Homotopy theory in terms of cylinder objects

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Denote the 'interval category' $\cdot \rightarrow \cdot \leftarrow \cdot$ by I. Consider a category $\mathcal{D}$ and a class E of distinguished morphisms in $\mathcal{D}$. In this talk we will study the associated homotopy category $Ho\mathcal{D} = \mathcal{D}[\mathrm{E}^{-1}]$, using a suitable cylinder functor $cyl : I^{op}\mathcal{D} \rightarrow I\mathcal{D}$ as basic tool. We focus our study in the following two points: first, provide an 'acceptable' description of $Ho\mathcal{D}$ and second, induce a (neither additive nor stable) triangulated structure on $Ho\mathcal{D}$. Examples of such cylinder functors are those induced by a suitable 'geometric realization' $\mathbf{s} : \Delta^{op}\mathcal{D} \rightarrow \mathcal{D}$, that is, in case $\mathcal{D}$ is a 'simplicial descent category'. Hence this techniques can be applied, in particular, to: Δ^{op}Sets, topological spaces, (filtered) complexes, commutative differential graded algebras, DG-modules and mixed Hodge complexes.