Program

Fifth Seminar on Categories and Applications

Seca V

Pontevedra, September 10-12, 2008

Program

	Wednesday 10	Thursday 11	Friday 12
9:00-9:50	Reception of Participants	Plenary Talk C. Blohmann	Plenary Talk M. Weber
9:55-10:45	Plenary Talk J.-L. Loday	Plenary Talk J. Elgueta	Plenary Talk F. Muro
10:50-11:15	Short Communications E. Faro	Short Communications J. A. Álvarez	Short Communications S. Lopes
11:20-11:40	Coffee Break	Coffee Break	Coffee Break
11:45-12:10	Short Communications Z. Skoda	Short Communications J. Gutiérrez	Short Communications A. del Río
12:15-13:05	Plenary Talk D. Gepner	Plenary Talk J. Bergner	Plenary Talk M. Müger
13:10-13:35	Official Opening	Short Communications J. Dolan	Short Communications M. Pérez
13:40-14:00			
14:00-15:30	Lunch	Lunch	Lunch
15:30-16:20	Plenary Talk M. J. Souto	Cultural Visit	Plenary Talk D. Arias
16:25-16:50	Short Communications B. Rodríguez	Cultural Visit	Short Communications O. Raventós
16:55-17:15	Coffee Break	Cultural Visit	Coffee Break
17:20-17:45	Short Communications M. A. García-Muñoz	Cultural Visit	Short Communications H. Colman
17:50-18:40	Plenary Talk F. Neumann	Cultural Visit	Plenary Talk E. Vitale
18:45-19:00		Cultural Visit	Closing Session
20:00	Guided Visit	Cultural Visit	
21:30	Congress Dinner		

Activity	Room	Floor
Reception	Hall	0
Plenary Talk	Room 6	2
Short Communications	Room 6	2
Official Opening	Salón de Actos	-1
Coffee Break	Hall	-1

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SCHEDULE

Wednesday, September 10th

Morning session

9:00-9:50 Reception of participant

9:55-10:45 Plenary talk: Jean-Louis Loday

Operadic geometry

10:50-11:15 Short Communications: Emilio Faro

Some thoughts and observations on the trace of an endofunctor of a small category

11:20-11:40 Coffee break

11:45-12:10 Short Communications: Zoran Skoda

Categorified equivariance

12:15-13:05 Plenary talk: David Gepner

On the motivic spectra representing algebraic K-theory and algebraic cobordism

13:10-13:35 Official Opening

Afternoon session

15:30-16:20 Plenary talk: María José Souto

Auslander-Reiten theory in complex categories

16:25-16:50 Short Communications: Beatriz Rodríguez

Homotopy theory in terms of cylinder objects

16:55-17:15 Coffee Break

17:20-17:45 Short Communications: Miguel Ángel García-Muñoz

Obtaining the Postnikov system of an arbitrary CW-complex algebraically

17:50-18:40 Plenary talk: Frank Neumann

Cohomology of moduli stacks of vector bundles on algebraic curves

Thursday, September 11th

Morning session

9:00-9:50 Plenary talk: Christian Blohmann
Groups and groupoids in higher categories

9:55-10:45 Plenary talk: Josep Elgueta
Linear representations of categorical groups

10:50-11:15 Short Communications: Jesús Antonio Álvarez
Distortions and bidistortions

11:20-11:40 Coffee break

11:45-12:10 Short Communications: Javier Gutiérrez
A model structure for coloured operads in symmetric spectra

12:15-13:05 Plenary talk: Julia Bergner
Diagrams of simplicial sets inducing algebraic structures

13:10-13:45 Short Communications: James Dolan
Hecke operators and groupoidification

Afternoon session

15:30-21:30 Cultural Visit

Friday, September 12th

Morning session

9:00-9:50 Plenary talk: Mark Weber
Higher operad algebras as enriched categories

9:55-10:45 Plenary talk: Fernando Muro
On determinants (as functors)

10:50-11:15 Short Communications: Samuel Lopes
Generalized down-up algebras and their automorphisms

11:20-11:40 Coffee break

11:45-12:10 Short Communications: Aurora del Río
K-theory for categorical groups

12:15-13:05 Plenary talk: Michael Müger
On isocategorical compact groups

13:10-13:35 Short Communications: Marta Pérez
Axiomatic stable homotopy: the derived category of quasi-coherent sheaves

Afternoon session

15:30-16:20 Plenary talk: Daniel Arias
Central extensions of precrossed and crossed modules

16:25-16:50 Short Communications: Oriol Raventós
On the commutativity of stabilization and linearization

16:55-17:15 Coffee Break

17:20-17:45 Short Communications: Hellen Colman
A Quillen model structure for orbifolds

17:50-18:40 Plenary talk: Enrico Vitale
Homotopy pullbacks for n-groupoids

18:45-19:00 Closing session

Fifth Seminar on Categories and Applications
Pontevedra, September 10-12, 2008

PLENARY TALKS

Operadic geometry

Jean-Louis Loday
CNRS, Université Louis Pateur, Strasbourg, France

Algebras, operads, computads. We will show the relationship of these three levels of sophistication in algebra/non-commutative geometry. We will present several open problems in this area.

On the motivic spectra representing algebraic K-theory and algebraic cobordism

David Gepner
University of Sheffield, UK

We show that algebraic K-theory and periodic algebraic cobordism are localizations of motivic suspension spectra obtained by inverting the Bott element, generalizing theorems of V. Snaith in the topological case. This yields an easy proof of the motivic Conner-Floyd theorem and also implies that algebraic K-theory is E-infinity as a motivic spectrum.

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Pontevedra, September 10-12, 2008

Auslander Reiten Theory in complex categories

María José Souto Salorio
Universidade de A Coruña, Spain

The classical Auslander-Reiten theory started in the seventies of the last century. It involves two relevant notions: the almost split sequences (a.s.s. for short) and the irreducible morphisms in $\operatorname{mod}\Lambda$ (the category of finitely generated modules over an Artin algebra). The existence of minimal left or right almost split maps is one of the main assertions of this theory. And, from it one can produces a wealth of irreducible morphism. Moreover, the translate functor which relates the two ends of an almost split sequence, yields a tool for obtaining new indecomposable modules from a given one.

In the mid eighties, Auslander-Reiten theory for triangulated categories has been initiated by D. Happel. He introduced Auslander-Reiten triangles and characterized their existence in the derived category $\mathbf{D}^b(\operatorname{mod}\Lambda)$.

On the other hand, while it is known what the terms of certain irreducible maps look like, we do not know in general so much about irreducible maps in $\mathbf{D}^b(\operatorname{mod}\Lambda)$.

In this talk, we present some results obtained in joint work with Raymundo Bautista. We prove the existence of a.s.s. in certain subcategories of complexes and show its relation with the existence of Auslander-Reiten triangles in $\mathbf{D}^b(\operatorname{mod}\Lambda)$. In turn, we present some properties of irreducible morphisms in complexes similar to the ones of irreducible morphisms in $\operatorname{mod}\Lambda$.

Cohomology of moduli stacks of vector bundles on algebraic curves

Frank Neumann
University of Leicester, UK

After giving an introduction into moduli problems and moduli stacks, I will describe the l-adic cohomology algebra of the moduli stack of vector bundles on a give algebraic curve in positive characteristic and will describe the action of the various geometric and arithmetic Frobenius morphisms. It turns out that using the higher categorical language of stacks instead of geometric invariant theory this becomes surprisingly easy and topological in flavour. Using the Lefschetz trace formula for algebraic stacks due to Behrend I will discuss the analogues of the Weil Conjectures for this moduli stack and will determine how many isomorphism classes of vector bundles on an algebraic curve in positive characteristic there are. This is joint work in progress with U. Stuhler (Goettingen).

Groups and groupoids in higher categories

Christian Blohmann
Universität Regensburg, Germany

What is a group object internal to a higher category? We expect this to be an object together with the 1-morphisms of the group structure, the 2-isomorphisms of the associator and unit constraints, and possibly yet higher morphisms, satisfying certain coherence relation. Since in a higher category the composition of 2-morphisms is itself not associative we expect the usual pentagon relation to be replaced by a more complicated diagram involving both the associator of the group and the associator of the category. The question of coherence relations is best tackled in the simplicial approach to higher categories. I will show how to define simplicial objects and Kan conditions internal to a quasi-category, and thus obtain natural notions of groups in a higher category. As example, the coherence relations for a 1-group in a weak 2-category are deduced from the simplicial formulation and spelled out explicitly. This example applies to presentations of group stacks and to hopfish algebras, which was the initial motivation for this work.

Linear representations of categorical groups

Josep Elgueta
Universitat Politècnica de Catalunya, Spain

After a few general facts about the representation theory of categorical groups, I'm going to talk about their representations as K-linear self-equivalences of Kapranov and Voevodsky 2-vector spaces. I shall describe the equivalence classes of such representations in terms of the homotopy invariants of the categorical group, and I shall give a geometric description of the corresponding categories of morphisms between two such representations. I shall also discuss some related points, in particular, the notion of regular representation and some of its properties.

Diagrams of simplicial sets inducing algebraic structures

Julia Bergner
University of California, Riverside, USA

Diagrams given by product-preserving functors from an algebraic theory to the category of simplicial sets essentially induce an algebraic structure on one of the simplicial sets in the diagram. We show how simpler diagrams can be used to encode monoid and group structures and how they can be generalized to relate simplicial categories to Segal categories (or their groupoid analogues).

Higher operad algebras as enriched categories

Mark Weber
Université Paris Diderot - Paris 7, France

(joint work with Michael Batanin) In the combinatorial approach to defining and working with higher categorical structures, one uses globular operads to say what the structures are in one go. However in the simplicial approaches to higher category theory, one proceeds inductively following the idea that a weak (n+1)-category is something like a category enriched in weak n-categories. In this talk the inductive content hidden within the globular operadic approach will be revealed. Doing this involves a rather fascinating interplay between the theory of parametric right adjoint monads and extensive categories.

On determinants (as functors)

Fernando Muro
Universitat de Barcelona, Spain

The elementary properties of determinants are better understood in terms of categories and functors. We will review different notions of determinant functors and their connection with K-theory. We will show how determinant functors can be used to prove results in K-theory and vice versa.

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Pontevedra, September 10-12, 2008

On isocategorical compact groups

Michael Müger
Radboud Universiteit Nijmegen, The Netherlands

It is well known and classical that non-isomorphic finite groups can have isomorphic representation rings or, equivalently, character tables. In the classical cases, however, such groups have inequivalent tensor categories of representations. A few years ago, Davydov, Etingof/Gelaki and Izumi/Kosaki discovered independently that there even exist non-isomorphic finite groups with representation categories that are equivalent as tensor categories. Such groups were called isocategorical. We provide a more categorical approach to the phenomenon that allows (a) extension to compact groups and (b) more complete classification results.

Central extensions of precrossed and crossed modules

Daniel Arias
Universidad de León, Spain

The notion of centrality for crossed modules was introduced by Norrie in her thesis [7], in which she studied the category of crossed modules $\mathcal{CM}$ from an algebraic point of view, showing suitable generalizations of group theoretic concepts and results.

Subsequently, Norrie's approach was followed by Carrasco, Cegarra and R.-Grandjeán. In [5] they proved that $\mathcal{CM}$ is an algebraic category (i.e. there is a tripleable underlying functor from $\mathcal{CM}$ to the category of sets). This fact leads to the construction of cotriple homology and cohomology theories for crossed modules.

In [3], Arias, Ladra and R.-Grandjeán began a similar study for the category of precrossed modules $\mathcal{PCM}$. In this work we extended to precrossed modules Norrie's definition of center, and we proved that $\mathcal{PCM}$ is also an algebraic category.

In the aforementioned works some results on central extensions of crossed modules were developed. For example, Norrie studied the existence of universal central extensions in $\mathcal{CM}$. Moreover, Carrasco, Cegarra and R.-Grandjeán also classified the central extensions in $\mathcal{CM}$ with their second cohomology group. The analogue results for precrossed modules can be found in [4] and [1].

This talk will cover some of these and other results developed in the last years concerning central extensions in $\mathcal{PCM}$ and $\mathcal{CM}$, with special attention to the connection between universal central extensions in both categories [2].

In general, universal central extensions in $\mathcal{PCM}$ and $\mathcal{CM}$ don't coincide for a perfect crossed module. For example, for a two-sided ideal I of a ring R there is a perfect crossed module $(E(I), E(R), i)$ of elementary matrices, with universal central extension in $\mathcal{PCM}$ $(K_2(I), K_2(R), \gamma) \rightarrowtail (St(I), St(R), \gamma) \twoheadrightarrow (E(I), E(R), i)$, while $(K_2(R, I), K_2(R), \overline{\gamma}) \rightarrowtail (St(R, I), St(R), \overline{\gamma}) \twoheadrightarrow (E(I), E(R), i)$ is its universal central extension in $\mathcal{CM}$, where $St(I)$ and $K_2(I)$ are the Stein relativizations of $St(R)$ and $K_2(R)$, and $St(R, I)$ and $K_2(R, I)$ denote the relative groups of Loday and Keune (see [4] or [6]).

References

[1] D. Arias and M. Ladra, *Central extensions of precrossed modules*, Appl. Categ. Structures **12** (2004) 339–354.

[2] D. Arias, J.M. Casas and M. Ladra, *On universal central extensions of precrossed and crossed modules*, J. Pure Appl. Algebra **210** (2007) 177–191.

[3] D. Arias, M. Ladra and A. R.-Grandjeán, *Homology of precrossed modules*, Illinois J. Math. **46** (3) (2002) 739–754.

[4] D. Arias, M. Ladra and A. R.-Grandjeán, *Universal central extensions of precrossed modules and Milnor's relative* K_2, J. Pure Appl. Algebra **184** (2003) 139–154.

[5] P. Carrasco, A.M. Cegarra and A. R.-Grandjeán, *(Co)Homology of crossed modules*, J. Pure Appl. Algebra **168** (2-3) (2002) 147–176.

[6] Loday, J.-L., Cohomologie et groupes de Steinberg relatifs, *J. Algebra* **54** (1) (1978), 178–202.

[7] K.J. Norrie, *Crossed modules and analogues of Group theorems*, Ph. D. Thesis, University of London, 1987.

Fifth Seminar on Categories and Applications
Pontevedra, September 10-12, 2008

Homotopy pullbacks for n-groupoids

Enrico Vitale
Institut de Mathématique, Université Catholique de Louvain, Belgium,

Abstract

In this talk, based on [5], I will have a glance to a convenient notion of pullback for n-categories and n-groupoids. The main application is the study of exact sequences for pointed n-groupoids.

The starting point is a result due to R. Brown [1]: given a functor $A \to B$ between pointed groupoids and its homotopy fibre $\mathcal{F}_*$, there is a 6-term exact sequence of pointed sets

$$\mathcal{F}_* \to \mathcal{A} \to \mathcal{B}$$

$$\pi_1\mathcal{F}_* \to \pi_1 A \to \pi_1 B \to \pi_0\mathcal{F}_* \to \pi_0 A \to \pi_0 B$$

More recently, starting from a morphism $A \to B$ of pointed 2-groupoids and using convenient notions of homotopy fibre $\mathcal{F}_*$ and exactness, a 6-term exact sequence of pointed groupoids and a 9-term exact sequence of pointed sets have been obtained in [2] and [3]

$$\mathcal{F}_* \to A \to B$$

$$\pi_1\mathcal{F}_* \to \pi_1 A \to \pi_1 B \to \pi_0\mathcal{F}_* \to \pi_0 A \to \pi_0 B$$

$$\pi_1\pi_1\mathcal{F}_* \to \pi_1\pi_1 A \to \pi_1\pi_1 B \to \pi_1\pi_0\mathcal{F}_* \to \pi_1\pi_0 A \to \pi_1\pi_0 B \to \pi_0\pi_0\mathcal{F}_* \to \pi_0\pi_0 A \to \pi_0\pi_0 B$$

It is therefore quite natural to ask if, given a morphism $A \to B$ of pointed n-groupoids, is it possible to construct an $(n+1)$-level "ziqqurath" of exact sequences a 3-term exact sequence of pointed n-groupoids

a 6-term exact sequence of pointed $(n-1)$-groupoids

a 9-term exact sequence of pointed $(n-2)$-groupoids

....

a $3n$-term exact sequence of pointed groupoids

a $3(n+1)$-term exact sequence of pointed sets

Clearly, to make precise such a question we have to understand what is an homotopy fibre and more in general an homotopy pullback in the context of n-groupoids and what it means exactness for a sequence of pointed n-groupoids.

The main steps to work out in details the previous program are:
1) An inductive definition of the sesqui-category $nCat$ of n-categories, n-functors and lax n-transformations.
2) An inductive description of homotopy pullbacks in $nCat$.
3) An inductive definition of n-groupoids using equivalences of $(n-1)$-groupoids. The definition we give is equivalent to those given by Kapranov-Voevodsky [4] and by Street [6].
4) The construction of sesqui-functors $\pi_0,\ \pi_1 \colon nGrpd_* \to (n-1)Grpd_*$. The construction of π_1 is the most delicate point: one needs to consider n-modifications and to show that homotopy pullbacks in $nCat$ satisfy a universal property involving n-modifications.
5) A definition of exactness for a sequence of pointed n-groupoids, using homotopy fibers.

Once these definitions and constructions achieved, we get the desired results:
1) If $A \to B \to C$ is an exact sequence of pointed n-groupoids, then $\pi_0 A \to \pi_0 B \to \pi_0 C$ and $\pi_1 A \to \pi_1 B \to \pi_1 C$ are exact sequences of pointed $(n-1)$-groupoids.
2) Given an n-functor $A \to B$ between pointed n-groupoids together with its homotopy fibre $\mathcal{F}_*$, it is possible to construct a 6-term exact sequence of pointed $(n-1)$-groupoids $\pi_1\mathcal{F}_* \to \pi_1 A \to \pi_1 B \to \pi_0 \mathcal{F}_* \to \pi_0 A \to \pi_0 B$

3) Since π_0 and π_1 commute, $\pi_0 \cdot \pi_1 \simeq \pi_1 \cdot \pi_0$, applying π_0 and π_1 to the exact sequerce of the previous point one gets a 9-term exact sequence of pointed $(n-2)$-groupoids. By iteration, we have a ziqqurath of exact sequences of increasing length (from the top to the bottom) and decreasing dimension.

References

[1] R. Brown, Fibrations of groupoids, *Journal of Algebra* **15** (1970) 103–132.

[2] J.W. Duskin, R.W. Kieboom and E.M. Vitale, Morphisms of 2-groupoids and low-dimensional cohomology of crossed modules, *Fields Institut Comm.* **43** (2004) 227–241.

[3] K.A. Hardie, K.H. Kamps and R.W. Kieboom, Fibrations of bigroupoids, *Journal of Pure and Applied Algebra* **168** (2002) 35–43.

[4] M.M. Kapranov and V.A. Voevodsky, ∞-groupoids and homotopy types, *Cahiers de Topologie et Géométrie Différentielle Catégorique* **32** (1991) 29–46.

[5] G. Metere, The ziqqurath of exact sequences on n-groupoids, *Thesis, Università di Milano* (2008), available as arXiv:0802.0800

[6] R. Street, The algebra of oriented simplexes, *Journal of Pure and Applied Algebra* **49** (1987) 283–335.

Fifth Seminar on Categories and Applications
Pontevedra, September 10-12, 2008

SHORT COMMUNICATIONS

Some Thoughts and Observations on the Trace of an Endofunctor of a Small Category

Emilio Faro
Universidad de Vigo, Spain

The concept of trace of a square matrix can be generalized to the trace of an endofunctor of a small category. This general concept (from which the Linear Algebra traces are obtained when applied to the identity endofunctor of finite dimensional vector spaces) has been known to category theorists for almost 30 years and has many interesting applications in different areas. Yet, it is very poorly represented in the literature. In this paper we review the basic definitions of the general trace and give a new construction, the "pretrace category", to obtain the trace of (the identity of) a small category as the set of connected components of its pretrace. We show that this pretrace construction determines a finite-product preserving endofunctor of the category of small categories which has a natural comonad structure. Several results (known and new) follow from this simple construction.

References

[1] F. Borceux. *Handbook of Categorical Algebra.* Cambridge, 1994.

[2] M. Kelly and M. Laplaza. Coherence for compact closed categories. *University of Sydney*, 1978.

[3] S. Lang. *Algebra.* Addisin-Wesley, 1965.

[4] F. W. Lawvere. Metric spaces, generalized logic, and closed categories. *Rend. Sem. Fis. Milano*, 43:135–166, 1973.

[5] Representations lineaires des groupes finis. *Jean-Paul Serre.* Hermann, Paris, 1978.

[6] S. MacLane. *Categories for the Working Mathematicien.* Springer Verlag, 1971.

Categorified equivariance

Zoran Skoda
Institute Rudjer Boskovic, Zagreb, Croatia

In the first part of the talk I will show how to systematically introduce the 2-equivariant objects in a fibre of a 2-fibered 2-category over a pseudoalgebra over a pseudomonad in the base 2-category. The main case is when the pseudomonad is a categorical group, or only a monoidal category internal to the base 2-category. In the second part of the talk I will talk about applications like descent along 2-torsors over categorical groups and also the role of 2-equivariance in noncommutative algebraic geometry.

Homotopy theory in terms of cylinder objects

Beatriz Rodríguez González
Universidad de Sevilla, Spain

Denote the 'interval category' $\cdot \rightarrow \cdot \leftarrow \cdot$ by I. Consider a category $\mathcal{D}$ and a class E of distinguished morphisms in $\mathcal{D}$. In this talk we will study the associated homotopy category $Ho\mathcal{D} = \mathcal{D}[\mathrm{E}^{-1}]$, using a suitable cylinder functor $cyl : I^{op}\mathcal{D} \rightarrow I\mathcal{D}$ as basic tool. We focus our study in the following two points: first, provide an 'acceptable' description of $Ho\mathcal{D}$ and second, induce a (neither additive nor stable) triangulated structure on $Ho\mathcal{D}$. Examples of such cylinder functors are those induced by a suitable 'geometric realization' $\mathbf{s} : \Delta^{op}\mathcal{D} \rightarrow \mathcal{D}$, that is, in case $\mathcal{D}$ is a 'simplicial descent category'. Hence this techniques can be applied, in particular, to: Δ^{op}Sets, topological spaces, (filtered) complexes, commutative differential graded algebras, DG-modules and mixed Hodge complexes.

Fifth Seminar on Categories and Applications
Pontevedra, September 10-12, 2008

Obtaining the Postnikov system of an arbitrary CW-complex algebraically

Miguel Angel García-Muñoz
Universidad de Jaén, Spain

We determine the Postnikov tower of an arbitrary space X (CW-complex) as the geometric realization of the Postnikov tower of $\Pi(X)$, the fundamental simplicial groupoid of the singular complex of X, which is obtained in the category of simplicial groupoids by a purely algebraic method similar to the one used in [2] for crossed complexes. Eventually we obtain the Postnikov invariants from the fibrations in the Postnikov tower of $\Pi(X)$ (which are shown to represent cocycles in an algebraic cohomology) by defining an appropriate connecting map from the algebraic cohomology to singular cohomology.

Introduction

The Postnikov tower of a space X is a diagram of spaces $\cdots \to X_{n+1} \xrightarrow{\eta_{n+1}} X_n \xrightarrow{\eta_n} X_{n-1} \to \cdots \to X_0$, such that: (1) its limit is a space which is weak equivalent to X, (2) for each $n \geq 1$, X_n is a topological n-type, and (3) for each $n \geq 1$, η_n is a fibration whose fibres are Eilenberg-MacLane space. Associated with each fibration η_n there is a cohomology element $k_n \in H^{n+1}_{\text{sing}}\big(X_{n-1}, \pi_n(X)\big)$ in the singular cohomology of CW-complexes which is called the n^{th} Postnikov invariant of X and informs us about how the fibres of η_n are glued together to form X_n in the sense that $X_n = X_{n-1} \times_{k_n} K\big(\pi_n(X), n\big)$.

In the paper, [2], previously presented in a short communication by E. Faro in Seca I, we introduced a general algebraic procedure to obtain Postnikov towers and Postnikov invariants and developed it in the category of crossed complexes providing an algebraic calculation of the Postnikov towers and Postnikov invariants of filtered spaces which have the homotopy type of a crossed complex. In order to eliminate this restriction on the homotopy type of the spaces, we shall develop our algebraic approach in the category of simplicial groupoids, a category which models all homotopy types of CW-complexes.

1 Postnikov tower

The general scheme for the algebraic calculation of Postnikov towers introduced in [2] consists in "translating" the problem from a category $\mathcal{T}$ of CW-complexes to a "more algebraic" category Gd, the category of simplicial groupoids, which is related to $\mathcal{T}$ by means of an adjoint pair of functors *"fundamental simplicial groupoid of the singular complex of a space"*, $\Pi : \mathcal{T} \to \mathsf{Gd}$, and *"geometric realization of (the nerve of) a simplicial groupoid"*, $B : \mathsf{Gd} \to \mathcal{T}$, which induce an equivalence in the corresponding homotopy categories. Then, the geometric realization of the Postnikov tower of $\Pi(X)$ will be the Postnikov tower of X.

The calculation of the Postnikow towers of simplicial groupoids will proceed as follows: For each $n \geq 0$ we define, a subcategory Gd_n of Gd so that trivially $\mathsf{Gd}_n \subset \mathsf{Gd}_{n+1}$, an endofunctor $P_n\colon \mathsf{Gd} \to \mathsf{Gd}$ such that for every $\boldsymbol{\mathcal{G}} \in \mathsf{Gd}$, $P_n(\boldsymbol{\mathcal{G}}) \in \mathsf{Gd}_n$, P_n leaves Gd_n fixed, and $P_n P_{n+1} = P_n$, and a natural transformation $\delta^{(n)}\colon 1_{\mathsf{Gd}} \to P_n$ such that $\delta^{(n)} * P_n = P_n * \delta^{(n)} = 1_{P_n}$. Then we prove that for every simplicial groupoid $\boldsymbol{\mathcal{G}}$, the map $\eta^{(n+1)}_{\boldsymbol{\mathcal{G}}} = \delta^{(n)}_{P_{n+1}(\boldsymbol{\mathcal{G}})}$ is a fibration whose fibres have the homotopy type of a $K(\Pi, n+1)$, for each $n \geq 0$, and the cone formed by the maps $\delta^{(n)}_{\boldsymbol{\mathcal{G}}}$ over the chain of the $\eta^{(n+1)}_{\boldsymbol{\mathcal{G}}} = \delta^{(n)}_{P_{n+1}(\boldsymbol{\mathcal{G}})}$ is a limit diagram.

2 Postnikov invariants

The situation with respect to the Postnikov invariants is a little more involved. In the Postnikov tower of a space we have, for each n a fibration, η_{n+1}, from a $(n+1)$-type to a n-type. With a slight abuse of language and taking Gd_n as our category of n-types we can say that η_{n+1} is a fibration from a groupoid in Gd_n to an object in Gd_n. This, very closely resembles the concept of a 2-torsor in Gd_n, a gadget used by Duskin to represent 2-dimensional cotriple cohomology. In fact, if one makes a slightly different description of Duskin's 2-torsors (as given, for example, in [2, Appendix]) it is easy to show that by regarding the objects in Gd_{n+1} as groupoids in Gd_n, the fibration η_{n+1} becomes exactly a 2-torsor in Gd_n.

Thus, it seems that η_{n+1} should determine in a natural way an element of an algebraic 2-cohomology in Gd_n. It only remains to connect the resulting 2-cohomology with the $(n+2)$-cohomology in the category of all homotopy types, where the actual Postnikov invariant is supposed to live.

References

[1] M. Bullejos, E. Faro, and M. A. Garcia-Muñoz. Homotopy colimits and cohomology with local coefficients. *Cahiers Topologie Géom. Différentielle Catég.*, 44 (1):63–80, 2003.

[2] M. Bullejos, E. Faro, and M. A. Garcia-Muñoz. Postnikov invariants of crossed complexes. *Journal of Algebra*, 285:238–291, 2005.

[3] M. A. García-Muñoz. *Un acercamiento algebraico a la teoría de torres de Postnikov.* Ph.D. thesis, Universidad de Granada, 2003. ISBN: 9788433842206. Granada, 2007. http://adrastea.ugr.es/record=b1655506*spi G,

Fifth Seminar on Categories and Applications
Pontevedra, September 10-12, 2008

Distortions and bidistortions

Jesús A. Álvarez López
Universidad de Santiago de Compostela, Spain

The idea of "distortion" of a morphism between objects of an "higher level" category is formalized in a theory.

Introduction

This talk will be about a theory produced after several abstractions, beginning with the study of the geometry of leaves of foliated spaces [1]. It has the flavor of a combination of categories and metrics. There may be already combinations of this kind, but we hope that our point of view is new. The study of foliated spaces seems to be only one of the many applications that this very general theory could have.

1 Distortions

An *underlying functor* (or *u-functor*) $\mathcal{A}/\mathcal{B}$ is a functor $|\ | : \mathcal{A} \to \mathcal{B}$ whose restriction $\mathcal{A}(M,N) \to \mathcal{B}(|M|,|N|)$ is injective $\forall M,N \in \mathcal{A}_0$. Thus we can consider $\mathcal{A}$ as a subcategory of the category $\widetilde{\mathcal{B}}$ with $\widetilde{\mathcal{B}}_0 = \mathcal{A}$, $\widetilde{\mathcal{B}}(M,N) \equiv \mathcal{B}(|M|,|N|)$ and the operation induced by $\mathcal{B}$. The typical example of u-functor is provided by two species of structures, one subordinated to the other, with corresponding classes of morphisms.

Consider the following pre-order relation between sets of functions $X \to [0,\infty]$: $C \preccurlyeq D$ if, $\forall \epsilon > 0$, $\exists \delta > 0$ such that,

$$d(x) < \delta \ \forall d \in D,\ \forall x \in X \implies c(x) < \epsilon \ \forall c \in C,\ \forall x \in X\ .$$

A *distortion* of a u-functor $\mathcal{A}/\mathcal{B}$ is a map $\mathfrak{d} : \widetilde{\mathcal{B}}_1 \to [0, \infty]$ satisfying $\mathfrak{d}^{-1}(0) = \mathcal{A}_1$ and $\mathfrak{d}(f_n \cdots f_1) \preccurlyeq \mathfrak{d}(f_1)+\cdots+\mathfrak{d}(f_n)$. For $M, N \in \mathcal{A}_0$, let $\mathfrak{d}_{M,N} = \mathfrak{d}|_{\widetilde{\mathcal{B}}(M,N)}$. An induced distortion $\hat{\mathfrak{d}}$ is defined by $\hat{\mathfrak{d}}(f) = \inf\{\mathfrak{d}(f_1)+\cdots+\mathfrak{d}(f_n) \mid f_n \cdots f_1 = f\}$. Then a pseudo-metric $d_{\mathfrak{d}}$ on $\mathcal{A}_0$ (with possible infinite values) is given by $d_{\mathfrak{b}}(M, N) = \|(\inf \mathfrak{d}_{M,N}, \inf \mathfrak{d}_{N,M})\|$. It induces a pseudo-metric on the quotient set $\mathcal{A}\backslash\mathcal{A}_0$ of $\mathcal{A}_0$ defined by the equivalence relation $M \sim N$ when $\mathcal{A}(M, N) \neq \emptyset \neq \mathcal{A}(N, M)$. For instance, the best Lipschitz constant defines a distortion of Met / Set.

2 Bidistortions

An *involutive category* is a category $\mathcal{C}$ endowed with an *involution* (a contravariant functor $I : \mathcal{C} \to \mathcal{C}$ such that $I_0 = \mathrm{id}_{\mathcal{C}_0}$ and $I^2 = \mathrm{id}_{\mathcal{C}}$). The notation $\bar{f} = I(f)$ and $\overline{S} = \{\bar{f} \mid f \in S\}$ will be used. For example, any groupoid is involutive with $\bar{f} = f^{-1}$. Also, any category $\mathcal{B}$ induces an involutive category $\mathcal{B}^{\mathrm{in}}$ with $\mathcal{B}^{\mathrm{in}}_0 = \mathcal{A}_0$, $\mathcal{B}^{\mathrm{in}}(M, N) \equiv \mathcal{B}(M, N) \times \mathcal{B}(N, M)$, the operation induced by $\mathcal{B}$ and $\overline{(f, g)} = (g, f)$. The concept of *involutive* functor is now straightforward. Any u-functor $\mathcal{A}/\mathcal{B}$ induces an involutive u-functor $\mathrm{Iso}(\mathcal{A})/\mathcal{B}^{\mathrm{in}}$ given by $\phi \mapsto (|\phi|, |\phi^{-1}|)$.

A *crossing* of a category $\mathcal{D}$ is mapping $S \mapsto S^\times$, for sets S of parallel morphisms of $\mathcal{D}$, so that $\emptyset^\times = \emptyset$, $S \subset S^\times$ and $S \subset T \Rightarrow S^\times \subset T^\times$. For instance, $\mathcal{B}^{\mathrm{in}}$ has a crossing given by

$$S^\times = \{(f, g') \mid (f, g), (f', g') \in S \text{ for some } f', g \in \mathcal{B}_1\} .$$

Let $\mathcal{C}/\mathcal{D}$ be an involutive u-functor, and suppose that $\mathcal{D}$ is endowed with a crossing. A *bidistortion* $\mathfrak{b}$ of $\mathcal{C}/\mathcal{D}$ is a distortion satisfying:

$$\sup \mathfrak{b}(\{\sigma_1, \sigma_n\}^\times) \preccurlyeq \sup \mathfrak{b}(\{\sigma_1, \sigma_2\}^\times) \cup \{\sigma_2, \sigma_3\}^\times \cup \cdots \cup \{\sigma_{n-1}, \sigma_n\}^\times) ,$$

$$\sup \mathfrak{b}(\{\bar{\sigma}\sigma, \mathrm{id}\}^\times) \preccurlyeq \mathfrak{b}(\sigma) , \qquad \sup \mathfrak{b}(\overline{S}^\times) \preccurlyeq \sup \mathfrak{b}(S^\times) , \qquad \sup \mathfrak{b}(\{\sigma\}^\times) \preccurlyeq \mathfrak{b}(\sigma) ,$$

$$\sup \mathfrak{b}(S^\times) \preccurlyeq \sup \bigcup_{\sigma,\tau \in S} \mathfrak{b}(\{\sigma, \tau\}^\times) , \qquad \sup \mathfrak{b}((TS)^\times) \preccurlyeq \sup \mathfrak{b}(T^\times S^\times) .$$

If $\sup \mathfrak{b}(S)$ is small, consider $\sup \mathfrak{b}(S^\times)$ as some kind of "diameter" of S. With this idea, we introduce the concepts of (*a-commutative*) *a-functor* and *a-natural transformation*, where "*a*-" stands for *asymptotically* or *asymptotic*. They are defined as maps that "become" (commutative) functors or natural transformations at certain limit. In the same way, we introduced the equivalence relation of being *asymptotic* between a-natural transformations, whose equivalence classes form a groupoid $\overline{\mathcal{C}}$, called the *completion* of $\mathcal{C}$, and there is a canonical functor $\iota : \mathcal{C} \to \overline{\mathcal{C}}$. It is said that $\mathfrak{b}$ is *complete* if $\bar{\iota} : \mathcal{C}\backslash\mathcal{C}_0 \to \overline{\mathcal{C}}\backslash\overline{\mathcal{C}}_0$ is bijective.

Endow $\mathcal{C}_0$ with $d_\mathfrak{b}$, and $\overline{\mathcal{C}}_0$ with another pseudo-metric $\bar{d}_\mathfrak{b}$, similarly defined, which induces a pseudo-metric on $\overline{\mathcal{C}}\backslash\overline{\mathcal{C}}_0$. It turns out that $\iota_0 : \mathcal{C}_0 \to \overline{\mathcal{C}}_0$ is an isometry with dense image and $\overline{\mathcal{C}}_0$ is complete. $\mathcal{C}\backslash\mathcal{C}_0$ and $\overline{\mathcal{C}}\backslash\overline{\mathcal{C}}_0$ are Hausdorff when some kind of compactness or pre-compactness is satisfied.

3 Examples and applications

Any metric can be considered as a bidistortion. Bidistortions can be used to produce interesting spaces, like the Gromov space of isometry classes of pointed proper metric spaces, its subspace defined by complete connected Riemannian manifolds, and many "universal" foliated spaces. We also hope that bidistortions can be used to generalize our results on the generic geometry of leaves [1].

References

[1] J.A. Álvarez López & A. Candel. *Generic Geometry of Leaves*. To appear.

A model structure for coloured operads in symmetric spectra

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For a fixed set of colours C we describe a coloured operad S^C in *Sets* such that the S^C-algebras are precisely the C-coloured operads in *Sets*. Using this coloured operad and applying a theorem of Elmendorff and Mandell, we construct a model structure for operads in the category of symmetric spectra (with the positive model structure) in which fibrations and weak equivalences are defined levelwise.

Hecke operators and groupoidification

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"Groupoidification" is a variant of categorification in which vector spaces and linear operators between them are refined into groupoids and "spans" of group homomorphisms between them. Examples from and applications to the theory of Hecke operators are given.

Fifth Seminar on Categories and Applications
Pontevedra, September 10-12, 2008

Generalized down-up algebras and their automorphisms

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A generalization of down-up algebras was introduced by Cassidy and Shelton in [4], the so-called generalized down-up algebras. We describe the automorphism group of conformal Noetherian generalized down-up algebras $L(f, r, s, \gamma)$ such that r is not a root of unity, listing explicitly the elements of the group. Then we apply these results to Noetherian down-up algebras, thus obtaining a characterization of the automorphism group of Noetherian down-up algebras $A(\alpha, \beta, \gamma)$ for which the roots of the polynomial $X^2 - \alpha X - \beta$ are not both roots of unity.

Introduction

Generalized down-up algebras were introduced by Cassidy and Shelton in [4] as a generalization of the down-up algebras $A(\alpha, \beta, \gamma)$ of Benkart and Roby [2]. Generalized down-up algebras include all down-up algebras, the algebras *similar to the enveloping algebra of* $\mathfrak{sl}_2$ defined by Smith [10], Le Bruyn's *conformal* $\mathfrak{sl}_2$ *enveloping algebras* [8] and Rueda's algebras *similar to the enveloping algebra of* $\mathfrak{sl}_2$ [9].

Two of the most remarkable examples of down-up algebras are $U(\mathfrak{sl}_2)$ and $U(\mathfrak{h})$, the enveloping algebras of the 3-dimensional complex simple Lie algebra $\mathfrak{sl}_2$ and of the 3-dimensional nilpotent, non-abelian Heisenberg Lie algebra $\mathfrak{h}$, respectively. These algebras have a very rich structure and representation theory which has been extensively studied, having an unquestionable impact on the theory of semisimple and nilpotent Lie algebras. Nevertheless, a precise description of their symmetries, as given by the understanding of their automorphism group, is yet to be obtained (see [5, 6] and [7, 1]). The problem of describing the automorphism group seems to be considerably simpler when a deformation is introduced.

Fifth Seminar on Categories and Applications
Pontevedra, September 10-12, 2008

It is reasonable to think of a Noetherian generalized down-up algebra as a deformation of an enveloping algebra of a 3-dimensional Lie algebra. Working over an algebraically closed field of characteristic 0, we use elementary methods to compute the automorphism groups of Noetherian generalized down-up algebras, under certain assumptions. Specializing, our results to down-up algebras, we obtain a complete description of the automorphism groups of all Noetherian down-up algebras $A(\alpha, \beta, \gamma)$, under the restriction that at least one of the roots of the polynomial $X^2 - \alpha X - \beta$ is not a root of unity.

1 The main results

$\mathbb{K}$ denotes an algebraically closed field of characteristic 0 and $\mathbb{K}^*$ is the multiplicative group of units of $\mathbb{K}$.

Definition 1 *Let $f \in \mathbb{K}[X]$ be a polynomial and fix scalars $r, s, \gamma \in \mathbb{K}$. The generalized down-up algebra $L = L(f, r, s, \gamma)$ is the unital associative $\mathbb{K}$-algebra generated by d, u and h, subject to the relations:*

$$dh - rhd + \gamma d = 0, \quad hu - ruh + \gamma u = 0, \quad du - sud + f(h) = 0.$$

L is said to be conformal if there exists a polynomial $g \in \mathbb{K}[X]$ such that $f(X) = sg(X) - g(rX - \gamma)$.

We denote the group of algebra automorphisms of L by Aut. Consider the following subgroup of Aut: $\mathcal{H} = \{\phi \in \text{Aut} \mid \phi(h) = \lambda h, \text{ for some } \lambda \in \mathbb{K}^*\}$.

Theorem 1 *Let $L = L(f, r, s, \gamma)$ be a generalized down-up algebra. Assume r, $s \in \mathbb{K}^*$, r is not a root of unity and $f(X) = sg(X) - g(rX - \gamma)$ for some $g \in \mathbb{K}[X]$. Then the group* Aut *of algebra automorphisms of L is isomorphic to:*

1. $(\mathbb{K}^*)^3 \rtimes \mathbb{Z}/3\mathbb{Z}$ *if $f = 0$ and $s = r^{-1}$, where the generator $1 + 3\mathbb{Z}$ of $\mathbb{Z}/3\mathbb{Z}$ acts on the torus $(\mathbb{K}^*)^3$ via the automorphism $(\lambda_1, \lambda_2, \lambda_3) \mapsto (\lambda_3, \lambda_1, \lambda_2)$;*

2. $\mathbb{K} \rtimes (\mathbb{K}^*)^3$ *if* $f = 0$ *and* $s^\tau = r$*, where* $(\lambda_1, \lambda_2, \lambda_3) \in (\mathbb{K}^*)^3$ *acts on the additive group* $\mathbb{K}$ *via the automorphism* $t \mapsto \lambda_1^{-1} (\lambda_2\lambda_3)^\tau t$*;*

3. $\mathbb{K} \rtimes (\mathbb{K}^*)^2$ *if* $\deg(f) = 0$ *and* $s^\tau = r$*, where* $(\lambda_1, \lambda_2) \in (\mathbb{K}^*)^2$ *acts on the additive group* $\mathbb{K}$ *via the automorphism* $t \mapsto \lambda_1^{-1} t$*;*

4. $(\mathbb{K}^*)^2 \rtimes \mathbb{Z}/2\mathbb{Z}$ *if* $\deg(f) = 1$*,* $s = r^{-1}$ *and* $f(\frac{\gamma}{r-1}) = 0$*, where the generator* $1 + 2\mathbb{Z}$ *of* $\mathbb{Z}/2\mathbb{Z}$ *acts on the torus* $(\mathbb{K}^*)^2$ *via the automorphism* $(\lambda_1, \lambda_2) \mapsto (\lambda_1, \lambda_2^{-1}\lambda_1)$*;*

5. $\mathbb{K}^* \rtimes \mathbb{Z}/2\mathbb{Z}$ *if* $\deg(f) = 1$*,* $s = r^{-1}$ *and* $f(\frac{\gamma}{r-1}) \neq 0$*, where the generator* $1 + 2\mathbb{Z}$ *of* $\mathbb{Z}/2\mathbb{Z}$ *acts on the torus* $\mathbb{K}^*$ *via the automorphism* $\lambda \mapsto \lambda^{-1}$*;*

6. $\mathcal{H}$ *otherwise, where* $\mathcal{H}$ *can be explicitly described.*

Other classes of algebras to which our study applies are Le Bruyn's *conformal* $\mathfrak{sl}_2$ *enveloping algebras* [8], occuring as $L(bx^2 + x, r, s, \gamma)$, for $b \in \mathbb{K}$ and $rs \neq 0$, and some of Witten's seven parameter deformations of the enveloping algebra of $\mathfrak{sl}_2$ [11] (see also [3, Thm. 2.6] and [4, Ex. 1.4]).

For the convenience of those readers who are mostly interested in down-up algebras, we replace our usual generators x and y of L with the canonical generators d and u of A, respectively.

Theorem 2 *Let* $A = A(\alpha, \beta, \gamma)$ *be a down-up algebra, with* $\alpha = r + s$ *and* $\beta = -rs$*. Assume that* $\beta \neq 0$ *and that one of* r *or* s *is not a root of unity. The group* Aut(A) *of algebra automorphisms of* A *is described bellow.*

1. *If* $\gamma = 0$ *and* $\beta = -1$ *then* aut(A) $\simeq (\mathbb{K}^*)^2 \rtimes \mathbb{Z}/2\mathbb{Z}$*;*

2. *If* $\gamma = 0$ *and* $\beta \neq -1$ *then* Aut(A) $\simeq (\mathbb{K}^*)^2$*;*

3. *If* $\gamma \neq 0$ *and* $\beta = -1$ *then* aut(A) $\simeq \mathbb{K}^* \rtimes \mathbb{Z}/2\mathbb{Z}$*;*

4. *If* $\gamma \neq 0$ *and* $\beta \neq -1$ *then* Aut(A) $\simeq \mathbb{K}^*$.

In all cases, the 2-torus $(\mathbb{K}^*)^2$ acts diagonally on the generators d and u, $\mu \in \mathbb{K}^*$ acts as multiplication by μ on d and as multiplication by μ^{-1} on u, and the generator of the finite group $\mathbb{Z}/2\mathbb{Z}$ interchanges d and u.

References

[1] J. Alev, *Un automorphisme non modéré de* $U(\mathfrak{g}_3)$, Comm. Algebra **14** (1986), no. 8, 1365–1378.

[2] G. Benkart and T. Roby, *Down-up algebras*, J. Algebra **209** (1998), no. 1, 305–344.

[3] G. Benkart, *Down-up algebras and Witten's deformations of the universal enveloping algebra of* $\mathfrak{s}l_2$, Contemp. Math. **224** (1999), 29–45.

[4] T. Cassidy and B. Shelton, *Basic properties of generalized down-up algebras*, J. Algebra **279** (2004), no. 1, 402–421.

[5] J. Dixmier, *Sur les algèbres de Weyl*, Bull. Soc. Math. France **96** (1968), 209–242.

[6] J. Dixmier, *Quotients simples de l'algèbre enveloppante de* $\mathfrak{s}l_2$, J. Algebra **24** (1973), 551–564.

[7] A. Joseph, *A wild automorphism of* Usl(2), Math. Proc. Cambridge Philos. Soc. **80** (1976), no. 1, 61–65.

[8] L. Le Bruyn, *Conformal* sl_2 *enveloping algebras*, Commun. Algebra **23** (1995), no. 4, 1325–1362.

[9] S. Rueda, *Some algebras similar to the enveloping algebra of* $sl(2)$, Commun. Algebra **30** (2002), no. 3, 1127–1152.

[10] S.P. Smith, *A class of algebras similar to the enveloping algebra of sl(2)*, Trans. Am. Math. Soc. **322** (1990), no. 1, 285–314.

[11] E. Witten, *Gauge theories, vertex models, and quantum groups*, Nuclear Phys. B **330** (1990), 285–346.

Fifth Seminar on Categories and Applications
Pontevedra, September 10-12, 2008

K-theory for categorical groups

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Quillen defined the n-th algebraic K-group of a ring R as $\pi_n(BGL(R)^+)$. Using the notion of homotopy categorical groups of any pointed space, that are defined via the fundamental groupoid of iterated loop spaces, in this talk, we introduce the concept of K-categorical groups $\mathbb{K}_i R$ of any ring R. We also show the existence of a fundamental categorical crossed module associated to any fibre homotopy sequence. This fact, allows us to characterize $\mathbb{K}_1 R$ and $\mathbb{K}_2 R$, respectively, as the homotopy cokernel and kernel of the fundamental categorical crossed module associated to the fibre homotopy sequence $F(R) \xrightarrow{d_R} BGL(R) \xrightarrow{q_R} BGL(R)^+$.

References

[1] A. J. Berrick, An approach to algebraic K-theory, Resarch Notes in Mathematics 56, Pitma Advanced Publ. Comp. 1982.

[2] P. Carrasco, A.M. Cegarra and A.R. Garzón, The homotopy categorical crossed module of a CW-complex, Topology and its Applications 154 (2007) 834-847.

[3] P. Carrasco, A.M. Cegarra and A.R. Garzón, The clasifying space of a categorical crossed module, Math. Nachr. (to appear)

[4] P. Carrasco, A.R. Garzón and E.M. Vitale, On categorical crossed modules, Theory and Applications of Categories 16, No. 22 (2006) 585-618.

[5] M. Grandis, E.M. Vitale, A higher dimensional homotopy sequence. Homology Homotopy Appl. 4 (2002), no. 1, 59-69

Fifth Seminar on Categories and Applications
Pontevedra, September 10-12, 2008

Axiomatic stable homotopy: the derived category of quasi-coherent sheaves

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Hovey, Palmieri and Strickland have defined the concept of stable homotopy category in [HPS]. It consists of a list of additional properties and structure for a triangulated category. This concept arises in several contexts of algebraic geometry and topology, being two essential examples, $\mathsf{D}(R)$, the derived category of complexes of modules over a commutative ring R, and ${}^{\circ}sfHoSp$, the category of (non-connective) spectra up to homotopy.

In this talk we will show that for a quasi-compact and semi-separated (non necessarily noetherian) scheme X, the derived category of quasi-coherent sheaves over X, $\mathsf{D}(\mathsf{A}_{\mathrm{qct}}(X))$, is a stable homotopy category. We will also deal with the analogous result for formal schemes, namely, if $\mathfrak{X}$ is a noetherian semi-separated formal scheme the derived category of sheaves with quasi-coherent torsion homologies, $\mathsf{D}_{\mathrm{qct}}(\mathfrak{X})$ (*cfr* [AJL]), is a stable homotopy category. These results are included in [AJPV].

References

[AJL] Alonso Tarrío, L.; Jeremías López, A.; Lipman, J.: Duality and flat base change on formal schemes, in *Studies in duality on noetherian formal schemes and non-noetherian ordinary schemes.* Providence, RI: American Mathematical Society. Contemp. Math. **244**, 3–90 (1999).

[AJPV] Alonso Tarrío, L.; Jeremías López, A.; Pérez Rodríguez, M.; Vale Gonsalves, M. J.: The derived category of quasi-coherent sheaves and axiomatic stable homotopy. *Advances in Mathematics* **218**, 1224–1252 (2008).

[HPS] Hovey, M.; Palmieri, J. H.; Strickland, N. P.: Axiomatic stable homotopy theory. *Mem. Amer. Math. Soc.* **128** (1997), no. 610.

Fifth Seminar on Categories and Applications
Pontevedra, September 10-12, 2008

On the commutativity of stabilization and localization

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We show that, for a Morava K-theory spectrum $K(n)$, the localization on the category of spectra with respect to $K(n)$ is not equivalent to the stabilization of the $K(n)$-localization on the category of simplicial sets.

Introduction

The stabilization $Sp\,(\mathcal{M}, T)$ of a model category $\mathcal{M}$ with respect to an endofunctor T was constructed by Hovey in [4].

We fix a spectrum E and, following [2], we consider the Bousfield localization $L_E(Sp)$ on the model category of spectra with respect to the homology theory defined by E. On the other hand, following [1], we can also consider the Bousfield localization $L_E(sSet)$ on simplicial sets with respect to the homology theory defined by E, and, then, construct the stabilization $Sp\,(L_E(sSet), \Sigma)$ with respect to the suspension functor. The problem that we study is whether or not their homotopy categories agree, i.e.

$$Ho(Sp\,(L_E(sSets), \Sigma)) \cong Ho(L_E(Sp))?$$

We use a characterization by Dugger [3] to express the stabilization of a Bousfield localization on a model category as another Bousfield localization on the stabilization of the original model category.

Then we show that the answer to our problem is negative when E is taken to be a Morava K-theory.

References

[1] A. K. Bousfield, *The localization of spaces with respect to homology*, Topology **14** (1975), 133–150.

[2] A. K. Bousfield, *The localization of spectra with respect to homology*, Topology **18** (1979), 257–281.

[3] D. Dugger, *Spectral enrichments of model categories*, Homology Homotopy Appl. **8**, no. 1 (2006), 1–30.

[4] M. Hovey, *Spectra and symmetric spectra in general model categaries*, J. Pure Appl. Algebra **165** (2001), 63–127.

A Quillen model structure for orbifolds

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We discuss the existence of Quillen model structures on the category of (certain kind of) topological groupoids. Our weak equivalences will be the essential equivalences and the homotopy essential equivalences. Fibrant objects will be described in terms of orbifolds as groupoids [2] . We present a bicategorical approach which relates the bicategory of fractions of Pronk [3] and the notion of a model Cat-category for 2-categories of Lack [1].

This is join work with Cristina Costoya.

References

[1] S. Lack, Homotopy-theoretic aspects of 2-monads, *J. Homotopy Relat. Struct.* 2 (2007), 229–260.

[2] I. Moerdijk, Orbifolds as groupoids, in Orbifolds in mathematics and physics, 205–222, Contemp. Math. 310, Amer. Math. Soc., Providence, RI, 2002.

[3] D. A. Pronk, Etendues and stacks as bicategories of fractions, *Compositio Mathematica* 102 no. 3 (1996), 243–303.

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