Groups and groupoids in higher categories

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What is a group object internal to a higher category? We expect this to be an object together with the 1-morphisms of the group structure, the 2-isomorphisms of the associator and unit constraints, and possibly yet higher morphisms, satisfying certain coherence relation. Since in a higher category the composition of 2-morphisms is itself not associative we expect the usual pentagon relation to be replaced by a more complicated diagram involving both the associator of the group and the associator of the category. The question of coherence relations is best tackled in the simplicial approach to higher categories. I will show how to define simplicial objects and Kan conditions internal to a quasi-category, and thus obtain natural notions of groups in a higher category. As example, the coherence relations for a 1-group in a weak 2-category are deduced from the simplicial formulation and spelled out explicitly. This example applies to presentations of group stacks and to hopfish algebras, which was the initial motivation for this work.