

Some Thoughts and Observations on the Trace of an Endofunctor of a Small Category

Emilio Faro

Universidad de Vigo, Spain

The concept of trace of a square matrix can be generalized to the trace of an endofunctor of a small category. This general concept (from which the Linear Algebra traces are obtained when applied to the identity endofunctor of finite dimensional vector spaces) has been known to category theorists for almost 30 years and has many interesting applications in different areas. Yet, it is very poorly represented in the literature. In this paper we review the basic definitions of the general trace and give a new construction, the “pretrace category”, to obtain the trace of (the identity of) a small category as the set of connected components of its pretrace. We show that this pretrace construction determines a finite-product preserving endofunctor of the category of small categories which has a natural comonad structure. Several results (known and new) follow from this simple construction.

References

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