Axiomatic stable homotopy: the derived category of quasi-coherent sheaves

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Hovey, Palmieri and Strickland have defined the concept of stable homotopy category in [HPS]. It consists of a list of additional properties and structure for a triangulated category. This concept arises in several contexts of algebraic geometry and topology, being two essential examples, $\mathsf{D}(R)$, the derived category of complexes of modules over a commutative ring R, and ${}^{o}sfHoSp$, the category of (non-connective) spectra up to homotopy.

In this talk we will show that for a quasi-compact and semi-separated (non necessarily noetherian) scheme X, the derived category of quasi-coherent sheaves over X, $\mathsf{D}(\mathsf{A}_{\mathrm{qct}}(X))$, is a stable homotopy category. We will also deal with the analogous result for formal schemes, namely, if $\mathfrak{X}$ is a noetherian semi-separated formal scheme the derived category of sheaves with quasi-coherent torsion homologies, $\mathsf{D}_{\mathrm{qct}}(\mathfrak{X})$ (*cfr* [AJL]), is a stable homotopy category. These results are included in [AJPV].

References

[AJL] Alonso Tarrío, L.; Jeremías López, A.; Lipman, J.: Duality and flat base change on formal schemes, in *Studies in duality on noetherian formal schemes and non-noetherian ordinary schemes*. Providence, RI: American Mathematical Society. Contemp. Math. **244**, 3–90 (1999).

[AJPV] Alonso Tarrío, L.; Jeremías López, A.; Pérez Rodríguez, M.; Vale Gonsalves, M. J.: The derived category of quasi-coherent sheaves and axiomatic stable homotopy. *Advances in Mathematics* **218**, 1224–1252 (2008).

[HPS] Hovey, M.; Palmieri, J. H.; Strickland, N. P.: Axiomatic stable homotopy theory. *Mem. Amer. Math. Soc.* **128** (1997), no. 610.