On the commutativity of stabilization and localization

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We show that, for a Morava K-theory spectrum K(n), the localization on the category of spectra with respect to K(n) is not equivalent to the stabilization of the K(n)-localization on the category of simplicial sets.

Introduction

The stabilization $Sp(\mathcal{M}, T)$ of a model category \mathcal{M} with respect to an endofunctor T was constructed by Hovey in [4].

We fix a spectrum E and, following [2], we consider the Bousfield localization $L_E(Sp)$ on the model category of spectra with respect to the homology theory defined by E. On the other hand, following [1], we can also consider the Bousfield localization $L_E(sSet)$ on simplicial sets with respect to the homology theory defined by E, and, then, construct the stabilization $Sp(L_E(sSet), \Sigma)$ with respect to the suspension functor. The problem that we study is whether or not their homotopy categories agree, i.e.

$$Ho(Sp(L_E(sSets), \Sigma)) \cong Ho(L_E(Sp))?$$

We use a characterization by Dugger [3] to express the stabilization of a Bousfield localization on a model category as another Bousfield localization on the stabilization of the original model category.

Then we show that the answer to our problem is negative when E is taken to be a Morava K-theory.

References

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