Central extensions of precrossed and crossed modules

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The notion of centrality for crossed modules was introduced by Norrie in her thesis [7], in which she studied the category of crossed modules $\mathcal{CM}$ from an algebraic point of view, showing suitable generalizations of group theoretic concepts and results.

Subsequently, Norrie's approach was followed by Carrasco, Cegarra and R.-Grandjeán. In [5] they proved that $\mathcal{CM}$ is an algebraic category (i.e. there is a tripleable underlying functor from $\mathcal{CM}$ to the category of sets). This fact leads to the construction of cotriple homology and cohomology theories for crossed modules.

In [3], Arias, Ladra and R.-Grandjeán began a similar study for the category of precrossed modules $\mathcal{PCM}$. In this work we extended to precrossed modules Norrie's definition of center, and we proved that $\mathcal{PCM}$ is also an algebraic category.

In the aforementioned works some results on central extensions of crossed modules were developed. For example, Norrie studied the existence of universal central extensions in $\mathcal{CM}$. Moreover, Carrasco, Cegarra and R.-Grandjeán also classified the central extensions in $\mathcal{CM}$ with their second cohomology group. The analogue results for precrossed modules can be found in [4] and [1].

This talk will cover some of these and other results developed in the last years concerning central extensions in $\mathcal{PCM}$ and $\mathcal{CM}$, with special attention to the connection between universal central extensions in both categories [2].

In general, universal central extensions in $\mathcal{PCM}$ and $\mathcal{CM}$ don't coincide for a perfect crossed module. For example, for a two-sided ideal I of a ring R there is a perfect crossed module $(E(I), E(R), i)$ of elementary matrices, with universal central extension in $\mathcal{PCM}$

$$(K_2(I), K_2(R), \gamma) \rightarrowtail (St(I), St(R), \gamma) \twoheadrightarrow (E(I), E(R), i)\,,$$

while

$$(K_2(R,I), K_2(R), \overline{\gamma}) \rightarrowtail (St(R,I), St(R), \overline{\gamma}) \twoheadrightarrow (E(I), E(R), i)$$

is its universal central extension in $\mathcal{CM}$, where $St(I)$ and $K_2(I)$ are the Stein relativizations of $St(R)$ and $K_2(R)$, and $St(R,I)$ and $K_2(R,I)$ denote the relative groups of Loday and Keune (see [4] or [6]).

References

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