiago de Compostela

In this talk, we will define a non abelian tensor product for two restricted Lie superalgebras acting on each other, following the steps previously given for Lie algebras ([1]), restricted Lie algebras ([3]) and Lie superalgebras ([2]). We will present it as a functor between a category of pairs of Lie superalgebras acting on each other, and the one of restricted Lie superalgebras; we will also offer an explicit construction as a quotient of a particular free restricted Lie superalgebra. Moreover, we will analyse its connection with the usual tensor product of the underlying spaces, as well as its applications to central extensions of restricted Lie superalgebras.
The author is supported by the project MTM2016-79661-P of the Ministerio de Economía y Competitividad (Spain), European FEDER fund included.

[1] G. J. Ellis. *A nonabelian tensor product of Lie algebras.* Glasg. Math. J. **33** (1991) 101-120.
[2] X. García-Martínez, E. Khmaladze, M. Ladra. *Non-abelian tensor product and homology of Lie superalgebras.* J. Algebra **440** (2015) 464-488.
[3] R. Kurdiani. *Non-abelian tensor product of restricted Lie Algebras.* Bull. Georgian Acad. Sci. **164** (2001) 32-34.

On the centralization of quandle extensions

Andrea Montoli

Università degli Studi di Milano

We show that quandle coverings in the sense of Eisermann form a (regular epi)-reflective subcategory of the category of surjective quandle homomorphisms, both by using arguments coming from categorical Galois theory and by constructing concretely a centralization congruence. Joint work with Mathieu Duckerts-Antoine and Valérian Even.

Conspectus of internal fibrations

Igor Bakovic

Stellenbosch University

The purpose of the talk is to describe the theory of internal fibrations in bicategories. There are three different approaches to define internal fibrations. The first and the most obvious one is the representable approach derived from the work of Grothendieck who originally introduced fibrations of categories. The second approach was developed by Street who introduced two-sided fibrations in 2-categories, followed by two-sided fibrations in bicategories. These fibrations are defined as algebras over certain 2-monads on 2-categories, or "hyperdoctrines" on bicategories respectively, and Chevalley's internal characterization of fibrations was obtained as a byproduct. The third approach was developed by Johnstone who provided an alternative way of defining fibrations in a bicategory, which is closer than Street's to Grothendieck's original definition. Johnstone established the equivalence of his definition with the representable one. However, two important aspects of the theory were missing; the internal notion of a cartesian functor between fibrations and the comparison between Johnstone's and Street's notion. We fill this gap by introducing semi-oplax pullbacks. These are special kind of bipullbacks with an additional universal property defining local terminal objects which first appeared in the work of Kasangian, Kelly and Rossi in automata theory. Finally we introduce internal cartesian functors between fibrations and we show that this notion coincide with the usual one in the special case of the 2-category of categories.